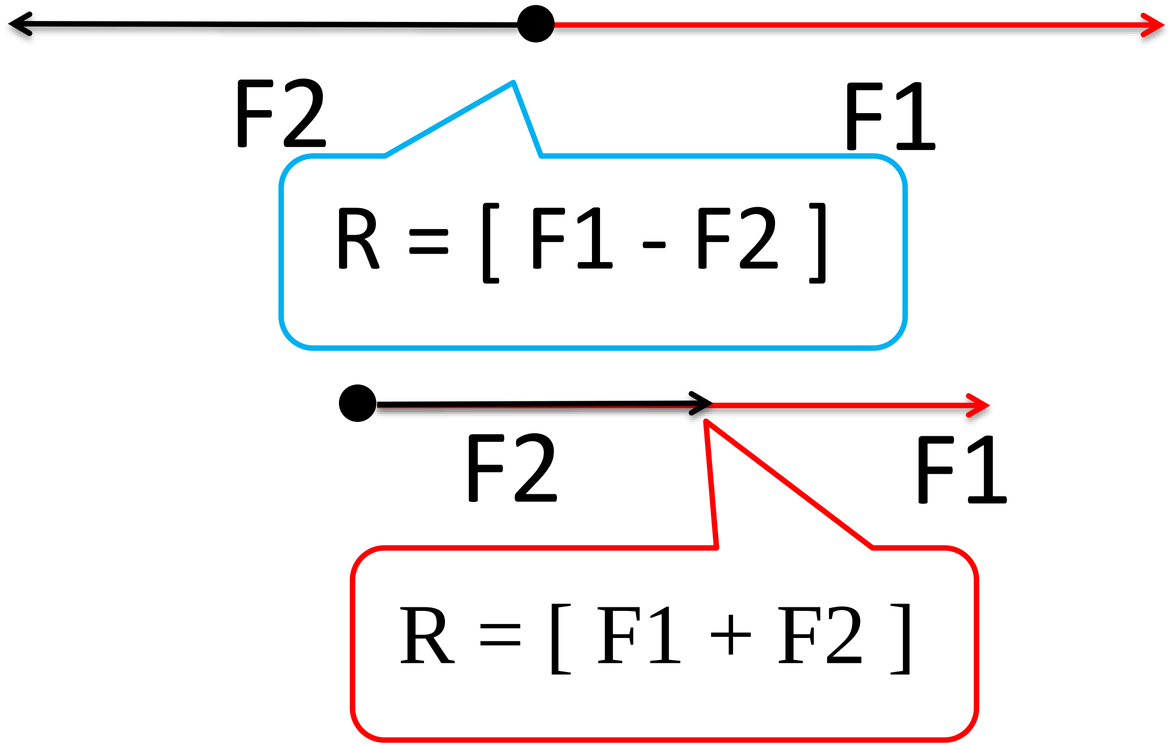
Resultants of force systems 1D, 2D and 3D

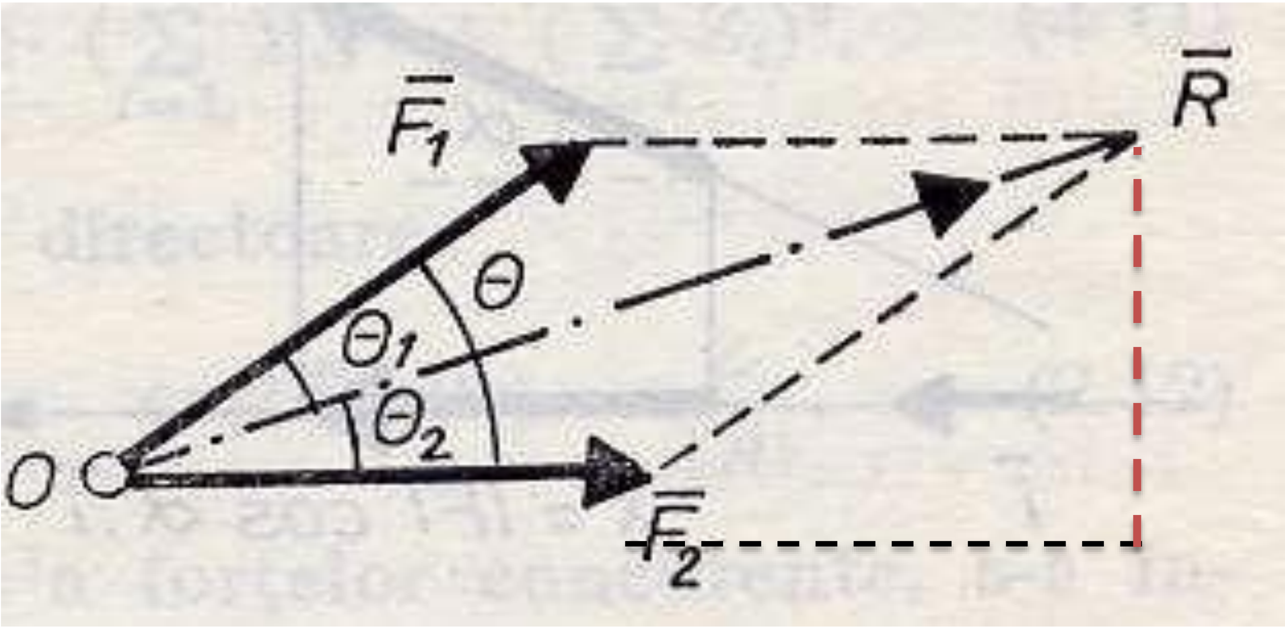
The resultant of a system of forces on a particle is the single force which has the same effect as the system of forces.

1 DIMENSION/RESULTANT



Please Review Trigonometry

2 DIMENSIONS/RESULTANT

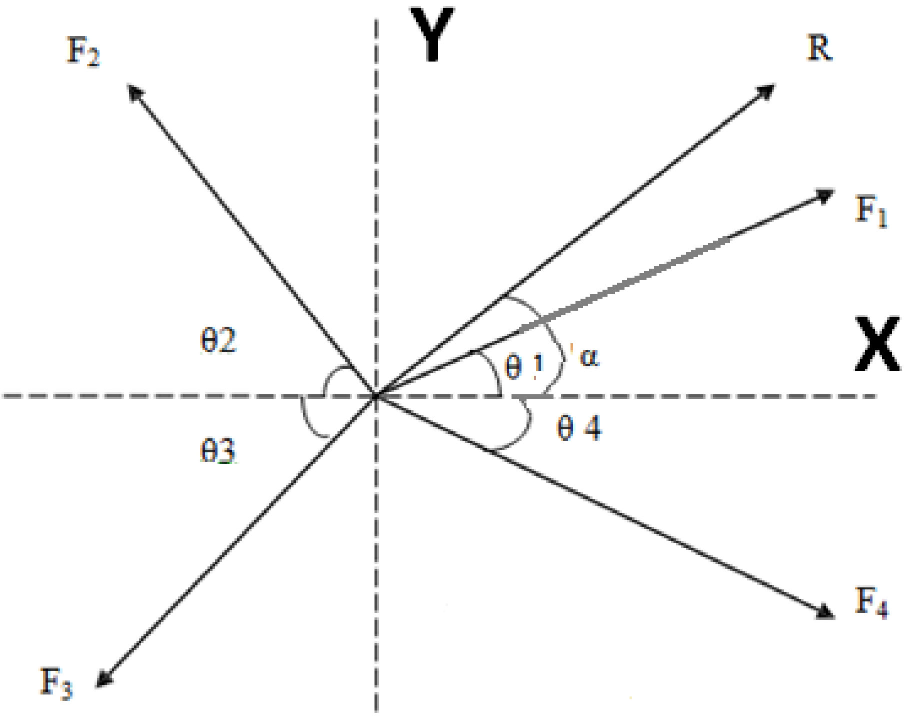


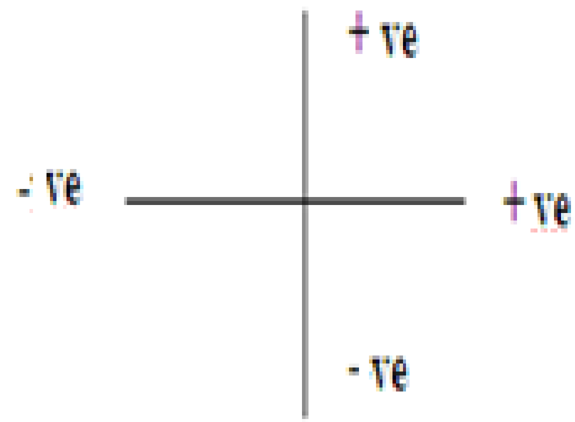
$$\bar{R} = \bar{F}_1 + \bar{F}_2$$

$$|\bar{R}| = \sqrt{|\bar{F}_1|^2 + |\bar{F}_2|^2 + 2|\bar{F}_1| \cdot |\bar{F}_2| \cos\theta}$$

$$\operatorname{tg}\theta_2 = \frac{|\bar{F}_1| \sin\theta}{|\bar{F}_2| + |\bar{F}_1| \cos\theta}.$$

Composition of forces by method of resolution





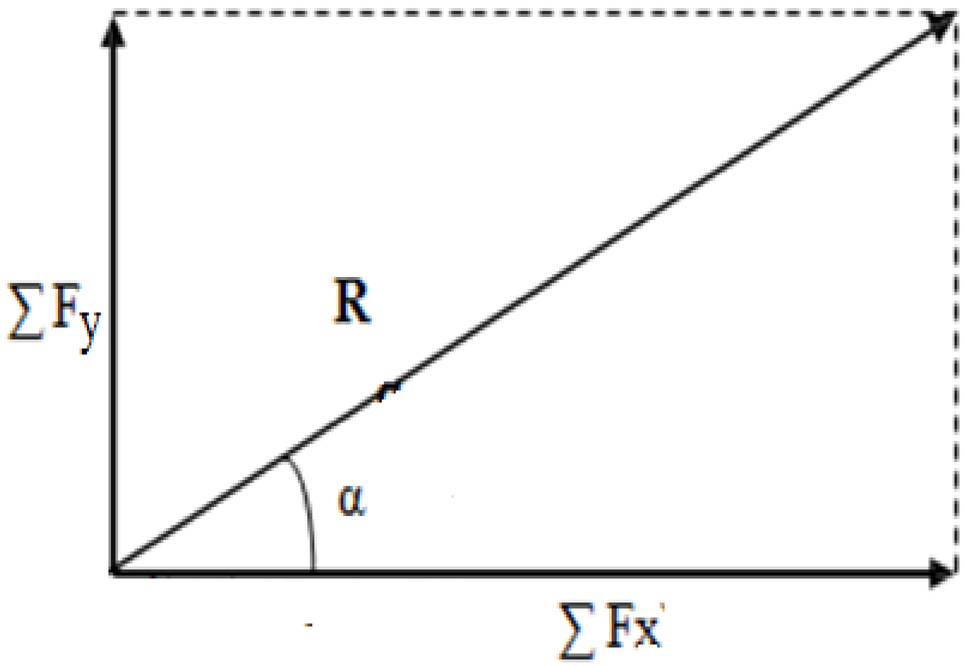
Let $\sum F_x$ be the algebraic sum of component forces in an x-direction

$$\sum Fx = f_{x1} + f_{x2} + f_{x3} + f_{x4}$$

Let $\sum FY$ be the algebraic sum of component forces in an Y-direction

$$\sum F_y = f_{y1} + f_{y2} + f_{y3} + f_{y4}$$

Composition of forces by method of resolution



The magnitude of the resultant is given as

$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$$

The direction of resultant can be obtained if the angle α made by the resultant with x direction is determined here, $\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

Composition of forces by method of resolution

The steps to solve the problems in the coplanar concurrent force system are, therefore as follows.

