UNIVERSITY of RWANDA

COLLEGE OF SCIENCE AND TECHNOLOGY

School of Engineering

Civil, Environmental & Geomatics Engineering (CEGE)

Course Lecturer: Ir. Philbert Habimana (MSc. & BSc.)

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Course name: MECHANICS OF MATERIALS

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Lecture xvii – *Combined direct and bending stresses*

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xvii. COMBINED DIRECT AND BENDING STRESSES (1 of 26)

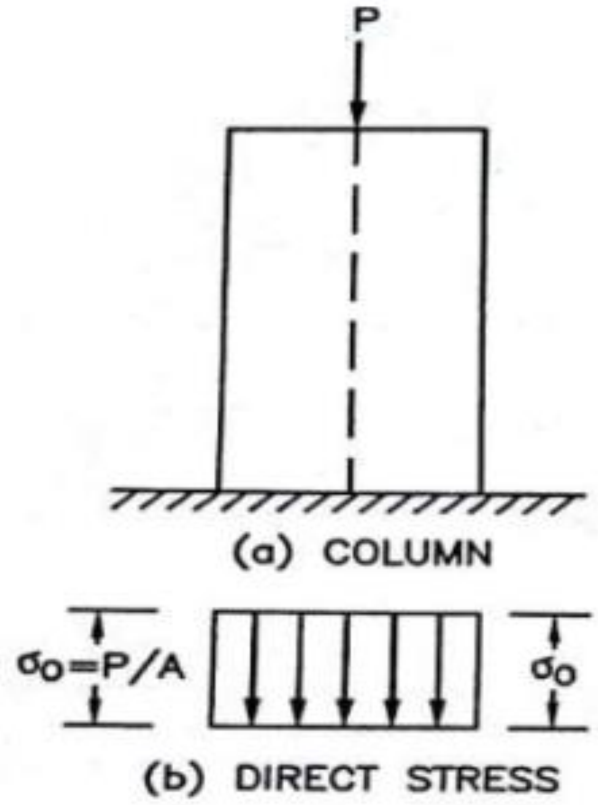
xvii. 1. INTRODUCTION (1 of 2)

Axial load: When load is acting along the longitudinal axis of column. It produces compressive stress in column.

Eccentric load: A load whose line of action does not coincide with the axis of the column. It produces direct stress and bending stress.

Eccentricity (e): The horizontal distance between the longitudinal axis of column and line of action of load.

In axially loaded column e =0.

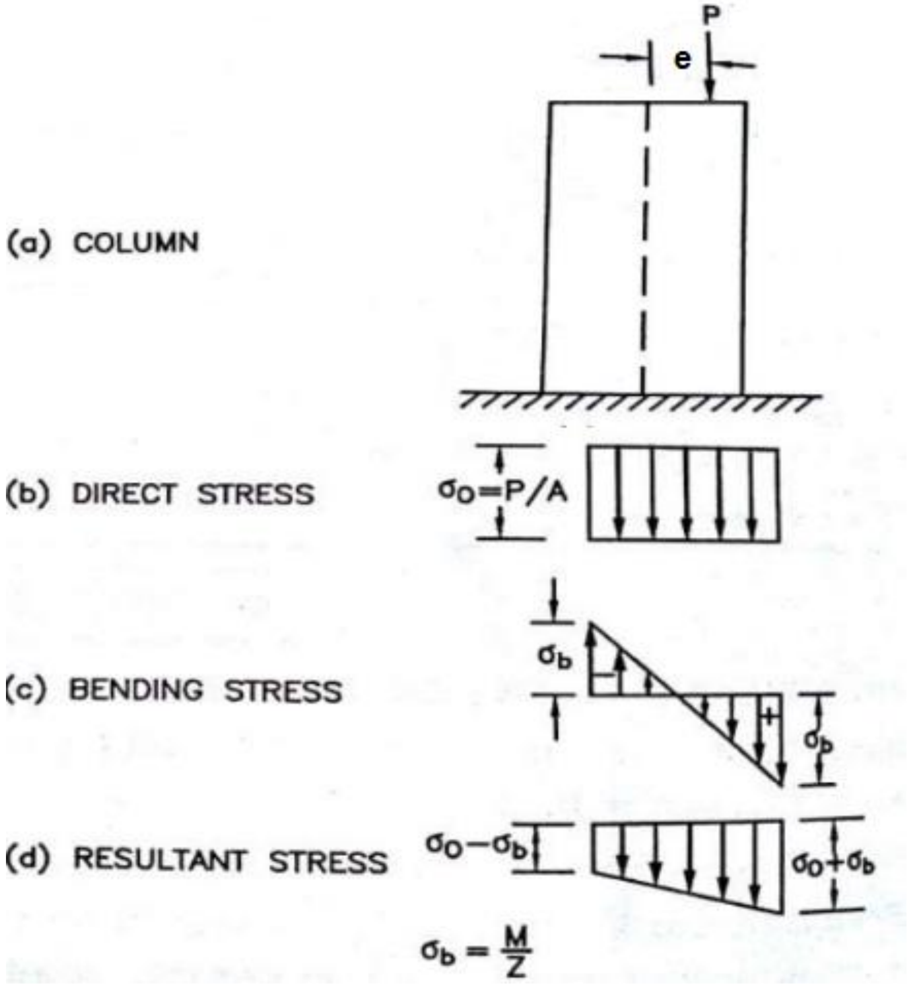


xvii. 1. INTRODUCTION (2 of 2)

- Consider the case of a column subjected by a compressive load P whose line of action is at a distance of *e* from the axis of the column.
- Here *e* is known as *eccentricity of the load*.
- The eccentric load will cause direct stress and bending stress.