1. Calculate the algebraic sum of all the forces acting in the x- direction (ie. $\sum Fx$) and also in the y- direction (ie. $\sum Fy$)
2. Determine the magnitude of the resultant using the formula

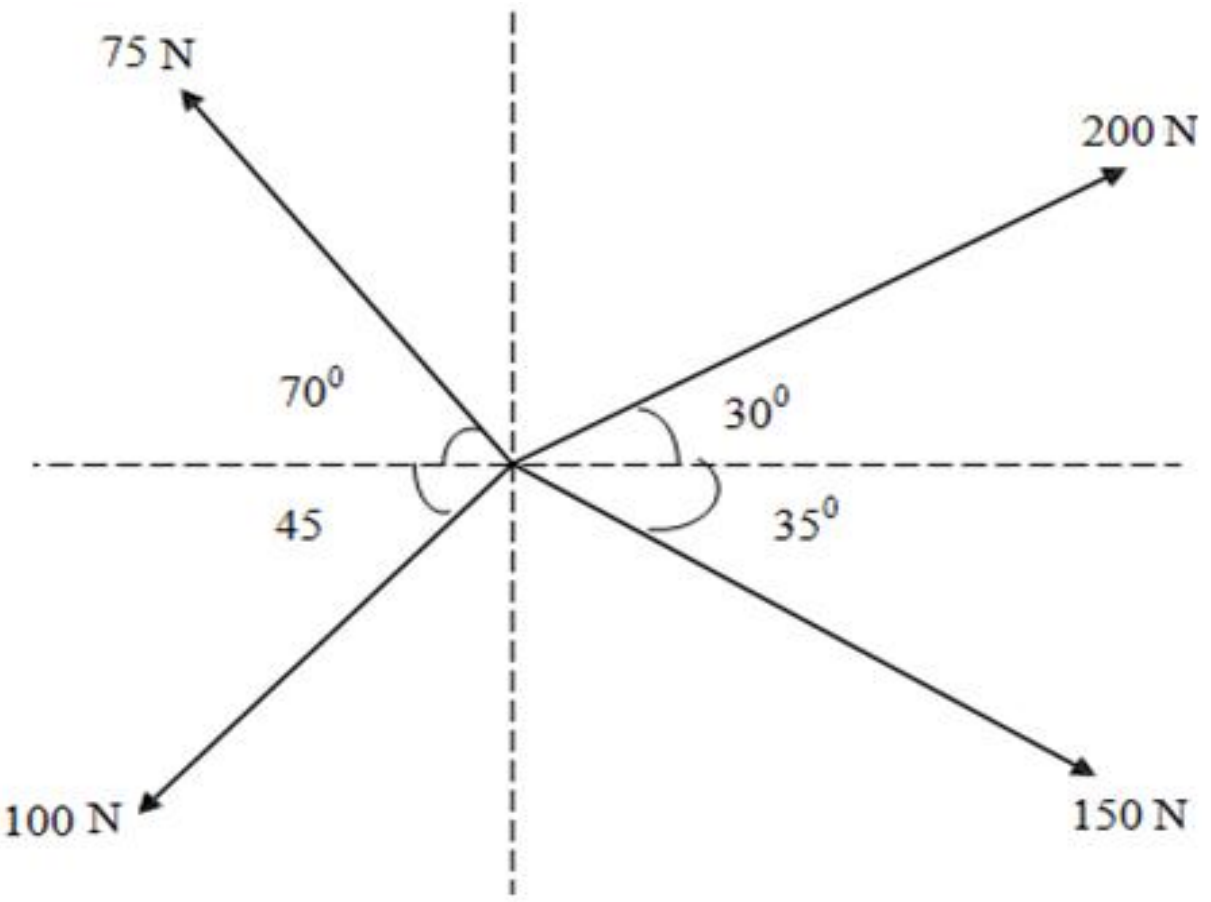
$$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$$

3. Determine the direction of the resultant using the formula

$$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$$

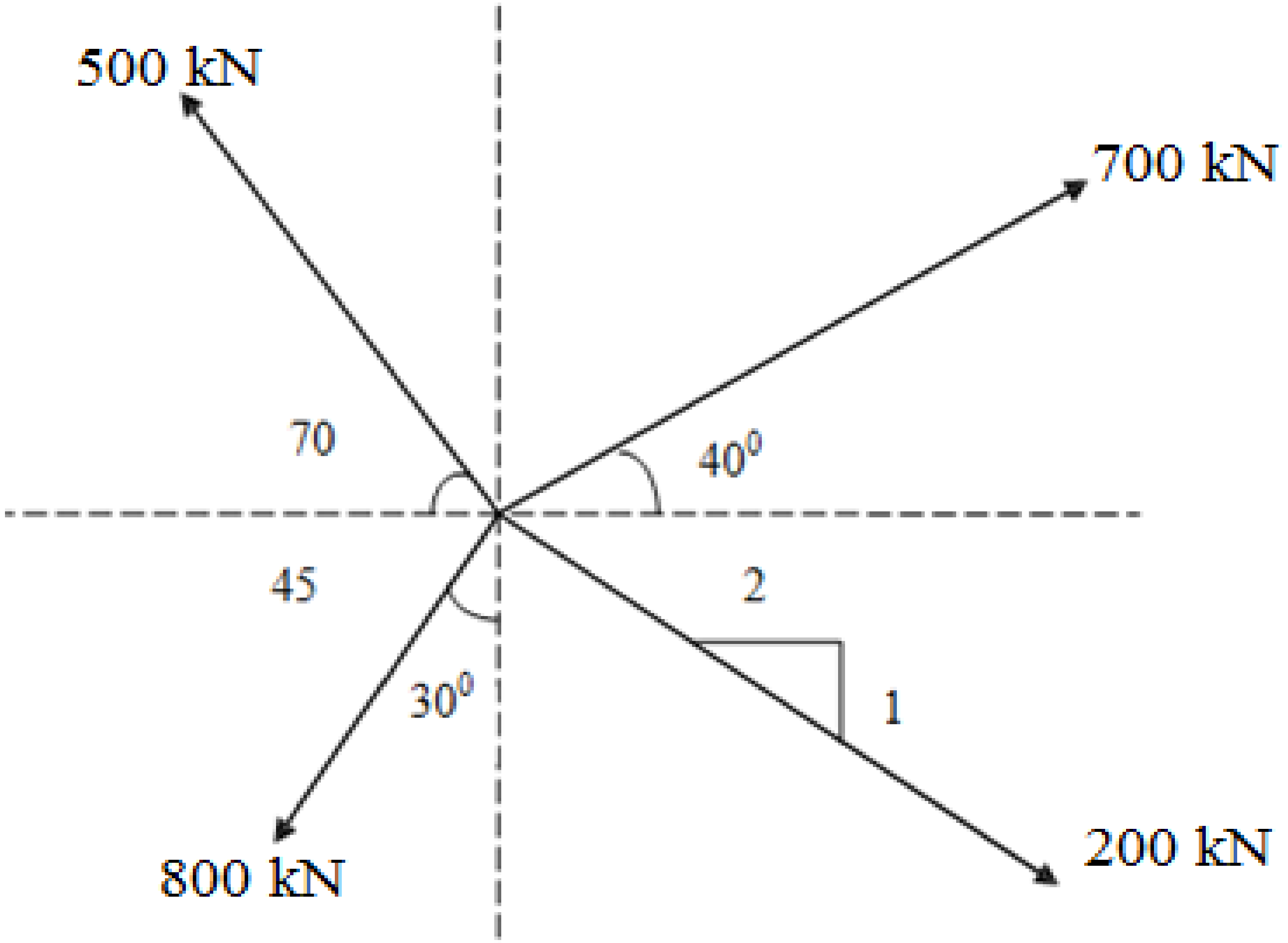
Tutorials

1. Determine the magnitude & direction of the resultant of the coplanar concurrent force system shown in figure below.



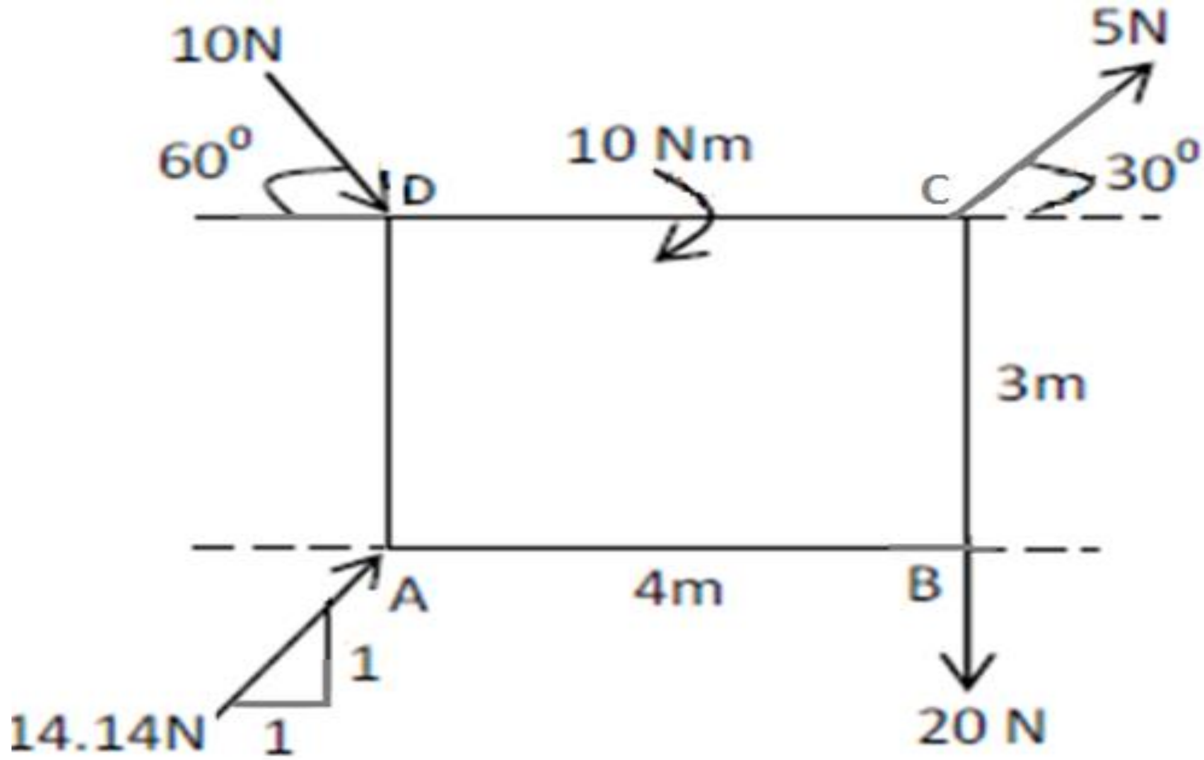
Tutorials

2. Determine the magnitude & direction of resultant of the concurrent force system shown in figure.



Tutorials

3. Determine the resultant of the force system acting on the plate. Also determine the direction of the resultant force as shown in figure given below with respect to AB and AD.



Tutorials

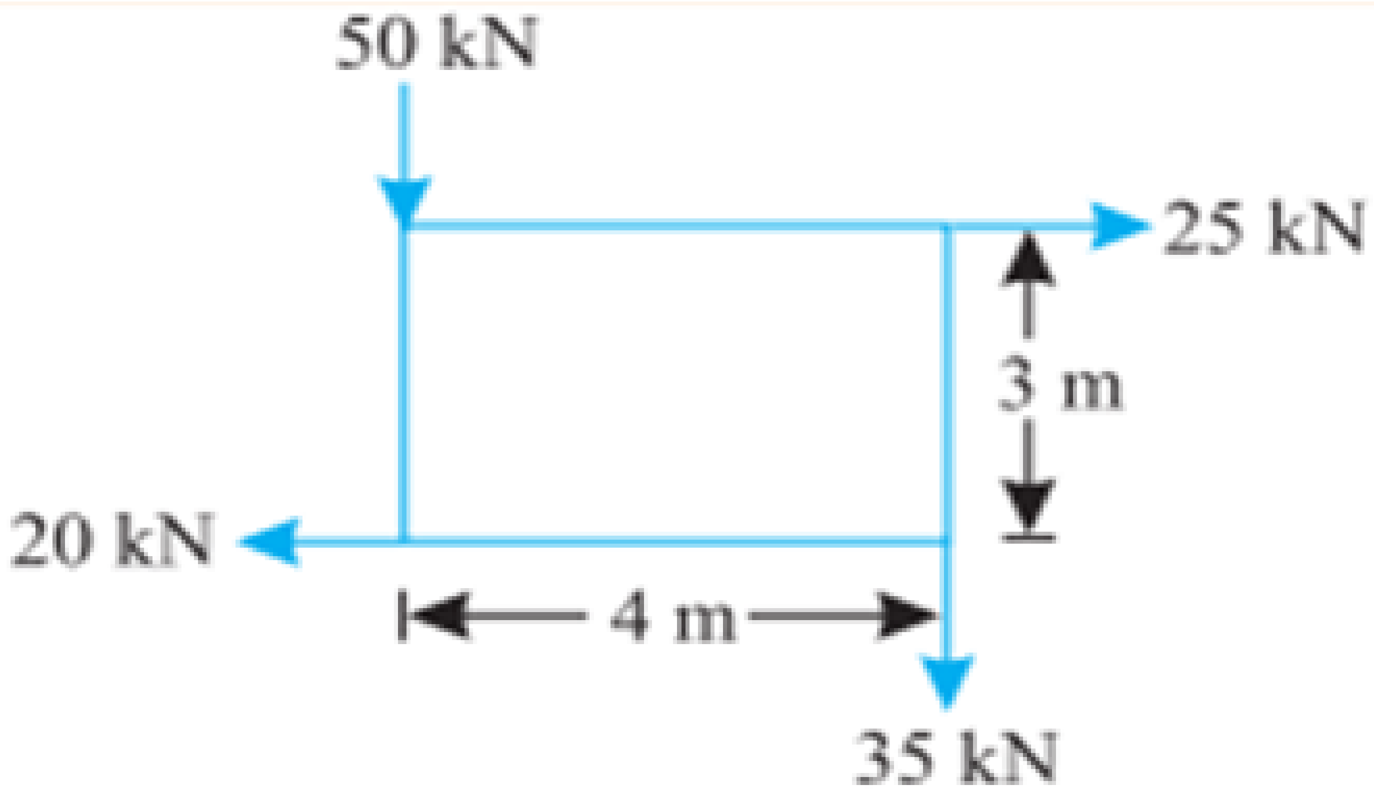
4. Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}N$. But if they act at 60^{o}, their resultant is $\sqrt{13}N$.

5. A horizontal line PQRS is 12m long, where PQ=QR=RS=4m. Forces of 1000N, 1500N, 1000N and 500N act at P, Q, R and S respectively downward orientation. The lines of action of these forces make angle of 90^{o}, 60^{o}, 45^{o} and 30^{o} respectively with PS. Find the magnitude, direction and position of the resultant force.

6. A triangle ABC has its side AB=40mm along the negative x-axis and side BC=30mm along positive y-axis. Three forces of 40N, 50N and 30N act along the sides AB, BC and CA respectively. Determine the magnitude of the resultant of such a system of forces.

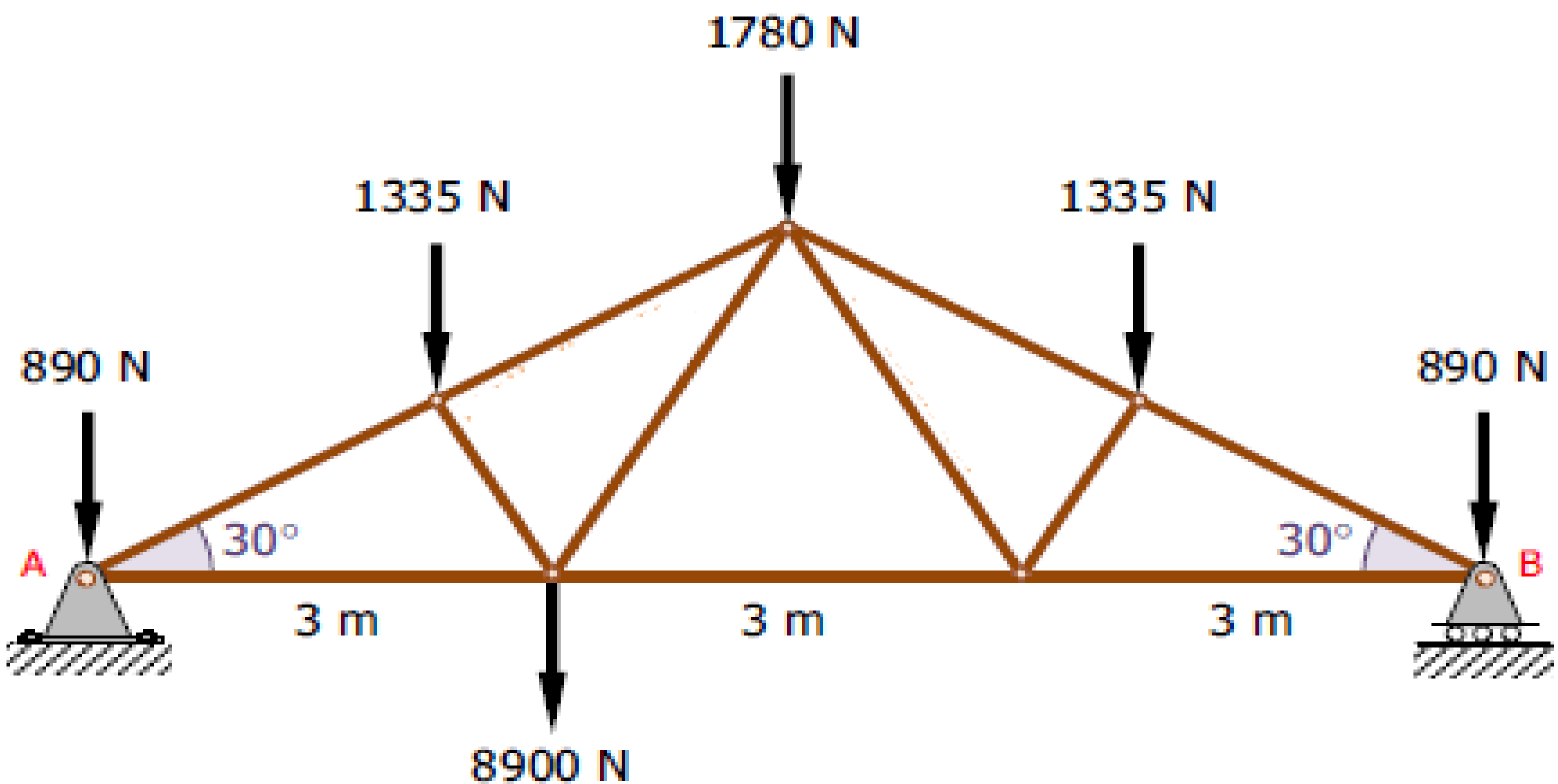
Tutorials

7. A system of forces are acting at the corners of a rectangular block as shown in figure below. Determine the magnitude and the direction of the resultant force.



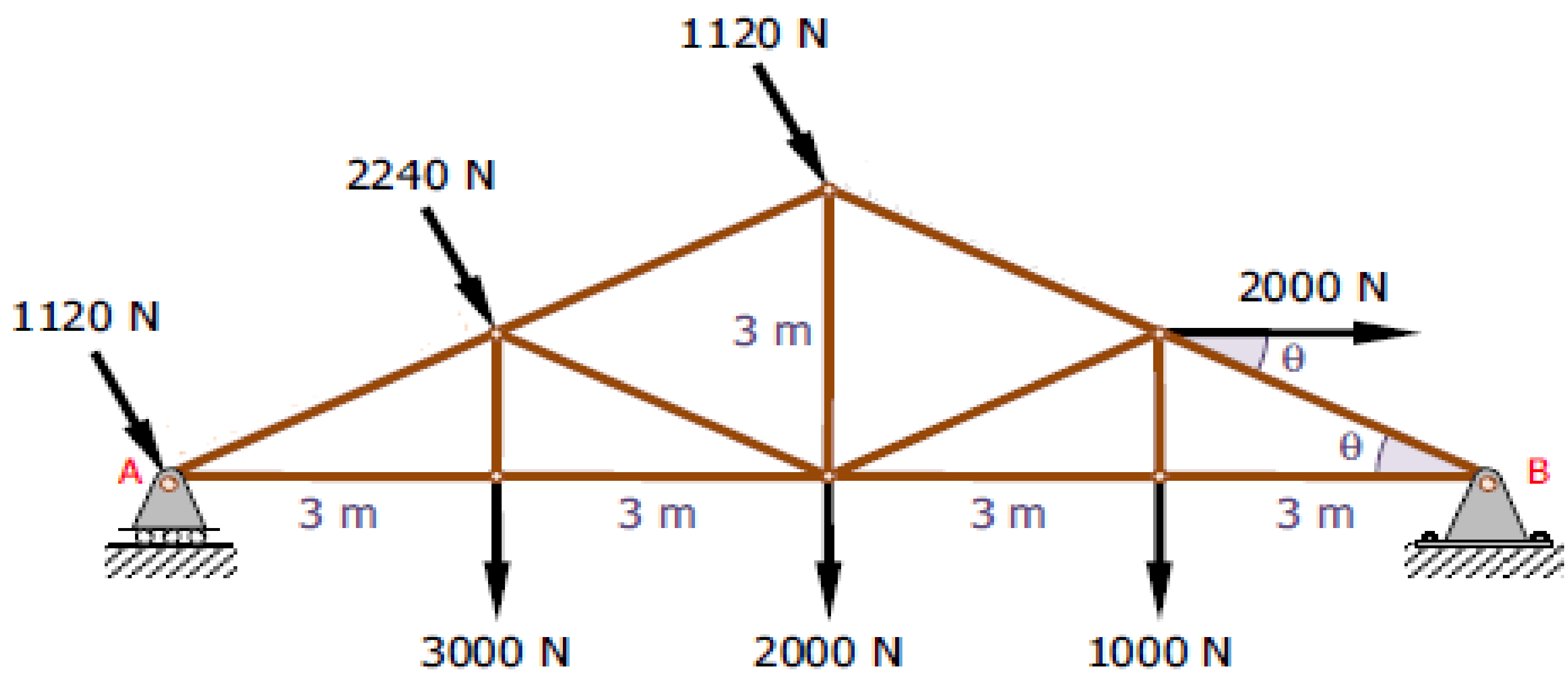
Tutorials

8. Locate the amount and position of the resultant of the loads (external forces) acting on the Frank truss shown in figure below.