Direct stress $= \sigma_o = \frac{P}{A}$

Bending stress $= \sigma_b = \frac{M}{S} = \frac{M}{I} \cdot y \quad \therefore \quad S = \frac{I}{y}$



xvii. 2. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO AN ECCENTRIC LOAD (1 of 6)

- Let the load is eccentric with respect to the axis y – y
- Let P – Eccentric load on column

  e – Eccentricity of the load

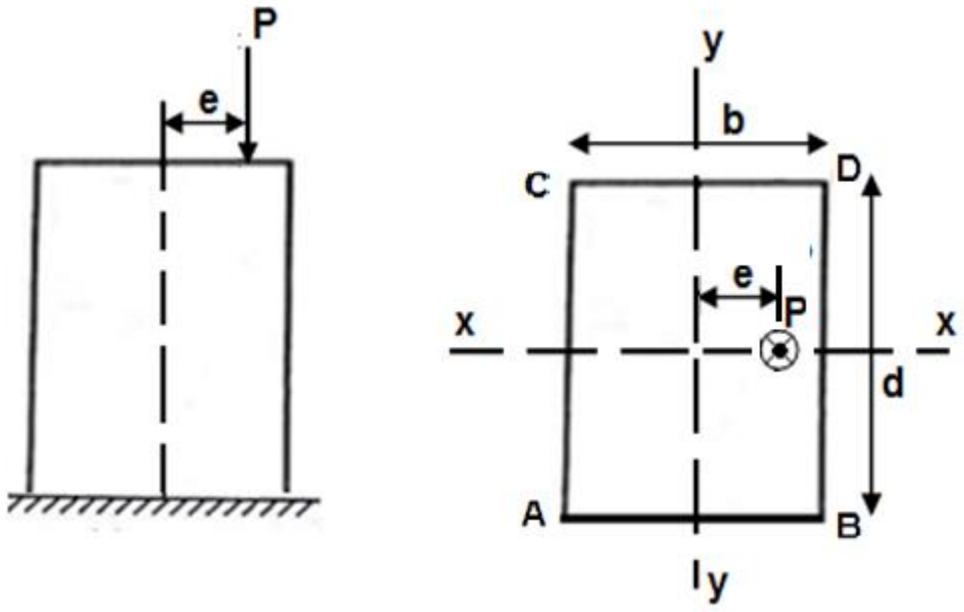
  σ_o – Direct stress

  σ_b – Bending stress

  b – Width of column

  d – Depth of column

  A – Area of column



xvii. 2. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO AN ECCENTRIC LOAD (2 of 6)

Now moment due to eccentric load P = P x e

The direct stress $\sigma_0 = \frac{P}{A}$ (1) and it is uniform along the cross – section.

The bending stress σ_b due to moment at any point of the column section at a distance y from the neutral axis y – y is given by

$$\frac{M}{I} = \frac{\sigma_b}{\pm y} \quad \text{then} \quad \sigma_b = \pm \frac{M}{I} \times y \qquad (2)$$

Where I - moment of inertia of the column section about the neutral axis y – y

$$= \frac{db^3}{12}$$

xvii. 2. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO AN ECCENTRIC LOAD (3 of 6)

Substituting the value of I in equation (2), we get $\sigma_b = \pm \frac{M}{\frac{db^3}{12}} \times y = \pm \frac{12M}{db^3} \times y$

The bending stress depends upon the value of y from the axis y – y

The bending stress at the extreme is obtained by substituting y = b/2

$$\sigma_b = \pm \frac{12M}{db^3} \times \frac{b}{2} = \pm \frac{6M}{db^2} = \pm \frac{6P \times e}{db^2} = \pm \frac{6P \times e}{d \times b \times b} = \pm \frac{6Pe}{Ab}$$

- The resultant stress at any point will be the algebraic sum of direct stress and bending stress.

xvii. COMBINED DIRECT AND BENDING STRESSES (6 of 26)

xvii. 2. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO AN ECCENTRIC LOAD (4 of 6)

- If y is taken positive on the same side of y –y as the load, then bending stress will be of the same type as the direct stress. Here direct stress is compressive and hence bending stress will also be compressive towards the right of the axis y – y.
- Similarly bending stress will be tensile towards the left of the axis y – y.
- Taking compressive stress as positive and tensile stress as negative we can find the maximum and minimum stress at the extremities of the section. The stress will be maximum along layer BC and minimum along layer AD.

Let σ_{max} = Maximum stress (along BC)

σ_{min} = Minimum stress (along AD)

Then σ_{max} = Direct stress + Bending stress

xvii. 2. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO AN ECCENTRIC LOAD (5 of 6)

$$\sigma_{max} = \sigma_0 + \sigma_b$$

$$\sigma_{max} = \frac{P}{A} + \frac{6Pe}{Ab}$$

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6e}{b}\right) \qquad (3)$$

And σ_{min} = Direct stress - Bending stress

$$\sigma_{min} = \sigma_0 - \sigma_b$$

$$\sigma_{min} = \frac{P}{A} - \frac{6Pe}{Ab}$$

$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6e}{b}\right) \qquad (4)$$

If in equation (4) σ_{min} is negative, then the stress along the layer AD will be tensile. If σ_{min} is zero then there will be no tensile stress along the width of the column.

If σ_{min} is positive then there will be only compressive stress along width of the column.

xvii. 2. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO AN ECCENTRIC LOAD (6 of 6)

- When a column is subjected to eccentric load , the edge of column towards the eccentricity will be subjected to maximum stress (σ_{max}) and the opposite edge will be subjected to minimum stress (σ_{min}).

Maximum Stress (σ_{max})

σ_{max} = direct stress + bending stress

$$= \sigma_o + \sigma_b$$

$$= \frac{P}{A} + \frac{M}{S}$$

$$= \frac{P}{A}\left(1 + \frac{6e}{b}\right)$$

Minimum Stress (σ_{min})

σ_{min} = direct stress - bending stress

$$= \sigma_o - \sigma_b$$

$$= \frac{P}{A} - \frac{M}{S}$$

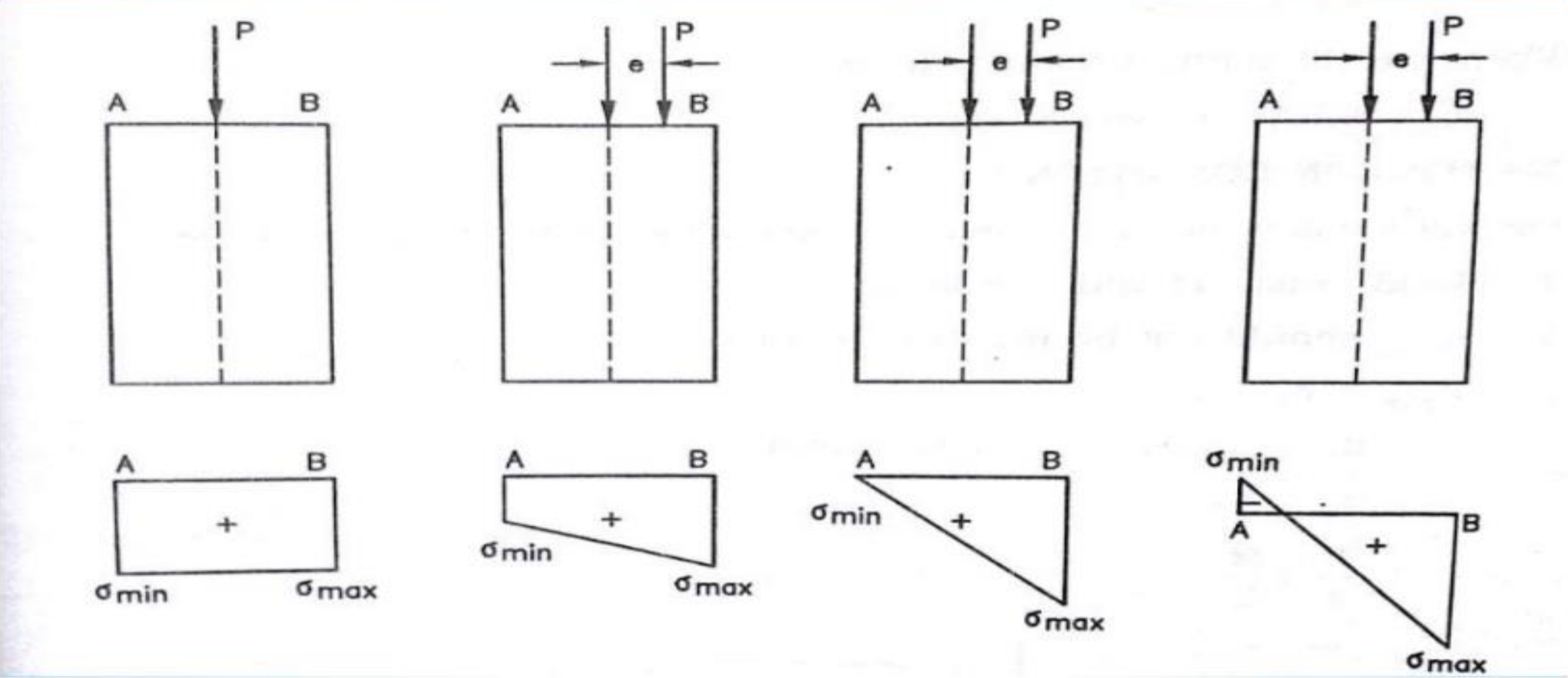
$$= \frac{P}{A}\left(1 - \frac{6e}{b}\right)$$