Tutorials

9.The Howe roof truss shown in figure below carries the given loads (external forces). The wind loads are perpendicular to the inclined members. Determine the magnitude of the resultant, its inclination with the horizontal and where it intersects with AB.



Tutorials

1. $\sum Fx = 200\cos 30^0 - 75\cos 70^0 - 100\cos 45^0 + 150\cos 35^0$

$\sum Fx = 199.72\,N$

$\sum Fy = 200\sin 30^0 + 75\sin 70^0 - 100\sin 45^0 - 150\sin 35^0$

$\sum Fy = 13.73\,N$

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

$R = 200.19\,N$

$\alpha = \tan^{-1}\left(\frac{\sum Fy}{\sum Fx}\right)$

$\alpha = \tan^{-1}(13.73/199.72) = 3.93^0$

Tutorials

2. $\sum Fx = 700\cos 40^0 - 500\cos 70^0 - 800\cos 60^0 + 200\cos 26.56^0$

$\sum Fx = 144.11 \text{ kN}$

$\sum Fy = 700\sin 40^0 + 500\sin 70^0 - 800\sin 60^0 - 200\sin 26.56^0$

$\sum Fy = 137.55\text{kN}$

$R = \sqrt{(\sum Fx)^2 + (\sum Fy)^2}$

$R = 199.21\text{N}$

$\alpha = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$\alpha = \tan^{-1}(137.55/144.11) = 43.66^0$

Tutorials

3. $\sum Fx = 5\cos 30^0 + 10\cos 60^0 + 14.14\cos 45^0$
$= 19.33N$

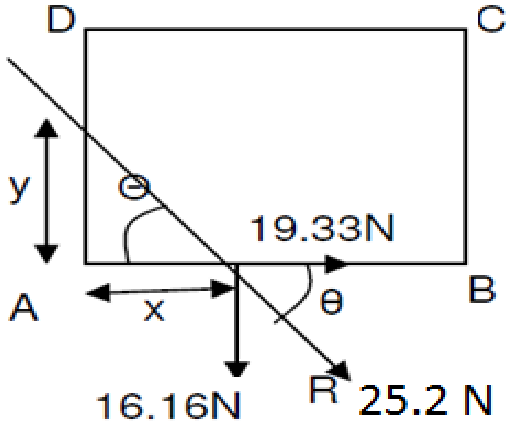
$\sum Fy = 5\sin 30^0 - 10\sin 60^0 + 14.14\sin 45^0 - 20$
$= -16.16N$

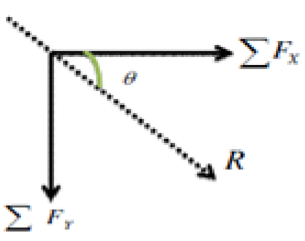
$R = \sqrt{(\sum Fx^2 + \sum Fy^2)} = 25.2N$

$\theta = Tan^{-1}(\sum Fy / \sum Fx)$

$\theta = Tan^{-1}(-16.16/19.33) = -39.89^0$

$\theta = 360^0 - 39.89^0 = 320.10^0$





Tutorials

4. **Solution.** Given : Two forces = F_1 and F_2.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

or $$10 = F_1^2 + F_2^2 \quad \text{...(Squaring both sides)}$$

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 60°}$$

$\therefore$ $$13 = F_1^2 + F_2^2 + 2F_1 F_2 \times 0.5 \quad \text{...(Squaring both sides)}$$

or $$F_1 F_2 = 13 - 10 = 3 \quad \text{...(Substituting } F_1^2 + F_2^2 = 10\text{)}$$

We know that $(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 6 = 16$

$\therefore$ $$F_1 + F_2 = \sqrt{16} = 4 \quad \text{...}(i)$$

Similarly $$(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$$

$\therefore$ $$F_1 - F_2 = \sqrt{4} = 2 \quad \text{...}(ii)$$

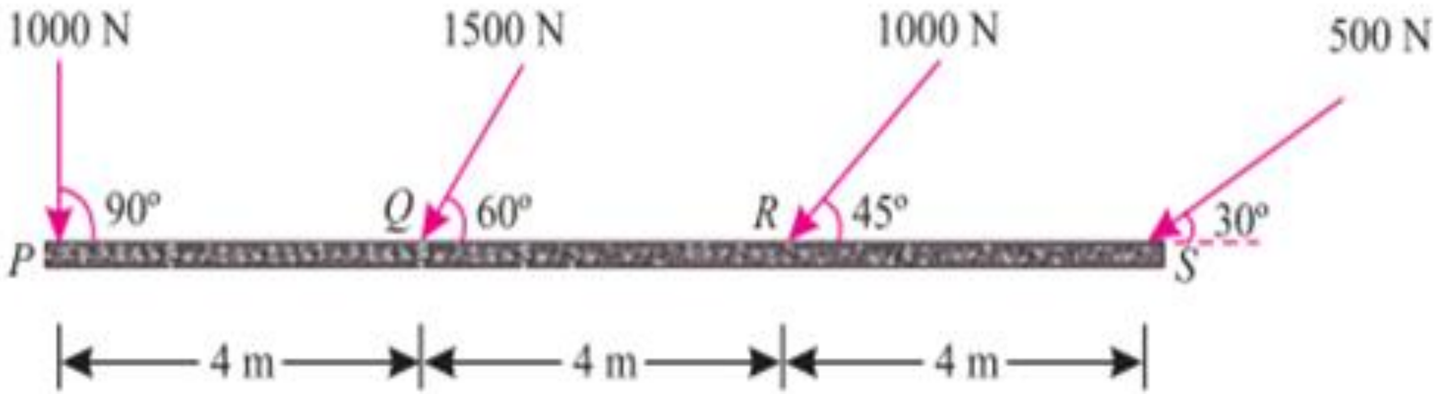
Solving equations (i) and (ii),

$$F_1 = 3 \text{ N} \quad \text{and} \quad F_2 = 1 \text{ N}$$

Tutorials

5.

Solution. The system of the given forces is shown in Figure below



Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = -(1500 \cos 60° + 1000 \cos 45° + 500 \cos 30°)\ \text{N}$$

$$= -((1500 \times 0.5) + (1000 \times 0.707) + (500 \times 0.866))\ \text{N}$$

$$= -1890\ \text{N}$$

Tutorials

and now resolving all the forces vertically,

$$\Sigma V = -(1000 \sin 90° + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30°)\ \text{N}$$

$$= -((1000 \times 1.0) + (1500 \times 0.866) + (1000 \times 0.707) + (500 \times 0.5))\ \text{N}$$

$$= -3256\ \text{N}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765\ \text{N}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with *PS*.

$$\therefore \qquad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722 \quad \text{or} \quad \theta = 59.8°$$

Note. Since both the values of ΣH and ΣV are –ve. therefore resultant lies between $180°$ and $270°$

$$\theta = 180° + 59.8° = 239.8°$$

Tutorials

Position of the resultant force

Let x = Distance between P and the line of action of the resultant force.

Now taking moments* of the vertical components of the forces and the resultant force about P, and equating the same,

$$3256\,x = (1000 \times 0) + (1500 \times 0.866)\,4 + (1000 \times 0.707)8 + (500 \times 0.5)12$$

$$= 13\,852$$

$$\therefore \quad x = \frac{13\,852}{3256} = 4.25 \text{ m}$$

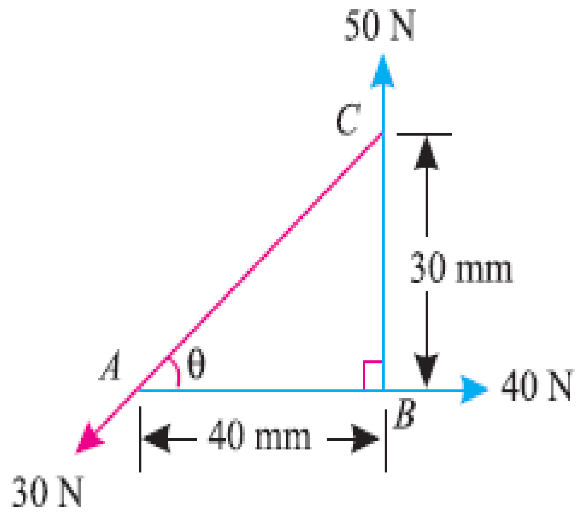
Tutorials

6. **Solution.** The system of given forces is shown in Figure below.

From the geometry of the figure, we find that the triangle *ABC* is a right angled triangle, in which the *side $AC = 50$ mm. Therefore

$$\sin\theta = \frac{30}{50} = 0.6$$

and $$\cos\theta = \frac{40}{50} = 0.8$$



Resolving all the forces horizontally (*i.e.*, along *AB*).

$$\sum H = 40 - 30\cos\theta$$
$$= 40 - (30 \times 0.8) = 16 \text{ N}$$

and now resolving all the forces vertically (*i.e.*, along *BC*)

$$\sum V = 50 - 30\sin\theta$$
$$= 50 - (30 \times 0.6) = 32 \text{ N}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8 \text{ N}$$

Tutorials

7.

Solution. Given : System of forces

Magnitude of the resultant force

Resolving forces horizontally,

$$\sum H = 25 - 20 = 5 \text{ kN}$$

and now resolving the forces vertically

$$\sum V = (-50) + (-35) = -85 \text{ kN}$$

∴ Magnitude of the resultant force

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(5)^2 + (-85)^2} = 85.15 \text{ kN} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle which the resultant force makes with the horizontal.