σ_o = direct stress, σ_b = bending stress, M = Moment = P . e, e = eccentricity

S = section modulus = $\frac{I}{Y}$, I = moment of Inertia, y = distance of extreme fibre from c.g. of column.

xvii. 3. STRESS DISTRIBUTION DUE TO THE POSITION OF THE APPLIED LOAD

Stress distribution in column as the load (P) moves from centre of column to the edge of column as shown in fig.



xvii. 4. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO A LOAD WHICH IS ECCENTRIC TO BOTH AXES (1 of 3)

Let **P – Eccentric load on column**

e_x – Eccentricity of load about X – X axis

e_y – Eccentricity of load about Y – Y axis

b – Width

d – Depth

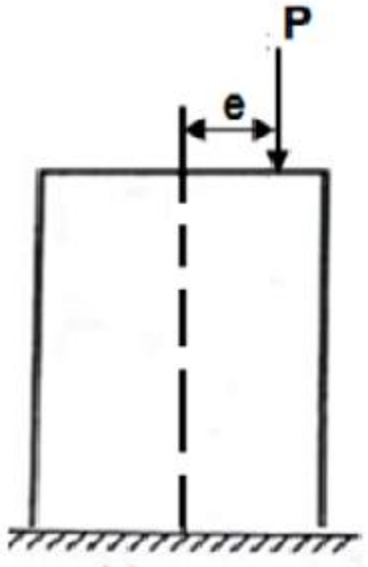
σ_0 – Direct stress

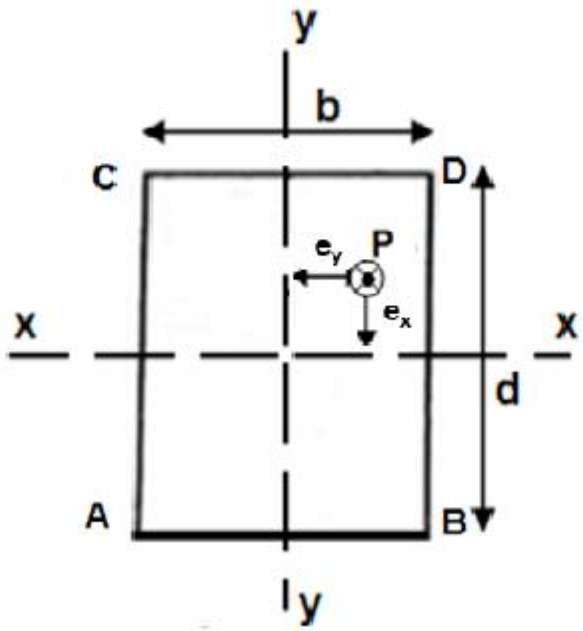
σ_{bx} – Bending stress due to eccentricity e_x

σ_{by} - Bending stress due to eccentricity e_y

M_x - Moment of load about X-X axis = P x e_x

M_y - Moment of load about Y-Y axis = P x e_y





xvii. 4. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO A LOAD WHICH IS ECCENTRIC TO BOTH AXES (2 of 3)

a. The direct stress $\sigma_0 = \frac{P}{A}$

b. The bending stress due to eccentricity e_y

$$\sigma_{by} = \frac{M_y \times x}{I_{yy}} = \frac{P \times e_y \times x}{I_{yy}}$$

Here x varies from $-\frac{b}{2}$ to $+\frac{b}{2}$

c. The bending stress due to eccentricity e_x

$$\sigma_{bx} = \frac{M_x \times y}{I_{xx}} = \frac{P \times e_x \times y}{I_{xx}}$$

Here y varies from $-\frac{d}{2}$ to $+\frac{d}{2}$

The resultant stress at any point of section

$$\sigma_R = \sigma_0 \pm \sigma_{by} \pm \sigma_{bx}$$

xvii. COMBINED DIRECT AND BENDING STRESSES (12 of 26)

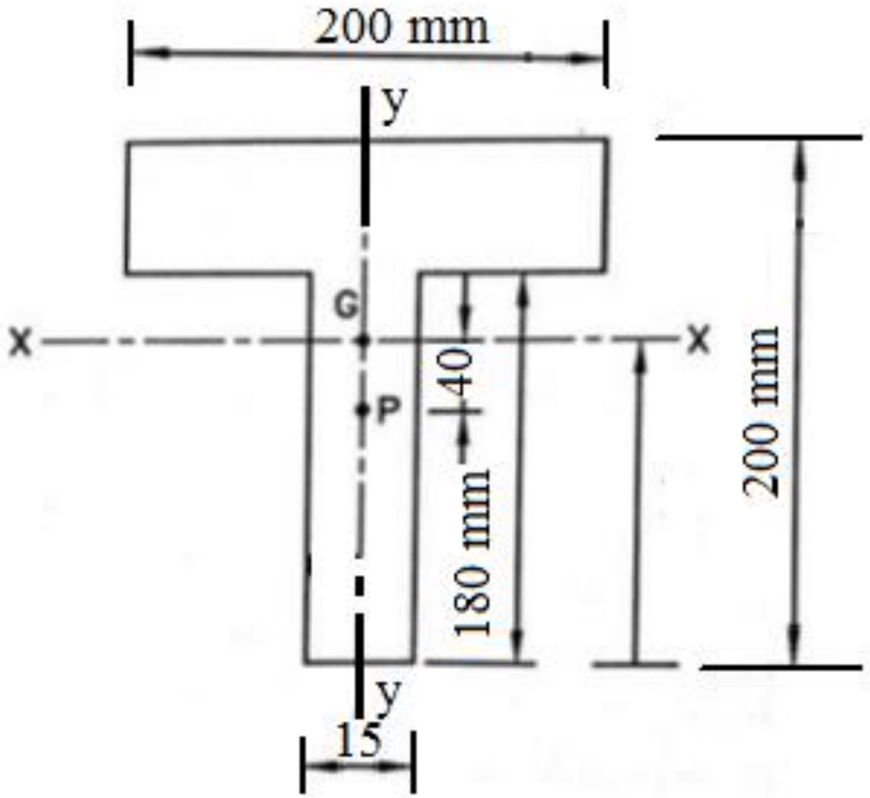
xvii. 4. RESULTANT STRESS WHEN A COLUMN OF RECTANGULAR SECTION IS SUBJECTED TO A LOAD WHICH IS ECCENTRIC TO BOTH AXES (3 of 3)