We know that

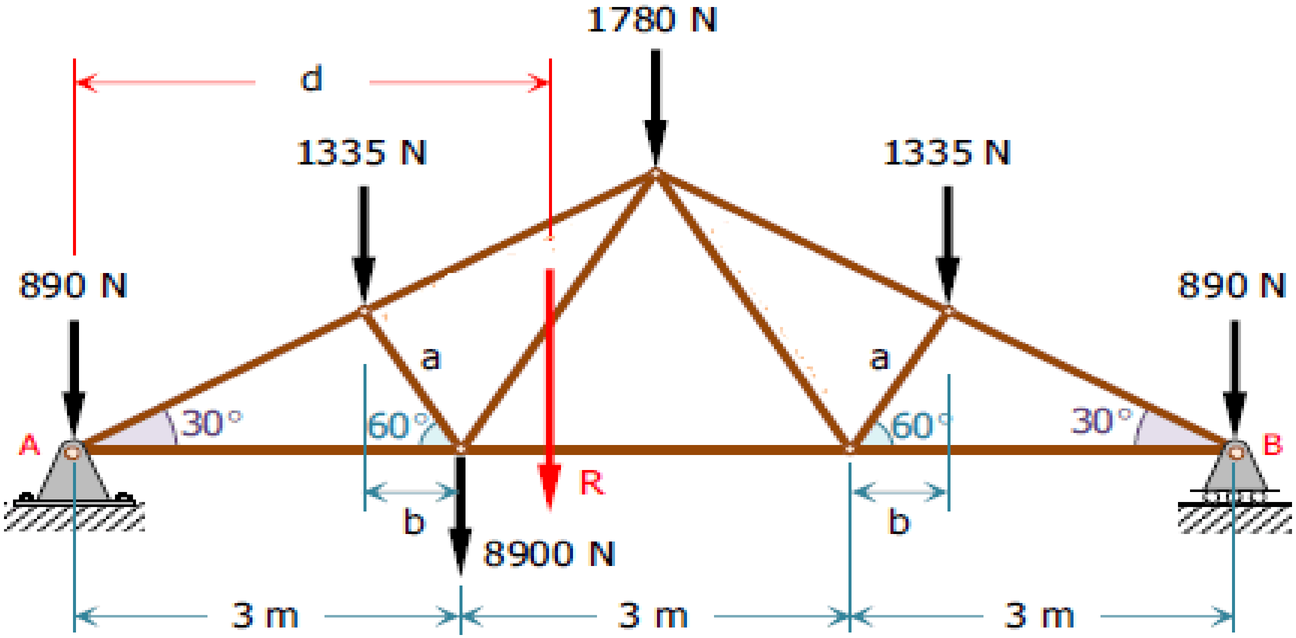
$$\tan\theta = \frac{\sum V}{\sum H} = \frac{-85}{5} = -17 \quad \text{or} \quad \theta = 86.6°$$

Since $\sum H$ is positive and $\sum V$ is negative, therefore resultant lies between 270° and 360°. Thus actual angle of the resultant force

$$= 360° - 86.6° = 273.4° \quad \textbf{Ans.}$$

Tutorials

8. Virtual FBD



$$\sin 30^\circ = \frac{a}{3}$$

$$a = 3\sin 30^\circ$$

$$a = 1.5\ \text{m}$$

and

$$\cos 60^\circ = \frac{b}{a}$$

$$b = a\cos 60^\circ$$

$$b = 1.5\cos 60^\circ$$

$$b = 0.75\ \text{m}$$

Tutorials

Magnitude of resultant

$$R = \Sigma F_v$$

$$R = 890 + 1335 + 8900 + 1780 + 1335 + 890$$

$$R = 15\,130 \text{ N} \qquad \text{downward}$$

Location of resultant

$$Rd = \Sigma Fx$$

$$Rd = 1335(3 - b) + 8900(3) + 1780(4.5) + 1335(6 + b) + 890(9)$$

$$15\,130d = 1335(3 - 0.75) + 8900(3) + 1780(4.5) + 1335(6 + 0.75) + 890(9)$$

$$15\,130d = 1335(2.25) + 8900(3) + 1780(4.5) + 1335(6.75) + 890(9)$$

$$15\,130d = 54\,735$$

$$d = 3.62 \text{ m} \qquad \text{to the right of A}$$

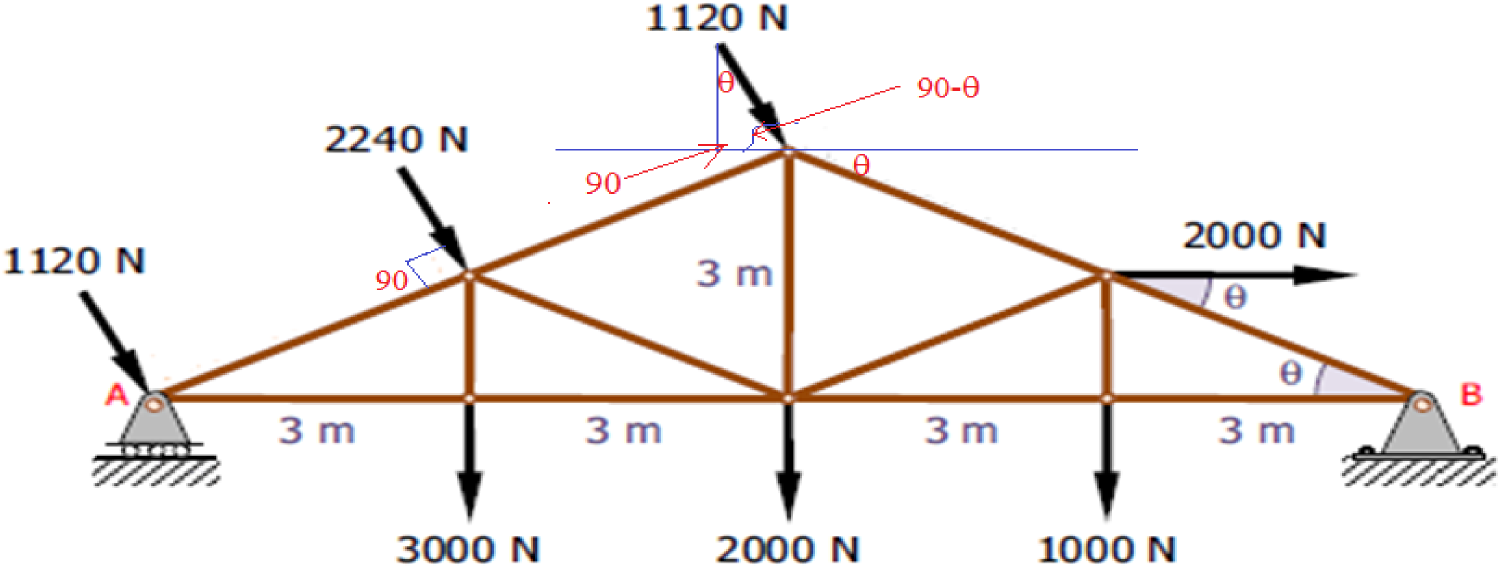
Tutorials

9. Calculation of slope

$$\frac{\mathbf{3}}{\mathbf{6}} = \frac{\mathbf{1}}{\mathbf{2}} ==> \sqrt{1^2 + 2^2} = \sqrt{5}$$

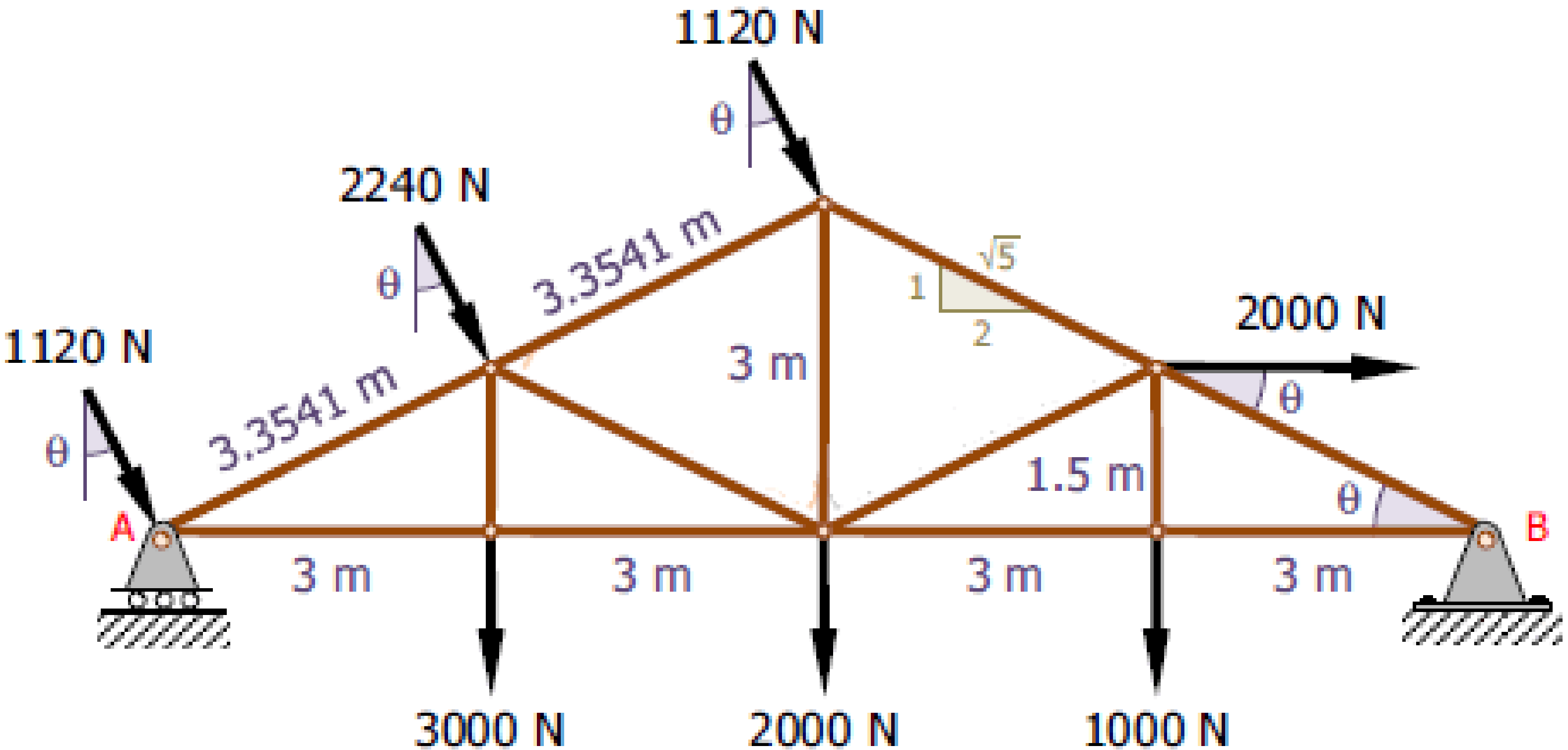
Calculation of distances

$$\frac{3}{6} = \frac{y}{3} ===> \quad y = \frac{3 \text{ x } 3}{6} = 1.5\, m \quad \text{and} \quad \sqrt{3^2 + 1.5} = 3.3541m$$



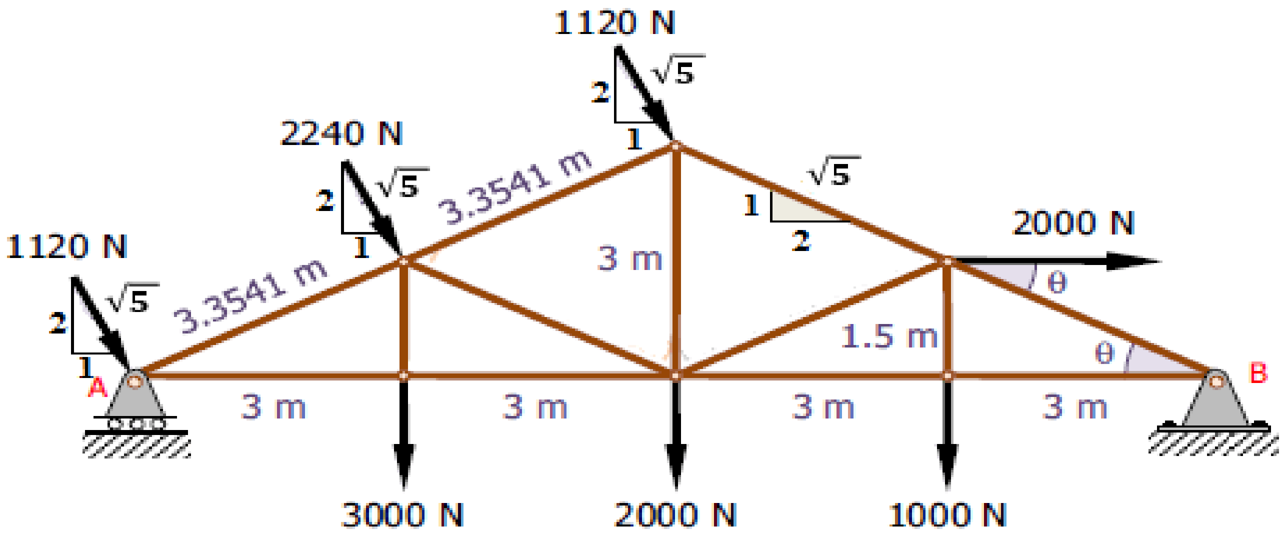
Tutorials

Virtual FBD (1)



Tutorials

Virtual FBD (2)



$$R_x = \Sigma F_x$$

$$R_x = (1120 + 2240 + 1120)(\frac{1}{\sqrt{5}}) + 2000$$

$$R_x = 4003.52 \text{ N to the right}$$

$$R_y = \Sigma F_y$$

$$R_y = (1120 + 2240 + 1120)(\frac{2}{\sqrt{5}}) + 3000 + 2000 + 1000$$

$$R_y = 10\,007.03 \text{ N downward}$$

Tutorials

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

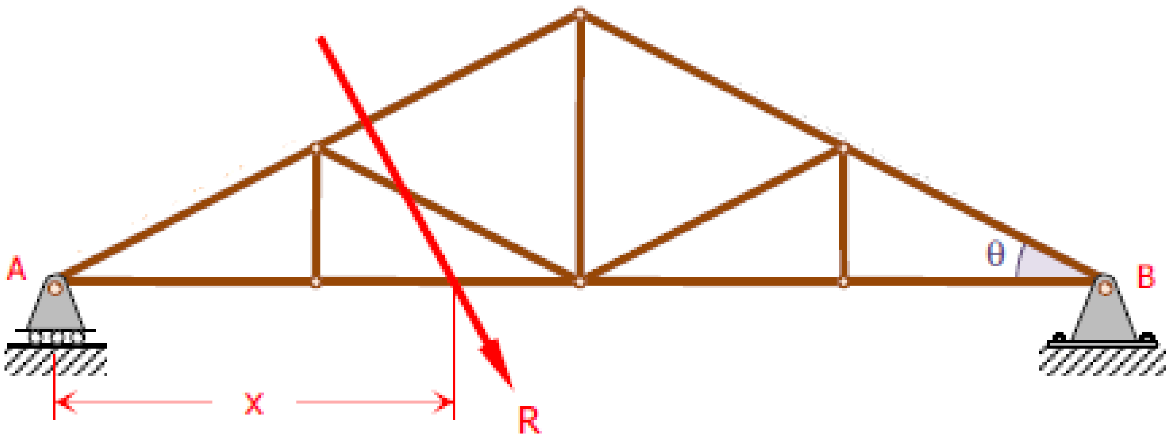
$$R = \sqrt{4003.52^2 + 10007.03^2}$$

$$R = 10\,778.16 \text{ N}$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{10007.03}{4003.52}\right) = 68.2°$$

Virtual FBD (3)



Tutorials

$$M_A = \Sigma F d$$

$$M_A = 2240(3.354) + 1120(3.354)(2) + 2000(1.5) + 3000(3) + 2000(6) + 1000(9)$$

$$M_A = 48\,026.37 \text{ N} \cdot \text{m clockwise}$$

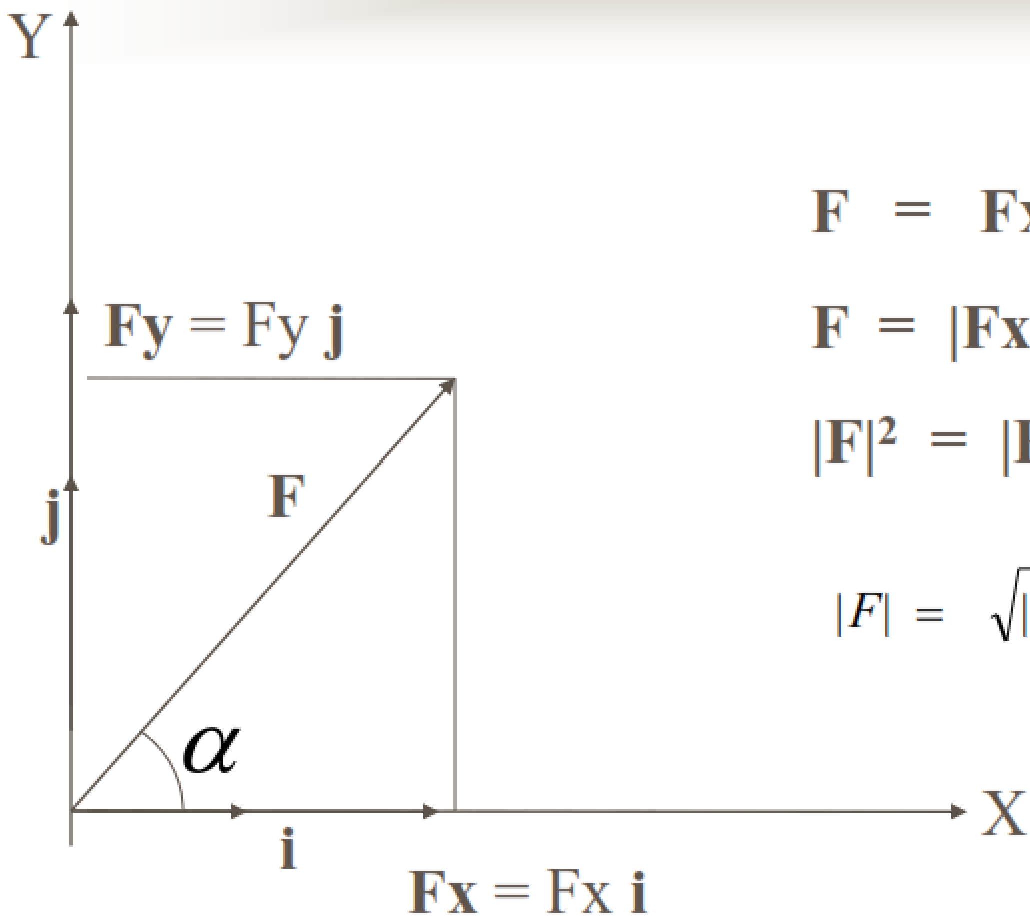
$$R_y x = M_A$$

$$10\,007.03x = 48\,026.37$$

$$x = 4.8 \text{ m to the right of A}$$

2D FORCES SYSTEMS

RECTANGULAR COMPONENTS OF FORCE



$$\mathbf{F} = \mathbf{Fx} + \mathbf{Fy}$$

$$\mathbf{F} = |\mathbf{Fx}| \cdot \mathbf{i} + |\mathbf{Fy}| \cdot \mathbf{j}$$

$$|\mathbf{F}|^2 = |\mathbf{Fx}|^2 + |\mathbf{Fy}|^2$$

$$|F| = \sqrt{|Fx|^2 + |Fy|^2}$$

- In many problems, it is desirable to resolve force **F** into two perpendicular components in the x and y directions.
- **Fx** and **Fy** are called rectangular vector components.
- In two-dimensions, the cartesian unit vectors **i** and **j** are used to designate the directions of x and y axes.
- **Fx** = Fx **i** and **Fy** = Fy **j**
- i.e. **F** = Fx **i** + Fy **j**
- Fx and Fy are scalar components of **F**

While the scalars, Fx and Fy may be positive or negative, depending on the sense of **Fx** and **Fy**, their absolute values are respectively equal to the magnitudes of the component forces **Fx** and **Fy**,

Scalar components of **F** have magnitudes:

$$F_x = F\cos\theta \quad \text{and} \quad F_y = F\sin\theta$$

F is the magnitude of force **F**.

$$F_x = F\cos\theta \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

Force along X- axis

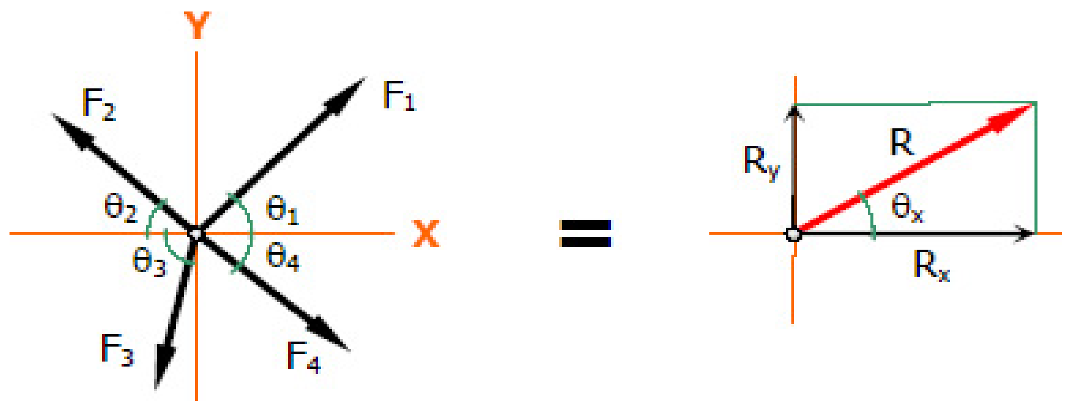
Force along Y- axis

Resultant force

Direction

Resultant of Coplanar Concurrent Force System

The line of action of each forces in coplanar concurrent force system are on the same plane. All of these forces meet at a common point, thus concurrent. In x-y plane, the resultant can be found by the following formulas:



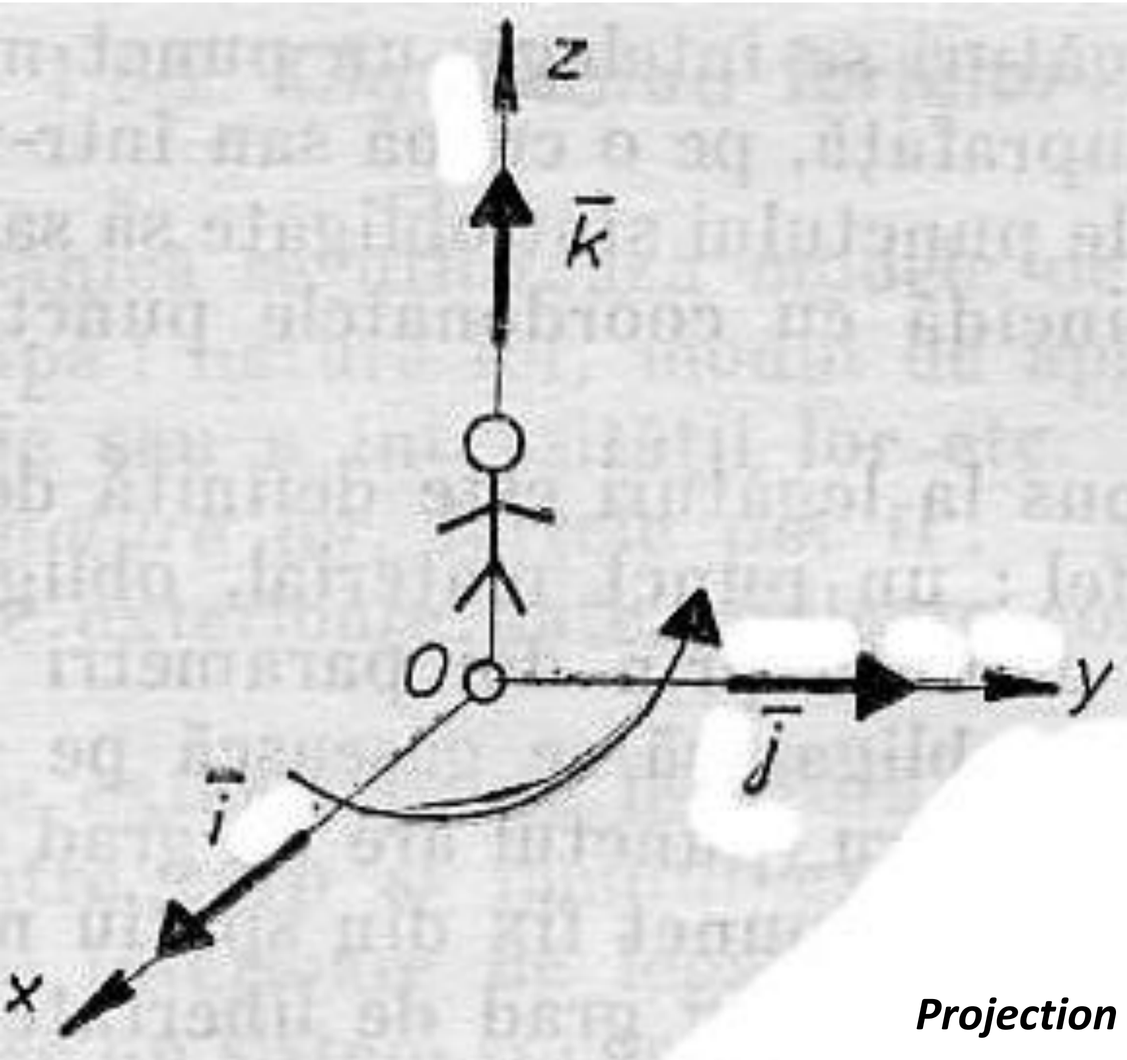
$$R_x = \Sigma F_x$$

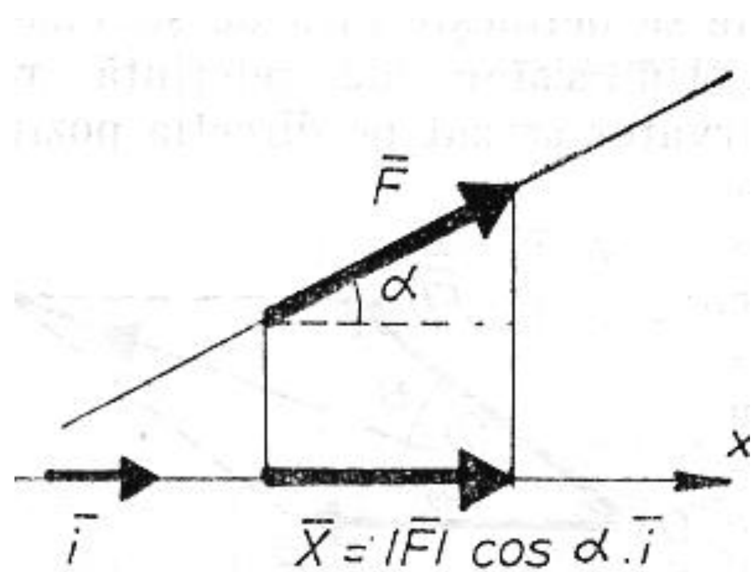
$$R_y = \Sigma F_y$$

$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

$$\tan\theta_x = \frac{R_y}{R_x}$$

RESULTANTS (2D,3D) RECTUNGULAR COMPONENT IN SPACE





Projection and component of force F on X axis

The *projection of the force F on an axis is a scalar* $X = F\cos\alpha$

The *component of the force F on an axis is a vector,* $\vec{X} = (F\cos\alpha)\vec{i}$

The resultant is given by the components of the vector R on the x, y, z axis:

$$\vec{R} = R_x\vec{i} + R_y\vec{j} + R_z\vec{k}$$

A given vector of force Fi in the system of forces has the expression:

$$\vec{F}_i = F_{xi}\vec{i} + F_{yi}\vec{j} + F_{zi}\vec{k}$$

The resultant R of the system of n forces expressed in terms of forces Fi:

$$\vec{R} = \vec{F}_1 + ... + \vec{F}_i + ... + \vec{F}_n = \left(\sum_{i=1}^{n} F_{xi}\right)\vec{i} + \left(\sum_{i=1}^{n} F_{yi}\right)\vec{j} + \left(\sum_{i=1}^{n} F_{zi}\right)\vec{k}$$

By identifying the two expressions of the resultant force, it results that:

$$R_x = \sum_{i=1}^{n} F_{xi}, R_y = \sum_{i=1}^{n} F_{yi}, R_z = \sum_{i=1}^{n} F_{zi}$$

In other words, this means that the projections of the resultant force R on the x, y, z axis are equal to the algebraic sum of the corresponding projections of the forces of the system (Theorem of projections).

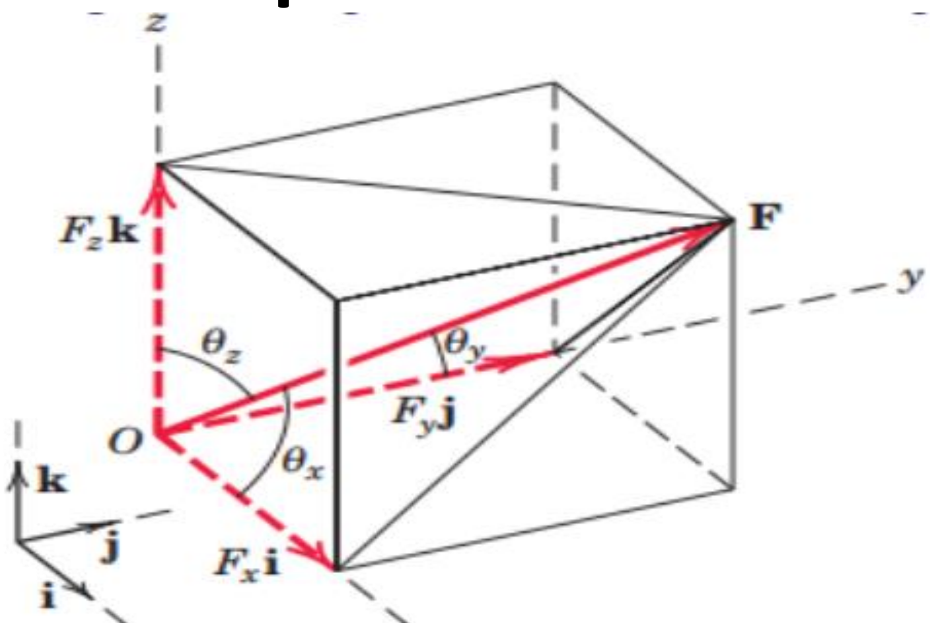
The magnitude of the resultant:

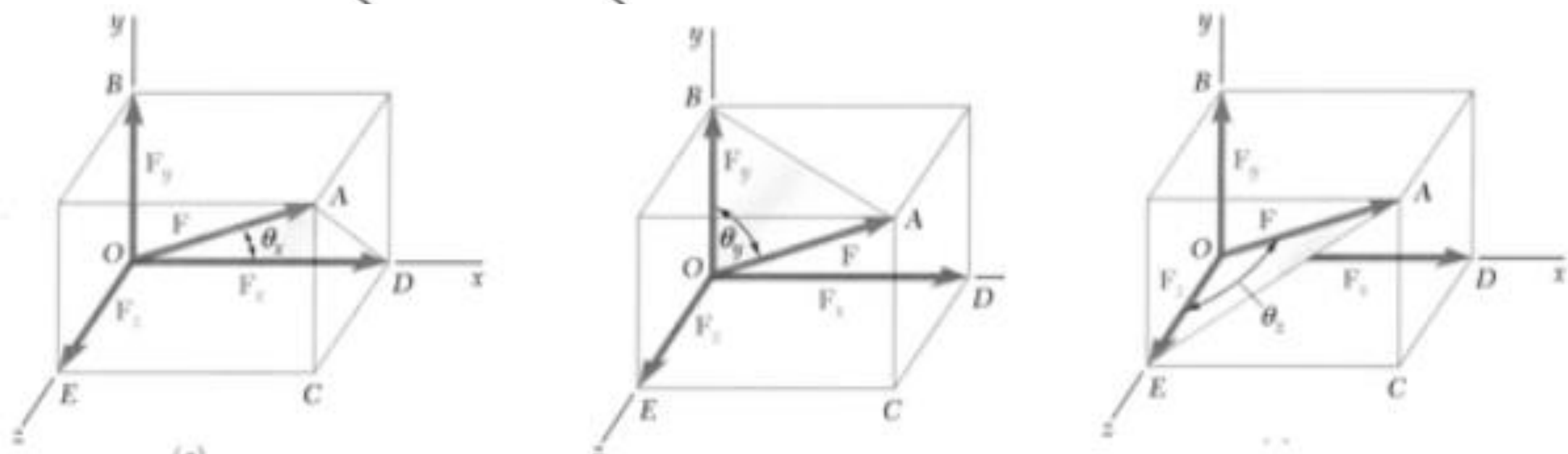
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

The angles that the resultant forms with the axes of coordinates:

$$\cos\theta_x = \frac{R_x}{R}, \cos\theta_y = \frac{R_y}{R}, \cos\theta_z = \frac{R_z}{R}$$

3D FORCES SYSTEMS
Rectangular Components of a Force in Space





$$\mathbf{F} = \mathbf{Fx} + \mathbf{Fy} + \mathbf{Fz}$$

$$\mathbf{F} = |\mathbf{Fx}| . \mathbf{i} + |\mathbf{Fy}| . \mathbf{j} + |\mathbf{Fz}| . \mathbf{k}$$

$$|\mathbf{F}|^2 = |\mathbf{Fx}|^2 + |\mathbf{Fy}|^2 + |\mathbf{Fz}|^2$$

$$|F| = \sqrt{|Fx|^2 + |Fy|^2 + |Fz|^2}$$

$$|Fx| = |F| \cos\theta_x \qquad |Fy| = |F| \cos\theta_y \qquad |Fz| = |F| \cos\theta_z$$

$Cos\theta_x$, $Cos\theta_y$ *and* $Cos\theta_z$ *are called direction cosines of angles* θ_x, θ_y *and* θ_z

$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F_y = F\cos\theta_y \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

$$F_z = F\cos\theta_z \qquad \mathbf{F} = F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$$

$$= F\vec{\lambda}$$

$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$

and $\cos\theta_x, \cos\theta_y, \text{and} \cos\theta_z$ are the direction cosines

$\vec{d}$ = vector joining M and N

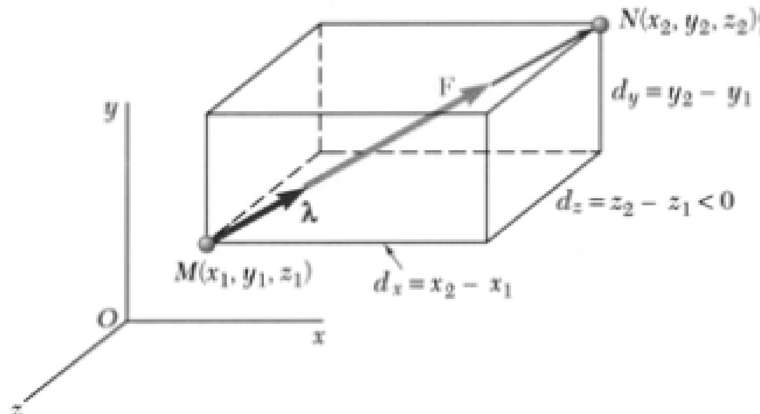
$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$