$$\sigma_R = \frac{P}{A} \pm \frac{M_y \times x}{I_{yy}} \pm \frac{M_x \times y}{I_{xx}}$$

- At the point D, the coordinates x and y are positive, σ_R is maximum
- At the point A, $x < 0$, $y < 0$, σ_R is minimum
- At the point B, $x > 0$, $y < 0$, $\sigma_R = \frac{P}{A} + \frac{M_y \times x}{I_{yy}} - \frac{M_x \times y}{I_{xx}}$
- At the point C, $x < 0$, $y > 0$, $\sigma_R = \frac{P}{A} - \frac{M_y \times x}{I_{yy}} + \frac{M_x \times y}{I_{xx}}$

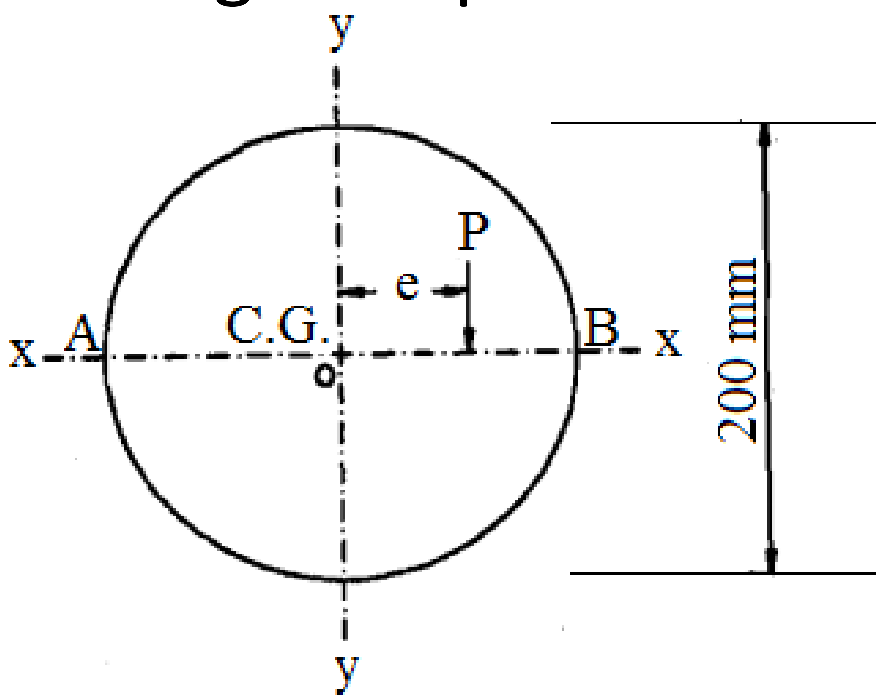
GIVEN EXAMPLES

1. A column of T-section shown in figure is subjected to a load (100 kN) at a point on the centroidal axis, 40mm below the centroidal x-x axis. Calculate the maximum stresses induced in the section.



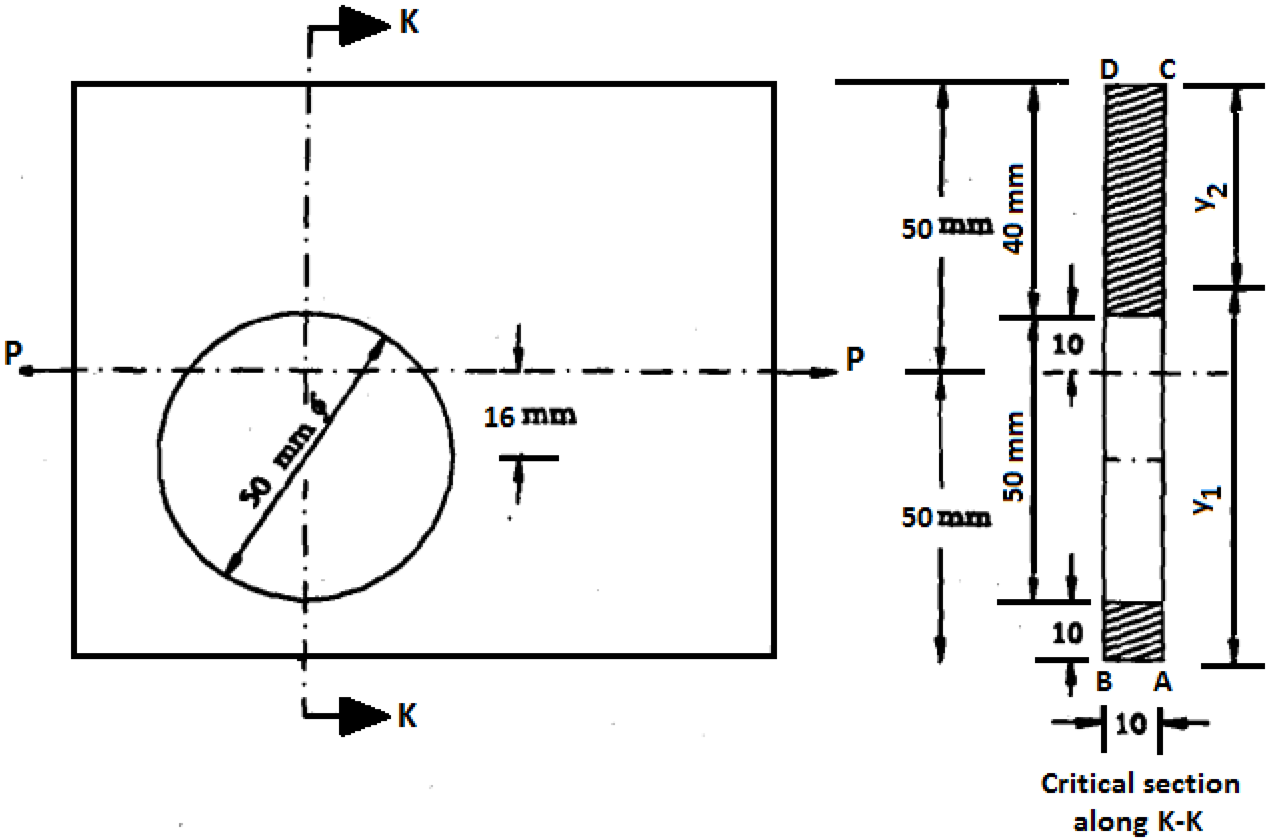
GIVEN EXAMPLES

2. A cast iron column of 200 mm diameter carries a vertical load 400 kN, at a distance of 40 mm from the centre. Determine the maximum and minimum stress developed in the section, along the diameter passing through the point of loading.



GIVEN EXAMPLES

3. A rectangular plate 10 mm thick with a hole of 50 mm diameter drilled on it as shown in figure aside. It is subjected to an axial pull of 45 kN. Determine the greatest and the least intensities of stress at the critical cross section of the plate.



GIVEN EXAMPLES

1. Sln

Solution: Data Given,

P = 100 kN

e = 40 mm

Part-1

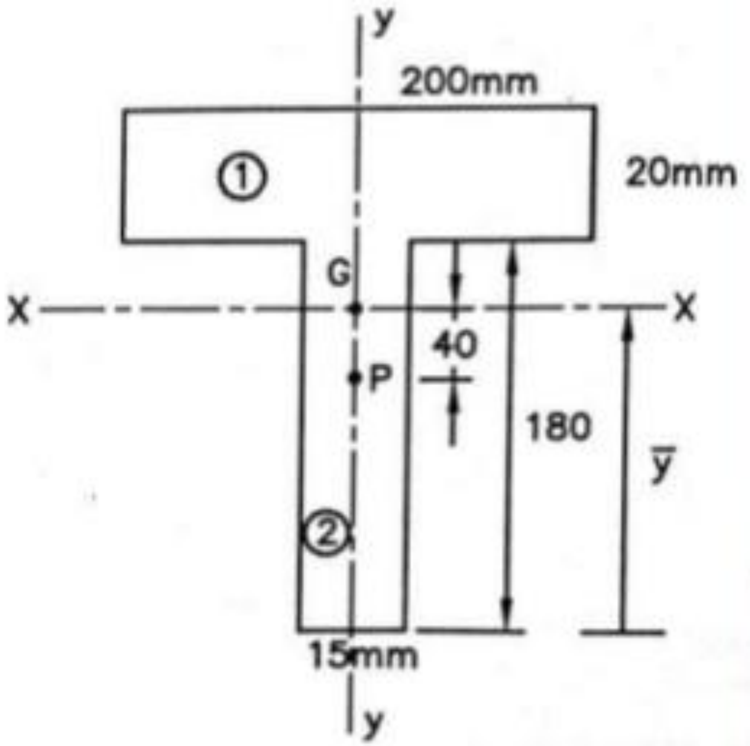
$a_1 = 200 \times 20 = 4000 mm^2$

$y_1 = 180 + 10 = 190 mm$

Part-2

$a_2 = 180 \times 15 = 2700 mm^2$

$y_2 = 90 = mm$



$$y = \frac{a1y1 + a2y2}{a1 + a2} = \frac{(4000 \times 190) + (2700 \times 90)}{(4000 + 2700)} = 149.70 \text{ mm}$$

GIVEN EXAMPLES

1. Sln

$$I_{xx1} = I_g + ah^2$$

$$= \frac{200 \times 20^3}{12} + 4000 \times (190 - 149.70)^2$$

$$= 6.629 \times 10^6 \ mm^4$$

$$I_{xx2} = I_g + ah^2$$

$$= \frac{15 \times 180^3}{12} + 2700 \times (149.70 - 90)^2$$

$$= 16.91 \times 10^6 \ mm^4$$

$$I_{xx} = 6.692 \times 10^6 + 16.91 \times 10^6$$

$$= 23.539 \times 10^6 mm^4$$

$$S_{xx} = \frac{I_{xx}}{y_{max}} = \frac{23.539 \times 10^6}{149.70} = 157241.15 mm^3$$

GIVEN EXAMPLES

1. Sln

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S} = \frac{100 \times 10^3}{6700} + \frac{100 \times 10^3 \times 40}{157241.15}$$

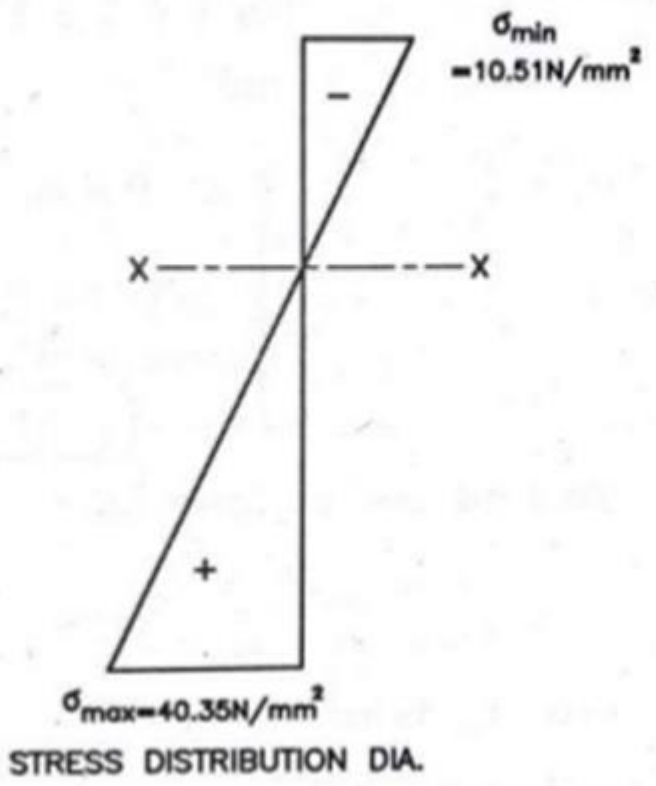
$$= 14.92 + 25.43$$

$$= 40.35\ N/mm^2\ \text{(compressive)}$$

$$\sigma_{min} = \frac{P}{A} - \frac{M}{S} = \frac{100 \times 10^3}{6700} - \frac{100 \times 10^3 \times 40}{157241.15}$$

$$= 14.92 - 25.43$$

$$= 10.51\ N/mm^2\ \text{(Tensile)}$$



GIVEN EXAMPLES

2. Given data: $\mathrm{P} = 400\,kN = 400 \text{ x } 10^3 N$

$\mathrm{D} = 200mm$

$e = 40mm$

Area of the section, $A = \frac{\pi D^2}{4} = \frac{\pi}{4} \text{x } (200)^2 = 31416\, mm^2$

Direct stress, $\sigma_0 = \frac{P}{A} = \frac{4 \text{ x } 10^5}{31416} = 12.732 N/mm^2$

Bending moment, $M = P \text{ x } e = (400 \text{ x } 10^3) \text{ x } 40 = 16 \text{ x } 10^6\; Nmm$

Section modulus, $S = \frac{\pi D^3}{32} = \frac{\pi}{32} \text{x } (200)^3 = 785.4 \text{ x } 10^3\; mm^3$

GIVEN EXAMPLES

2. Bending stress, $\sigma_b = \pm\frac{P \text{ x } e}{S} = \pm\frac{M}{S} = \frac{16 \text{ x } 10^6}{785.4 \text{ x } 10^3} = \pm 20.372 N/mm^2$

Resultant stress at the edge, $\mathrm{B} = \sigma_0 + \sigma_b = 12.732 + 20.372$

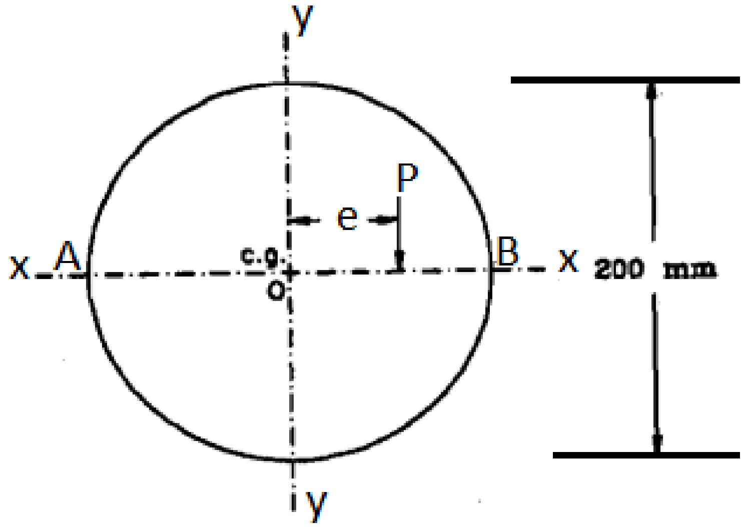
$= 33.104\ N/mm^2$ (Compressive)

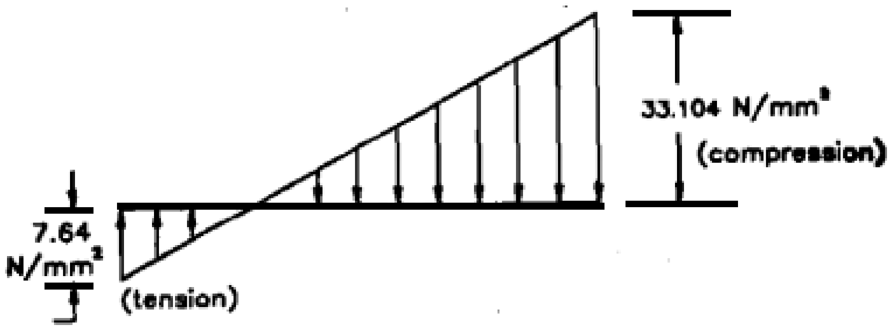
Resultant stress at the edge, $\mathrm{A} = \sigma_0 - \sigma_b = 12.732 - 20.372$

$= -7.640\ N/mm^2$ (Tensile)

GIVEN EXAMPLES

2. Stress diagram





Stress distribution along the diagonal i.e., the diameter

GIVEN EXAMPLES

3. Given data: $\mathrm{P} = 45\,kN = 45 \text{ x } 10^3 N$ (Tensile), There is an axial pulling force

$\mathrm{D} = 50\,mm$

$\mathrm{t} = 10\,mm$ (Plate's thickness)

Area of section at the weakest point, $\mathrm{A} = (10 \text{ x } 10) + (40 \text{ x } 10)$

$= 500mm^2$

GIVEN EXAMPLES

3. The location of the centroidal axes; taking moments about AB

$$y_1 = \frac{(a_1 \text{ x } s_1)+(a_2 \text{ x } s_2)}{a_1 + a_2} \text{ (Recall)}$$

With $s_1 = \frac{10}{2} = 5\,mm$ and $s_2 = 10 + 50 + \frac{40}{2} = 80\,mm$

$$y_1 = \frac{\left((10 \text{ x } 10) \text{ x } 5\right) + \left((40 \text{ x } 10) \text{ x } 80\right)}{(10 \text{ x } 10) + (40 \text{ x}10)} = 65\,mm \text{ (From the bottom edge)}$$

$$y_2 = H - y_1 = 100 - 65 = 35mm$$

GIVEN EXAMPLES

3. Moment of inertia about xx axis;

$$I_{xx} = \frac{b_1 \text{ x } h_1^3}{12} + a_1 \text{ x } (d_1)^2 + \frac{b_2 \text{ x } h_2^3}{12} + a_2 \text{ x } (d_2)^2$$

$$= \frac{10 \text{ x } 10^3}{12} + [(10 \text{ x}10) \text{ x } (65 - 5)^2] + \frac{10 \text{ x } 40^3}{12} + [(40 \text{ x}10) \text{ x } (35 - 20)^2]$$

$$= 50.42 \text{ x } 10^4 mm^4$$

Direct stress at the edge, $\sigma_0 = \frac{P}{A} = \frac{45 \text{ x } 10^3}{500} = 90N/mm^2$ (Tensile)

Eccentricity, $\text{e} = 15mm$

GIVEN EXAMPLES

3. Bending stress along edge AB, $\sigma_b = \frac{P \text{ x } e}{I_{xx}} \text{ x } y_1$

$$= \frac{\left((45 \text{ x } 10^3) \text{ x } 15\right)}{50.42 \text{ x } 10^4} \text{ x } (-65)$$

$$= -87N/mm^2 \text{ (Tensile)}$$

Bending stress along edge CD, $\sigma_b = \frac{P \text{ x } e}{I_{xx}} \text{ x } y_2$

$$= \frac{\left((45 \text{ x } 10^3) \text{ x } 15\right)}{50.42 \text{ x } 10^4} \text{ x } (35)$$

$$= 46.8N/mm^2 \text{ (Compressive)}$$

GIVEN EXAMPLES

3. Maximum stress along edge AB $= \sigma_0 + \sigma_b = -90 - 87$

$= -177 N/mm^2$ (Tensile)

Maximum stress along edge CD $= \sigma_0 + \sigma_b = -90 + 46.8$

$= -43.2 N/mm^2$ (Tensile)