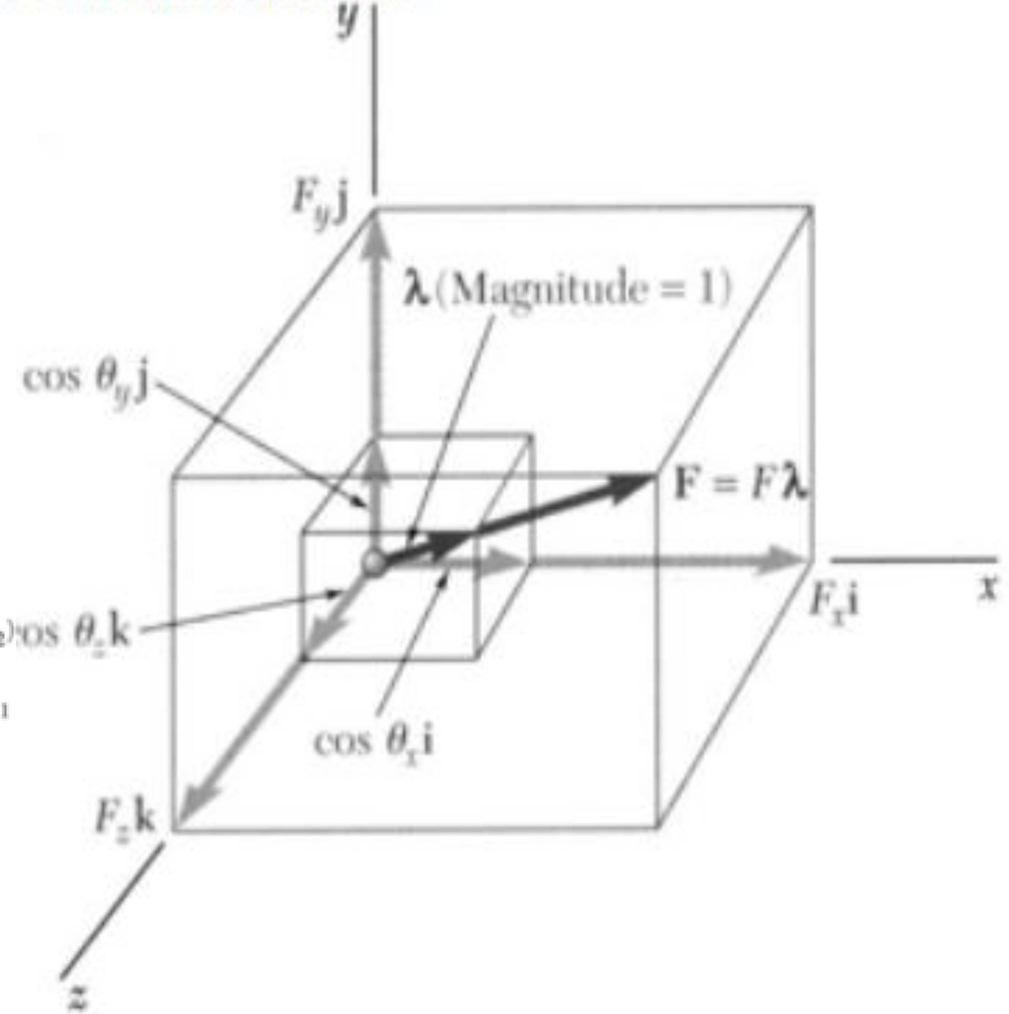
$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$

$\vec{F} = F\vec{\lambda}$

$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$

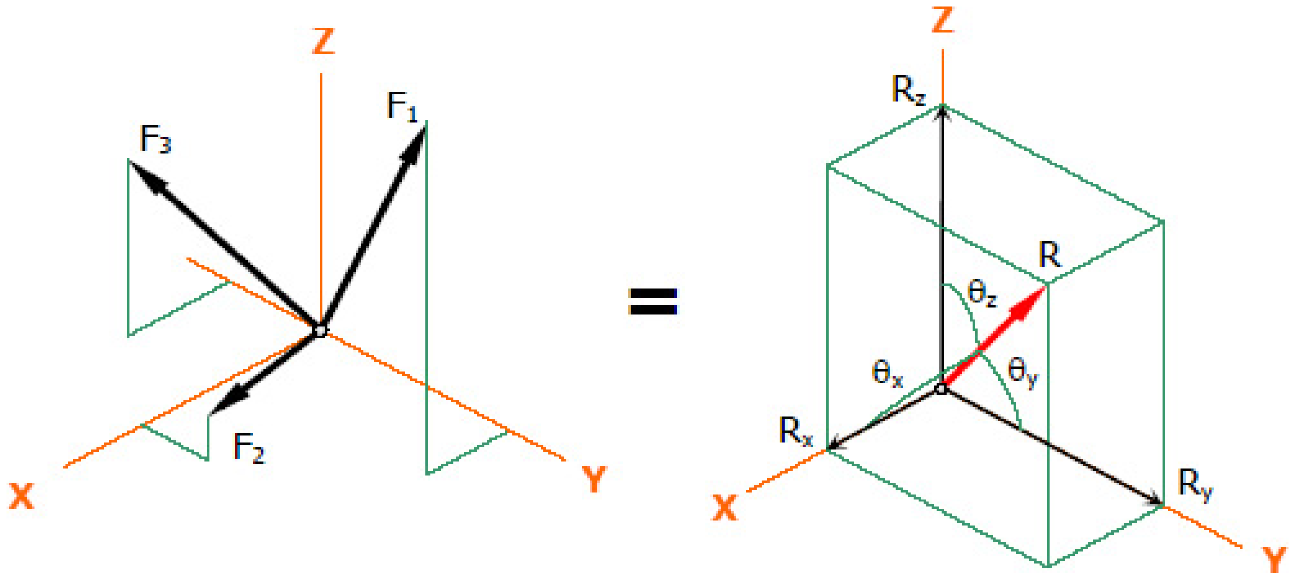
$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$





Resultant of Spatial Concurrent Force System

Spatial concurrent forces (forces in 3-dimensional space) meet at a common point but do not lie in a single plane. The resultant can be found as follows:



$$R_x = \Sigma F_x$$

$$R_y = \Sigma F_y$$

$$R_z = \Sigma F_z$$

$$R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2}$$

Direction Cosines

$$\cos\theta_x = \frac{R_x}{R}$$
$$\cos\theta_y = \frac{R_y}{R}$$
$$\cos\theta_z = \frac{R_z}{R}$$

Vector Notation of the Resultant

$\mathbf{R} = \Sigma\mathbf{F}$

$\mathbf{R} = (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} + (\Sigma F_z)\mathbf{k}$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k}$$

Where

$R_x = \Sigma F_x$

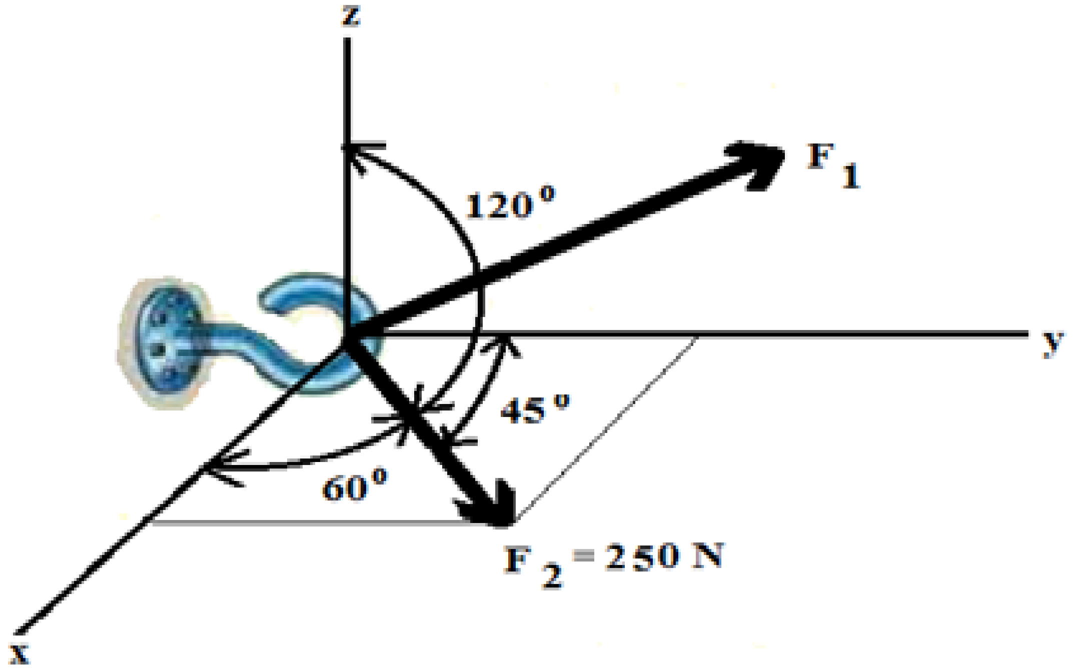
$R_y = \Sigma F_y$

$R_z = \Sigma F_z$

$R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2}$

Tutorial

Two forces act on the hook shown in Figure below. Calculate the magnitude of F_1 and its coordinate direction angles of F_1 that the resultant F_R acts along the positive y axis and has a magnitude of 550N. Also provide the final sketch (drawing) containing all the results.



Tutorial

It is necessary that $F_R = F_1 + F_2$

$$F_2 = F_2 \cos\alpha_2 i + F_2 \cos\beta_2 j + F_2 \cos\gamma_2 k \quad /$$

$$= 250\cos 60° i + 250\cos 45° j + 250\cos 120° k$$

$$F_2 = (125 i + 176.78 j - 125 k) N$$ And

$$F_1 = F_{1x} i + F_{1y} j + F_{1z} k$$

Since F_R has a magnitude of 550N and acts in the +j direction,

$$F_R = (550 N)(+j) = (550 j) N$$

We require $F_R = F_2 + F_1$

$$550 j = 125 i + 176.78 j - 125 k + F_{1x} i + F_{1y} j + F_{1z} k$$

$$550 j = (125 + F_{1x}) i + (176.78 + F_{1y}) j + (-125 + F_{1z}) k$$

Tutorial

To satisfy this equation the i, j, k components of F_R must be equal to the corresponding i, j, k components of (F_2+F_1). Hence,

$$0 = 125 + F_{1x} \quad \Rightarrow F_{1x} = -125N$$

$$550 = 176.78 + F_{1y} \quad \Rightarrow F_{1y} = 373.22N$$

$$0 = -125 + F_{1z} \quad \Rightarrow F_{1z} = 125N$$

The magnitude of F_1 is thus

$$F_1 = \sqrt{(-125N)^2 + (373.22N)^2 + (125N)^2}$$

$$F_1 = 412.97N$$

Tutorial

We can now determine direction angles of F_1

$$\cos\alpha_1 = \frac{-125}{412.97} \Rightarrow \alpha_1 = 107.62° \approx 108°$$

$$\cos\beta_1 = \frac{373.22}{412.97} \Rightarrow \beta_1 = 25.35 \approx 25°$$

$$\cos\gamma_1 = \frac{125}{412.97} \Rightarrow \gamma_1 = 72.38 \approx 72°$$

The results are shown on the figure below: