UNIVERSITY of RWANDA

COLLEGE OF SCIENCE AND TECHNOLOGY

School of Engineering

Civil, Environmental & Geomatics Engineering (CEGE)

MECHANICS OF MATERIALS

TRE1162 Mechanics of Materials

Instructor: Ir. Philbert Habimana

UR-CST/CEGE-Nyarugenge Campus

Introduction of the Instructor

Studies done:

MSc. degree in Civil Engineering Specialising in **Structural Engineering** from the University of Cape Town-South Africa (2013-2016).

BSc. degree in Civil Engineering & Environmental Technology from KIST-Rwanda (2006-2010).

Professional Certificate of Secondary Education (Public Works & Construction).

Subject

The **Mechanics of Materials** course delivered here combines the **essential** of both **Engineering Mechanics** and **Strength of Materials** previously delivered in the **old Civil Engineering program**.

Therefore, both **rigid bodies** and **deformable bodies** are studied taking into account all necessary details.

Subject

- Mechanics of Materials is a **15** credits course:

  Six contact hours per **week**
- Grads:
  - Assignments 20%
  - CATs and Quizzes 30%
  - Final exam 50% (3 Hours exam)
- Course Materials
  - Lecture notes (Soft Copy)
  - Various reading enrichments

Indicative resources

❑Engineering Mechanics

- Dietmar Gross et al., Engineering Mechanics 1: Statics
- Dietmar Gross et al., Engineering Mechanics 3: Dynamics
- R. C. Hibbeler, Engineering Mechanics: Statics, 12th Edition
- R. C. Hibbeler, Engineering Mechanics: Dynamics, 12th Edition
- R. C. Hibbeler, Engineering Mechanics: Statics & Dynamics, 12th Edition
- R. S. Khurmi, A textbook of Engineering Mechanics
- Shaum's Outline of Theory and Problems of Engineering mechanics: Statics & Dynamics, 5th Edition
- Any Book of Engineering Mechanics available in CST library

Indicative resources

❑Strength of Materials

- Ferdinand P. Beer et al., Mechanics of Materials, 5th Edition
- R. K. Bansal, Strength of Materials, 4th Edition
- Dietmar Gross et al., Engineering Mechanics 2: Mechanics of Materials
- R. S. Khurmi, Strength of Materials
- R. L. Timings, Engineering Materials
- Any Book of Strength Materials available in CST library

Course Syllabus

I. STATICS:

Force System

1. Force and its rectangular and oblique axis components (two and three dimensional systems).
2. Moment and resultant couple (two and three dimensional systems).

Equilibrium of a force

1. Concepts, Lami's theorem
2. Mechanical systems, isolation and equilibrium conditions for two and three dimensional systems.

Types of loads

Course Syllabus

I. STATICS:

Structures

1. Plane trusses.
2. Solution of plane trusses with method of joints and method of sections.
3. Frames.

Properties of areas

1. The centroid or centre of gravity
2. First moment of area or statical moment of area
3. Second moment of area or moment of inertia
4. Section modulus
5. Radius of gyration

Friction

1. Types of friction.
2. Application of friction

Course Syllabus

II. DYNAMICS:

- **Kinematics:** Rectilinear and curvilinear motion of Particle

- **Kinetics:** Equation of motion, angular momentum
- Work and Energy
- Impulse and momentum
- Torque and rotational inertia
- Velocity and acceleration diagrams
- Solid body rotation

Course Syllabus

III. Strength of Materials

- **Mechanical properties of materials:** introduction, various definition, properties of materials, behaviour of ductile and brittle materials.
- **Simples stresses and strains:** simple stresses, introduction, types of stresses, simples strains, types of strains, stress-strain diagram, thermal stresses, thermal stresses in bars of varying section, thermal stresses in composite bars. Numerical problems.
- **Elastic constant:** types, modulus of elasticity, Hooke's law, modulus of rigidity (shear modulus), bulk modulus, Poisson's ratio, relationship between three moduli, derivation of various formulae, typical elastic constant for different engineering materials, load and stress limit, allowable load/allowable stress, selection of factor of safety. Numerical problems.
- **Strain energy:** introduction, strain energy due to gradually applied load, strain energy due to suddenly applied load, strain energy due to shear stress. Numerical problems.

Course Syllabus

- **Geometrical properties of sections:** centroid, centroidal axes, moment of inertia, polar moment of inertia, radius of gyration, parallel and perpendicular axes theorems . Numerous & numerical problems.
- **Compound stresses and strains:** state of stress at a point (general 2D system), principal planes and principal stresses, biaxial direct stress, uniaxial direct stress, case of pure shear stress, Mohr's cycle for stresses, construction of Mohr's cycle, principal strains and strain energy due to principal stresses. Numerical problems.
- **Equilibrium of beams-Shear force (SF) and bending moment (BM):** introduction, types of loading, types of beams, shear force and bending moment diagrams, sign convention for shear force and bending moment, important points for drawing shear force (SF) and bending moment (BM) diagrams, methods for constructing the shear force and bending moment,

Course Syllabus

- **Equilibrium of beams-Shear force (SF) and bending moment (BM):** shear force (SF) and bending moment (BM) diagrams for various types of beams with different loading conditions, numerical examples, relationships between load, shear force and bending moment.
- **Bending and shear stresses in beams:** introduction, theory of simple bending with assumptions made, theory of simple bending, expression for bending stress, moment of resistance, bending stresses in symmetrical sections, section modulus, bending stresses in unsymmetrical sections, relationship between bending moment and bending stress, enhancing bending moment sign convention, some key observations, shear stress in beams . Numerical problems.

Course Syllabus

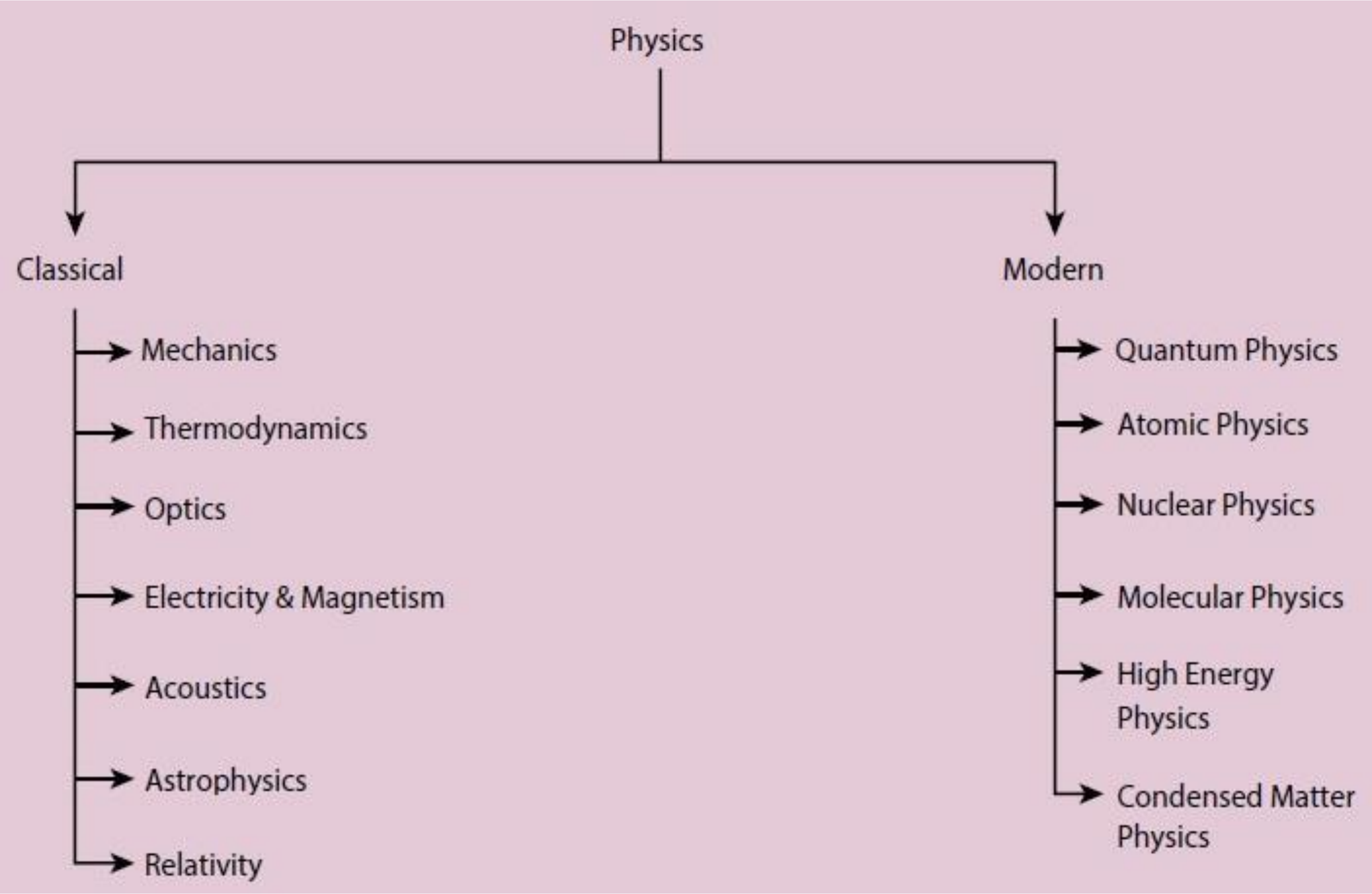
- **Combined direct and bending stresses:** introduction, resultant stress when a column of rectangular section is subjected to an eccentric load, stress distribution due to the position of the applied load, resultant stress when a column of rectangular section is subjected to a load which is eccentric on both axes. Numerical problems.
- **Deflection of beams:** introduction, relationships between loading, shear force (SF), bending moment (BM), slope and deflection of beam, formulae for slope and deflection of beams. Numerical problems.
- **Torsion:** introduction, pure torsion, polar moment of inertia, torsion rigidity, power transmitted by a shaft, composite shafts. Numerical problems.

Introduction to Mechanics

- **What is mechanics?**
  - Physical science deals with the state of rest or motion of bodies under the action of force.

- **Why we study mechanics?**
  - This science forms the groundwork for further study in the design and analysis of structures.

Introduction to Mechanics



Introduction to Mechanics

Classical Physics		Refers to traditional physics that was recognized and developed before the beginning of the 20^{th} century
	Branch	Major focus
1.	Classical mechanics	The study of forces acting on bodies whether at rest or in motion
2.	Thermodynamics	The study of the relationship between heat and other forms of energy
3.	Optics	The study of light
4.	Electricity and magnetism	The study of electricity and magnetism and their mutual relationship
5.	Acoustics	The study of the production and propagation of sound waves
6.	Astrophysics	The branch of physics which deals with the study of the physics of astronomical bodies
7.	Relativity	One of the branches of theoretical physics which deals with the relationship between space, time and energy particularly with objects moving in different ways .

Introduction to Mechanics

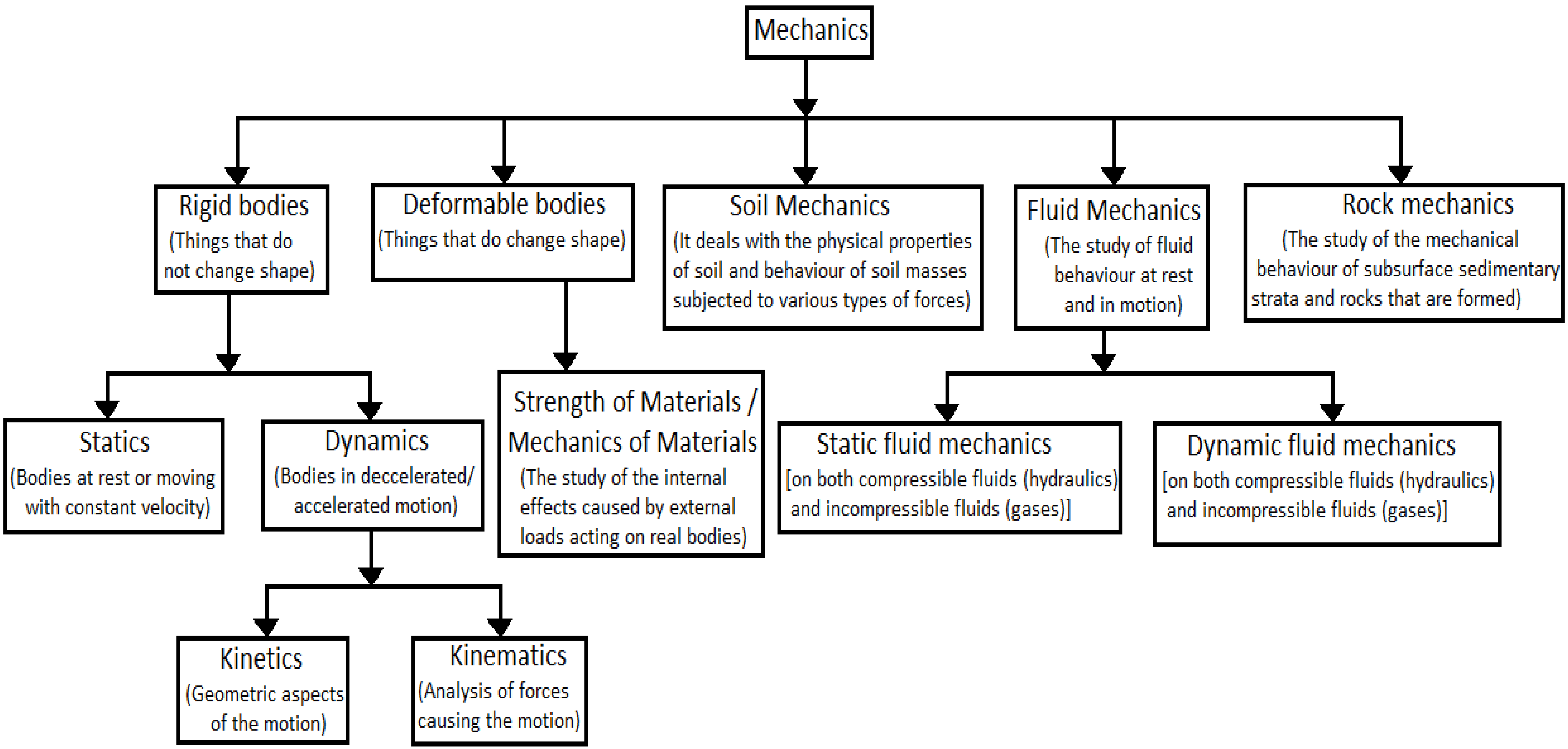
Modern Physics		Refers to the concepts in physics that have surfaced since the beginning of the 20^{th} century.
1.	*Quantum mechanics	The study of the discrete nature of phenomena at the atomic and subatomic levels
2.	Atomic physics	The branch of physics which deals with the structure and properties of the atom
3.	Nuclear physics	The branch of physics which deals with the structure, properties and reaction of the nuclei of atoms.
4.	Condensed matter physics	The study of the properties of condensed materials (solids, liquids and those intermediate between them and dense gas). It branches into various sub-divisions including developing fields such as nano science, photonics etc. It covers the basics of materials science, which aims at developing new material with better properties for promising applications.
5.	High energy physics	The study of the nature of the particles.

*Quantum mechanics is a broader approach; classical results can be reproduced in quantum mechanics also.

PARTS OF MECHANICS

Mechanics is the physical science which studies the effects of forces on objects.

What happens to a body when forces are applied to it?



Engineering Mechanics

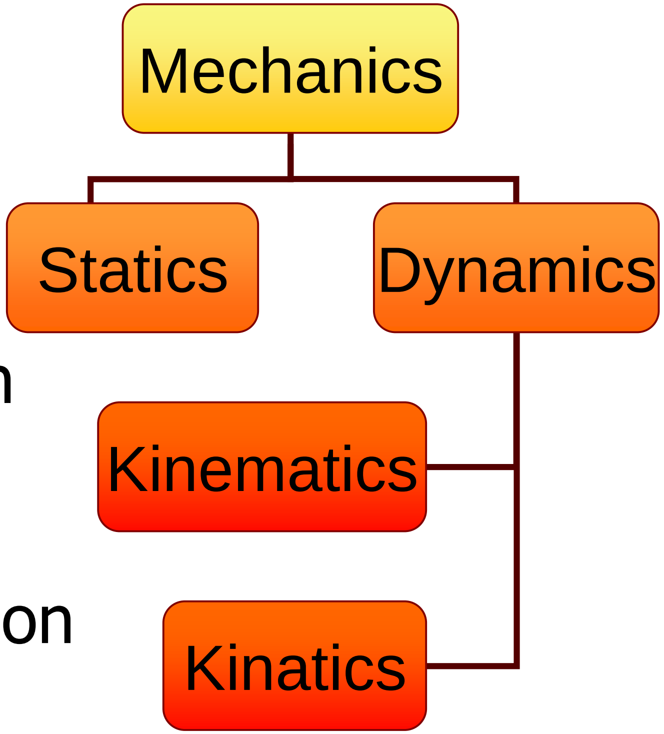
Statics:

Equilibrium of bodies at rest or with constant velocity

Dynamics:

Equilibrium of bodies moving with acceleration

Dynamics



Kinematics:

Treats geometric aspects of the motion

Kinetics:

Analysis of the forces causing the motion

Basic Terms

- Essential basic terms to be understood
  - **Statics:** dealing with the equilibrium of a rigid-body at rest
  - **Rigid body:** the relative movement between its parts are negligible
  - **Dynamics:** dealing with a rigid-body in motion
  - **Length:** applied to the linear dimension of a strait line or curved line
  - **Area:** the two dimensional size of shape or surface
  - **Volume:** the three dimensional size of the space occupied by substance

Basic Terms

- Essential basic terms to be understood
  - **Force:** the action of one body on another whether it's a push or a pull
  - **Mass:** the amount of matter in a body
  - **Weight:** the force with which a body is attracted toward the centre of the Earth
  - **Particle:** a body of negligible dimension

Fundamental Concepts

- ***Space*** - associated with the notion of the position of a point P given in terms of three coordinates measured from a reference point or origin.

- ***Time*** - definition of an event requires specification of the time and position at which it occurred.

- ***Mass*** - used to characterize and compare bodies, e.g., response to earth's gravitational attraction and resistance to changes in translational motion.

Fundamental Concepts

- ***Force*** - represents the action of one body on another. A force is characterized by its point of application, magnitude, and direction, i.e., a force is a **vector** quantity.

In Newtonian Mechanics, space, time, and mass are **absolute concepts**, independent of each other. Force, however, is not independent of the other three. The force acting on a body is related to the mass of the body and the variation of its velocity with time.

Problem solving

1- Read and correlate physical situation with theory studied

2- Draw necessary diagrams and tabulate data

3- Establish a coordinate system and apply the relevant principles in a mathematical form

4- Solve the necessary equations algebraically as far as practical

5- Study the answer using technical judgment and common sense to determine whether or not it seems reasonable

6- Review problem, try to think

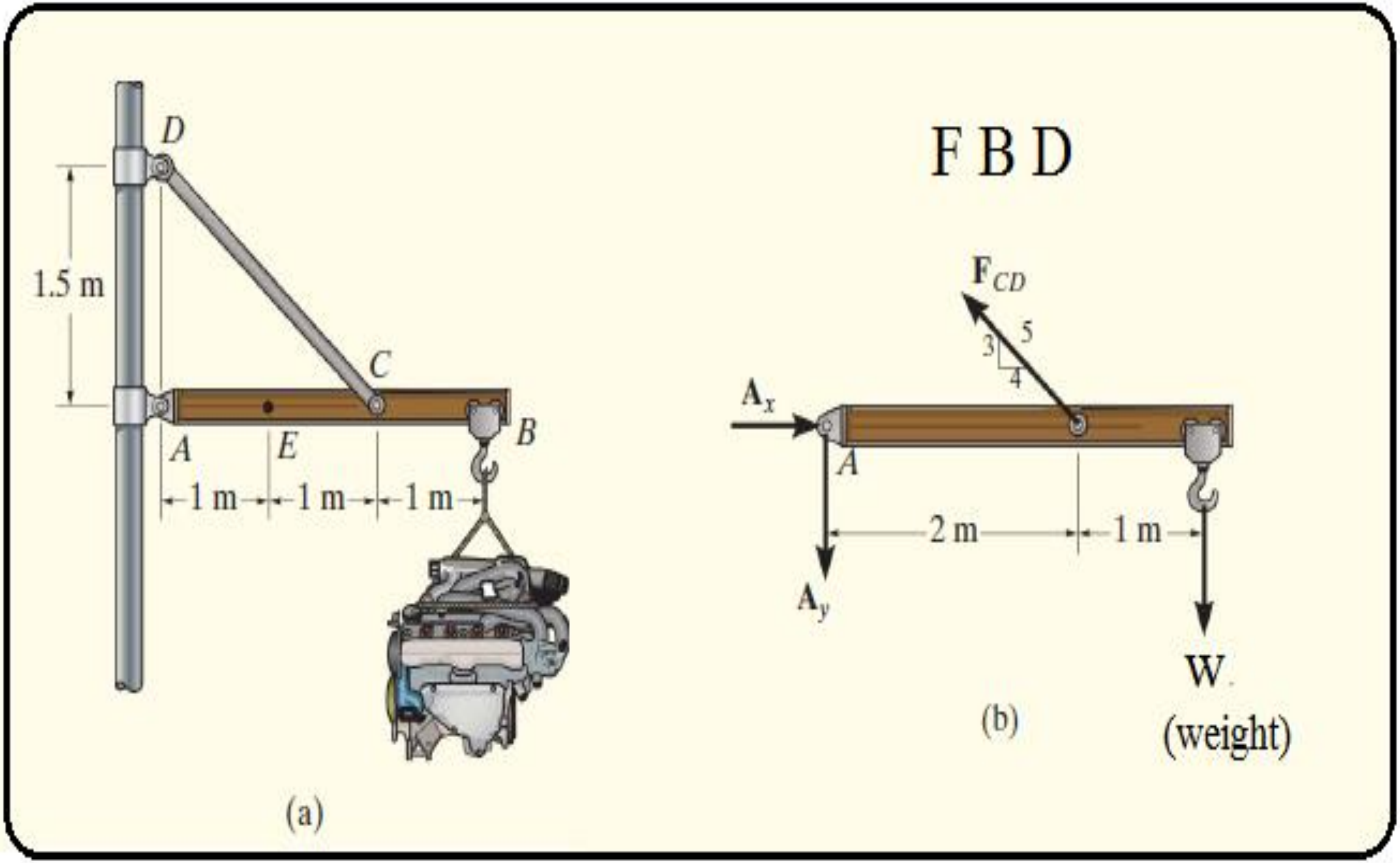
Method of Problem Solution

- ***Problem Statement*** **:**
  Includes given data, specification of what is to be determined, and a figure showing all quantities involved.

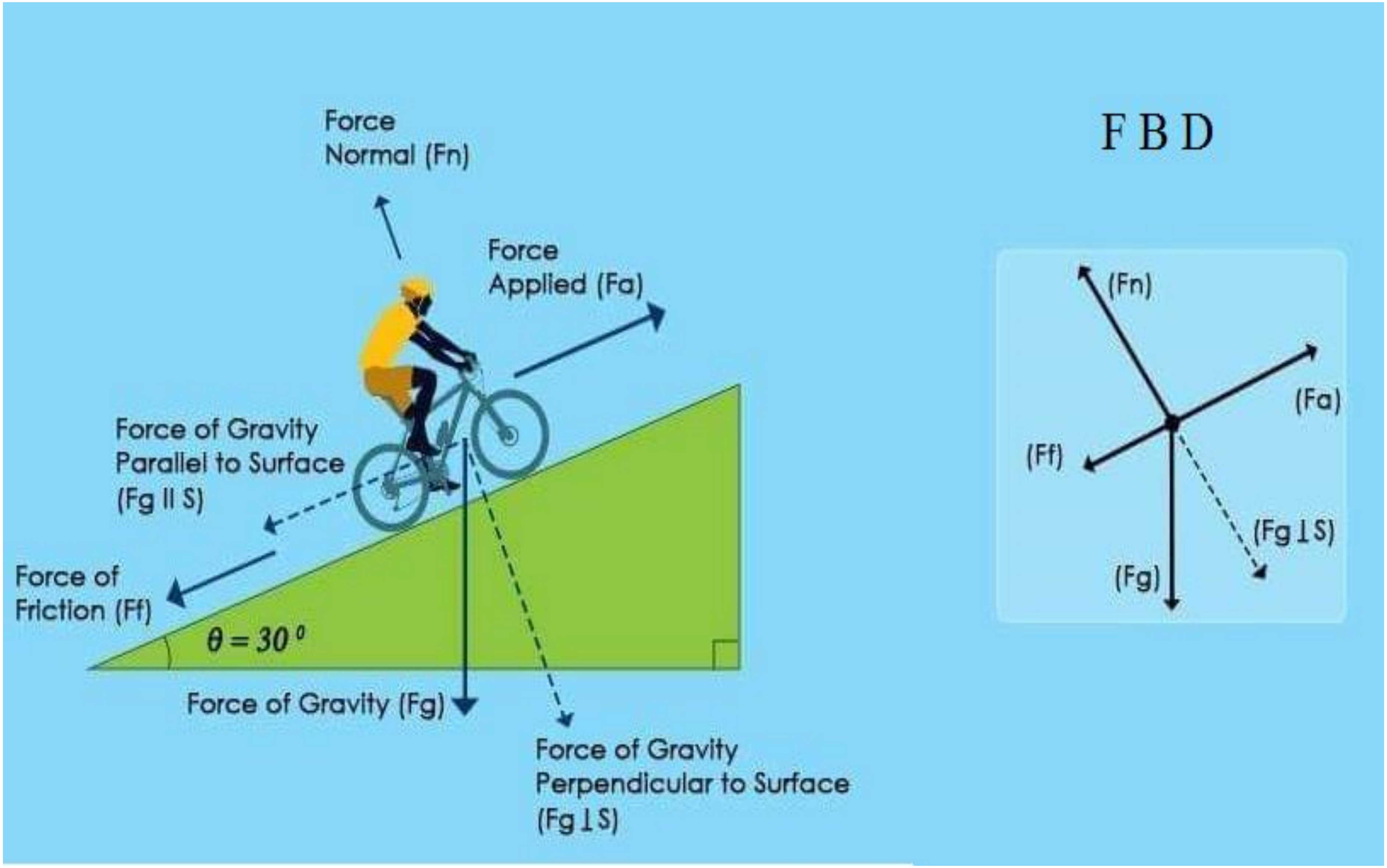
- ***Free-Body Diagrams*** **:**
  Create separate diagrams for each of the bodies involved with a clear indication of all forces acting on each body.

- ***Fundamental Principles*** **:**
  The six fundamental principles are applied to express the conditions of rest or motion of each body. The rules of algebra are applied to solve the equations for the unknown quantities.

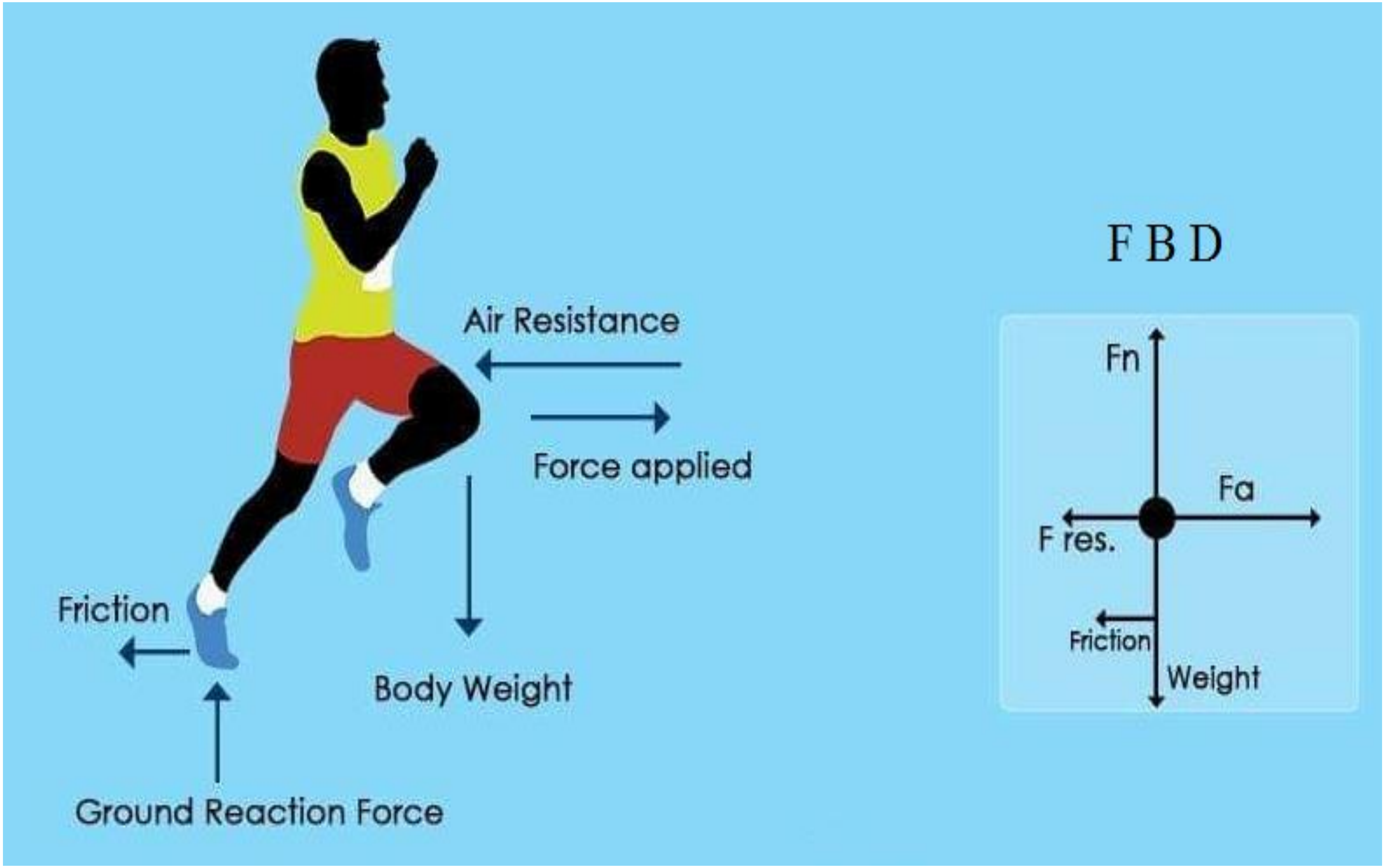
Method of Problem Solution



Method of Problem Solution



Method of Problem Solution



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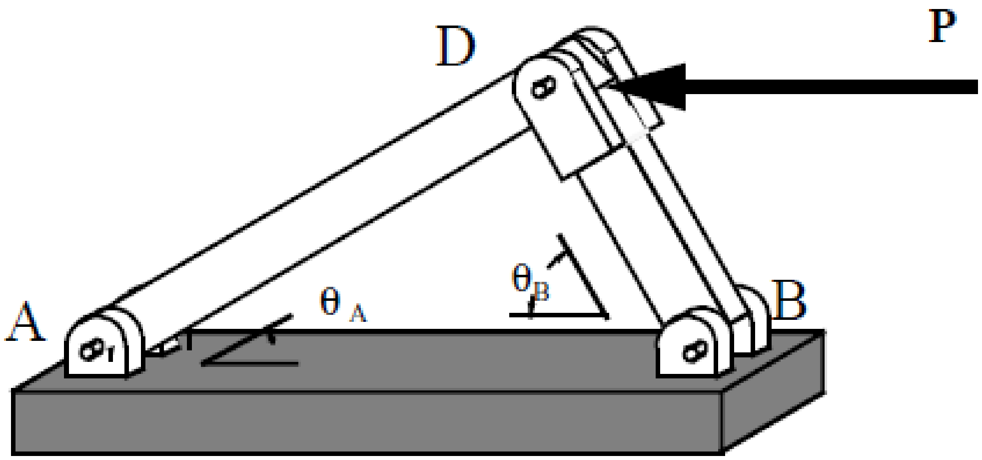
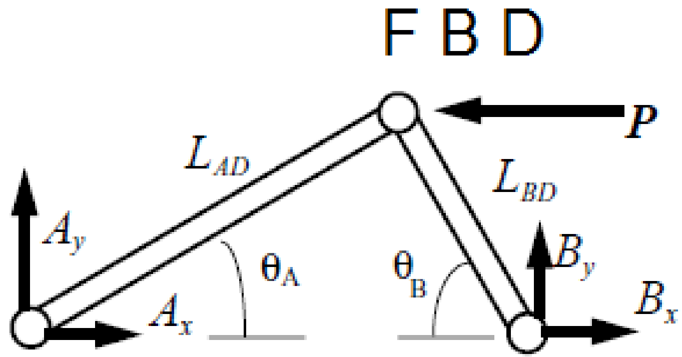
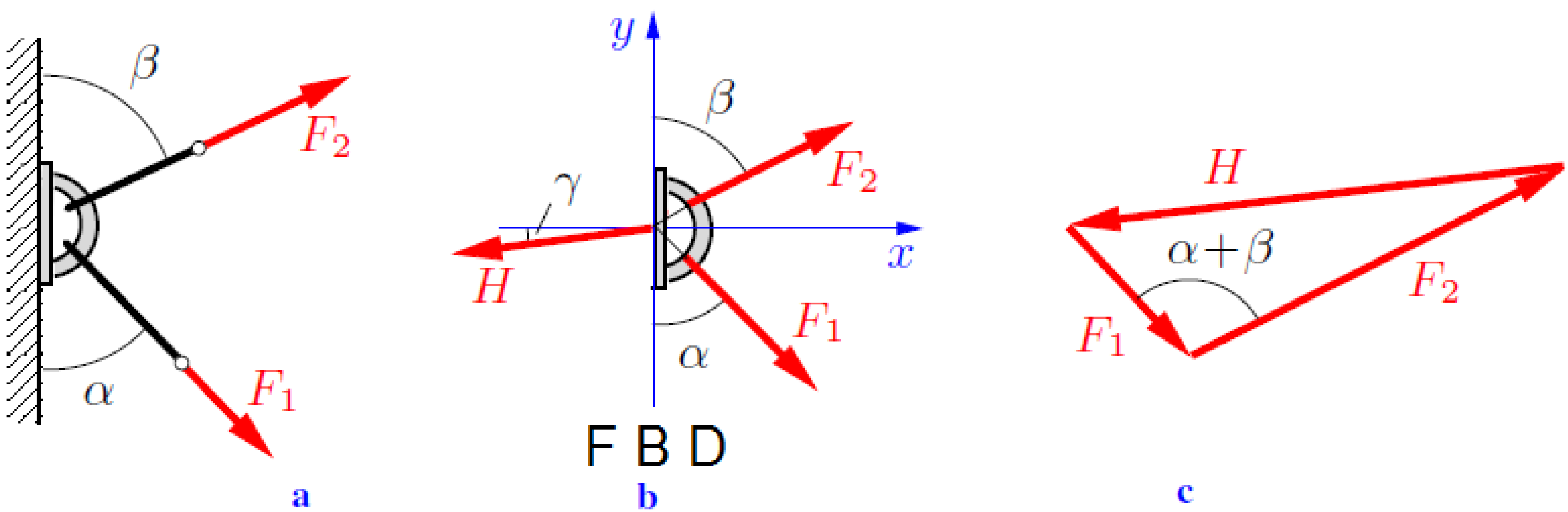
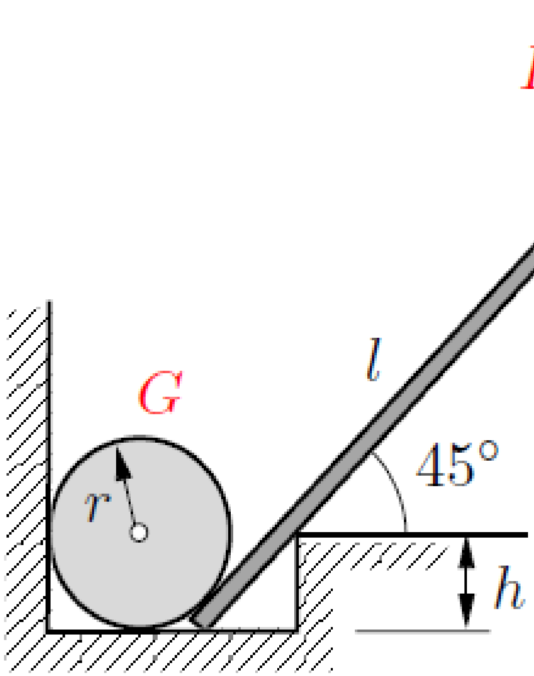
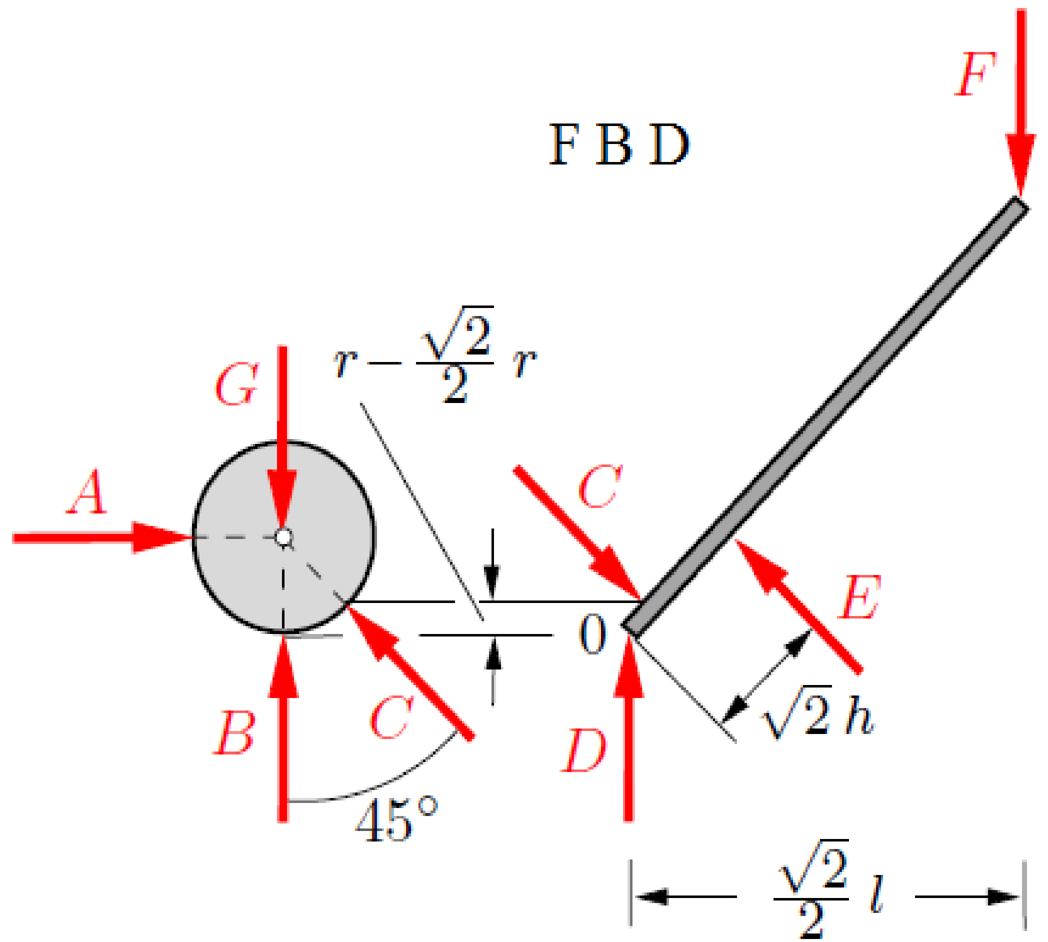






Method of Problem Solution





Method of Problem Solution

- ***Solution Check*:**
  -Test for errors in reasoning by verifying that the units of the computed results are correct,
  - Test for errors in computation by substituting given data and computed results into previously unused equations based on the six principles,
  - **<u>Always</u>** apply experience and physical intuition to assess whether results seem "reasonable"

Numerical Accuracy

- The accuracy of a solution depends on 1) accuracy of the given data, and 2) accuracy of the computations performed. The solution cannot be more accurate than the less accurate of these two.
  - The use of hand calculators and computers generally makes the accuracy of the computations much greater than the accuracy of the data. Hence, the solution accuracy is usually limited by the data accuracy.
- As a general rule for engineering problems, the data are seldom known with an accuracy greater than 0.2%. Therefore, it is usually appropriate to record parameters beginning with "1" with four digits and with three digits in all other cases, i.e., 40.2 lb and 15.58 lb.

Units of Measurement

- Four fundamental quantities in mechanics
  - Mass
  - Length
  - Time
  - Force
- Two different systems of units we dealing with during the course
  - U.S. Customary or British System of Units (FPS)
    - Length in feet (ft)
    - Time in Seconds (sec)
    - Force in Pounds (lb)
  - International System of Units or Metric Units (SI)
    - Length in metre (m)
    - Time in Seconds (s)
    - Force in Newton (N)

Units of Measurement

Summary of the four fundamental quantities in the two systems

Quantity	SI Units		FPS Units	
	Unit	Symbol	Unit	Symbol
Mass	kilogram	kg	slug	-
Length	meter	m	foot	ft
Time	second	s	second	sec
Force	newton	N	pound	lb

Units of Measurement

- **Comparison between two system of units (SI) and (FPS)**
  - SI System offers major advantages relative to the FPS system
    - Widely used throughout the world
    - Use one basic unit for length ☞ meter
    - SI based on multiples of 10, which makes it easier to use & learn
  - FPS uses many basic units ☞ inch, foot, yard, mile
    - FPS is complicated, for example;
      - SI system ➔ 1 meter = 100 centimeters, 1 kilometer = 1000 meters, etc.
      - FPS system ➔ 1 foot = 12 inches, 1 yard = 3 feet, 1 mile = 5280 feet, etc.

Units of Measurement

❑Comparison between two system of units (SI) and (FPS)

➢Metric System (SI)

- Newton's second law F = m x a
  - Thus the force (N) = mass (kg) × acceleration (m/s^2)
- Therefore 1 Newton is the force required to give a mass of 1 kg an acceleration of 1 m/s^2

➢U.S. Customary System (FPS)

- Force (lb) = mass (slugs) × acceleration (ft/sec^2)
  - Thus (slugs) = $lb.sec^2/ft$
- Therefore 1 slug is the mass which is given an acceleration of 1 ft/sec^2 when acted upon by a force of 1lb.

Units of Measurement

❑Conversion of Units

- Converting from one system of unit to another;

Quantity	FPS	Equals	SI
Force	**1 lb**	→	**4.448 N**
Mass	**1 slug**	→	**14.593 kg**
Length	**1 ft**	→	**0.304 m**

❑The standard value of g (gravitational acceleration)

- SI units $g = 9.806\ m/s^2$
- FPS units $g = 32.174\ ft/sec^2$

WHAT IS FORCE?

WHAT IS FORCE?

A <u>force</u> is a push or a pull !

When one object pushes or pulls another object, you say that the first object is exerting force on the second object.

Examples:

**You exert a force on a pen when you write

** On a book when you lift it

** On a ball when you throw it

** On a nail when you hammer it into a piece of wood

Force

- Force: a push or pull acting on a body that causes or tends to cause a change in the linear motion of the body.
  - Characteristics of a force
    - magnitude
    - direction
    - point of application.
    - line of action
    - *sense (push or pull along the line of action)*
- Net Force: resultant force (overall effect of multiple forces acting on a body)
  - Example: push from side and front = at angle

Vector is represented with an arrow

Force

- Force may also be defined as the product of a body's mass and the acceleration of that body resulting from the application of the force.

$$F = ma$$

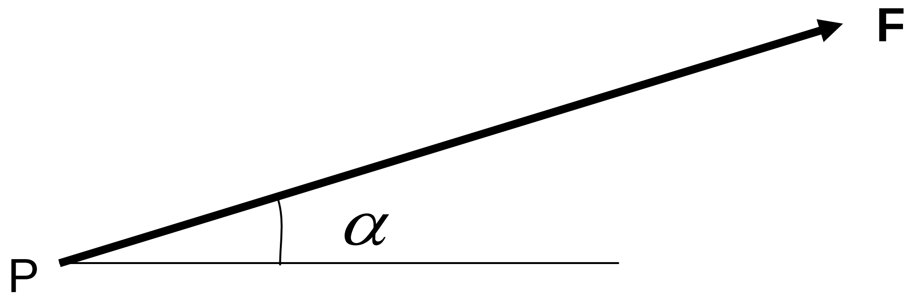
- Units of force are units of mass multiplied by units of acceleration.

Units of Force

- Metric system (système internationale -SI)
  - Newton (N)
    - the amount of force necessary to accelerate a mass of 1 kg at 1 m/s^2
- American/English system
  - pound (lb)
    - the amount of force necessary to accelerate a mass of 1 slug at 1 ft/s^2
    - equal to 4.45 N

FORCE ON A PARTICLE

- A Force is a Vector quantity and must have Magnitude, Direction and Line of action.



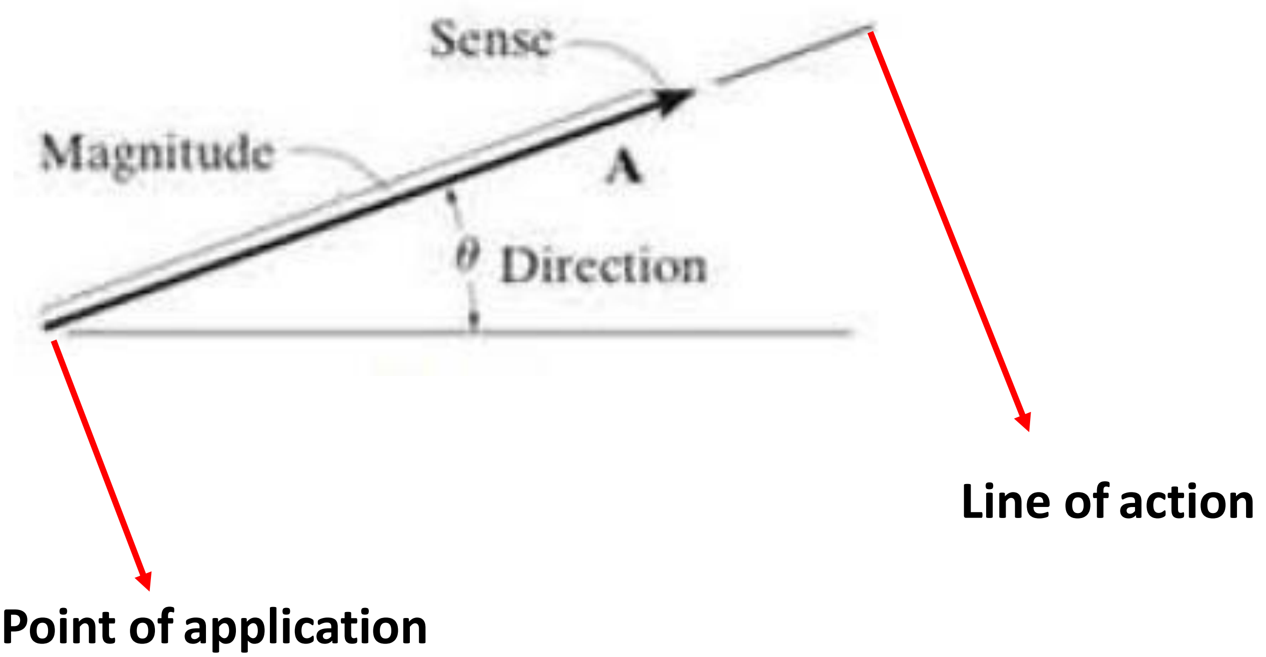
- **<u>Note:</u>** Point P is the point of application of force and PF and α are directions.

  To notify that **F** is a vector, it is printed in bold as in the text book.

- Its magnitude is denoted as |F| or simply F.

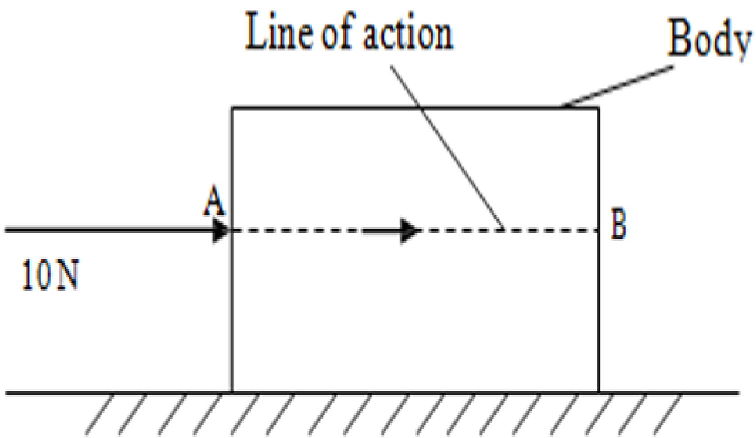
FORCE ON A PARTICLE

- A Force is a Vector quantity and must have Magnitude, Direction and Line of action.



Examples

Ex 1. Consider a body being pushed by a force of 10 N as shown in figure below.

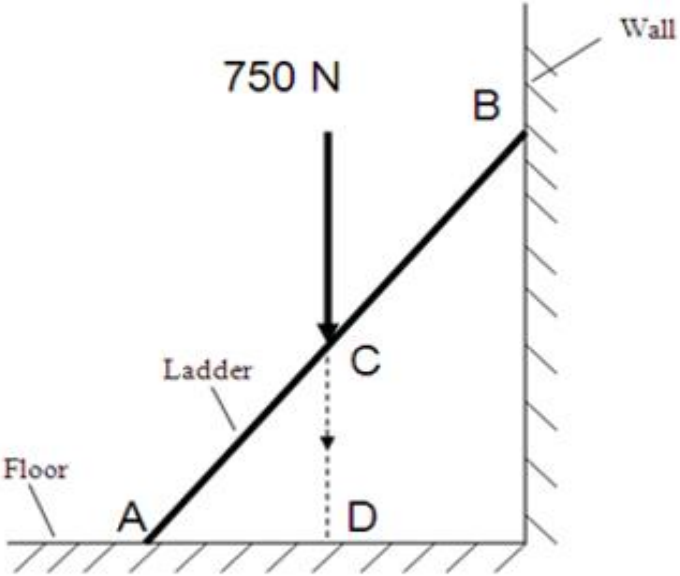


The characteristics of the force acting on the body are.

1. Magnitude is 10 N.
2. Point of application is A.
3. Line of action is A to B or AB.
4. Direction is horizontally from left to right.

Examples

Ex 2. Consider a ladder AB resting on a floor and leaning against a wall, on which a person weighing 750 N stands on the ladder at point C.



The characteristics of the force acting on the body are.

1. Magnitude is 750N.
2. Point of application is C.
3. Line of action is C to D or CD.
4. Direction is vertically downward.

Classifying Forces

- Internal Force: acts within the object or system whose motion is being investigated.
  - action / reaction forces both act on different parts of the system
    - **tensile**-internal *pulling* (**stretching**) forces when the structure is under compressive load.
    - **compressive**- internal *pushing* (**squeezing**) forces act on the ends of an internal structure when the is under tensile load.
  - do not accelerate the body
    - Orientate segments, maintain structural integrity
- Examples
  - Contraction of muscles
  - Do not accelerate the body

Classifying Forces

- External Force: acts on object as a result of interaction with the environment surrounding it.
  - **non-contact** - occur even if objects are not touching each other
    - gravity, magnetic
  - **contact** - occur between objects in contact
    - fluid (air & water resistance)
    - *reaction forces with another body* (ground, implement)
    - Vertical (normal) reaction force
      - acts perpendicular to bodies in contact
    - shear reaction force
      - acts parallel to surfaces in contact (friction)

Weight (external force)

- Weight - the amount of gravitational force exerted on a body. $Wt = ma_g$.
- Since weight is a force, units of weight are units of force - either N or lb.
- As the mass of a body increases, its weight increases proportionally.

Weight

- The factor of proportionality is the acceleration of gravity, which is -9.81m/s^2 or - 32 ft/s^2.
- The negative sign indicates that the acceleration of gravity is directed downward or toward the center of the earth.
- On the moon or another planet with a different gravitational acceleration, a body's weight would be different, although its mass would remain the same.

Weight

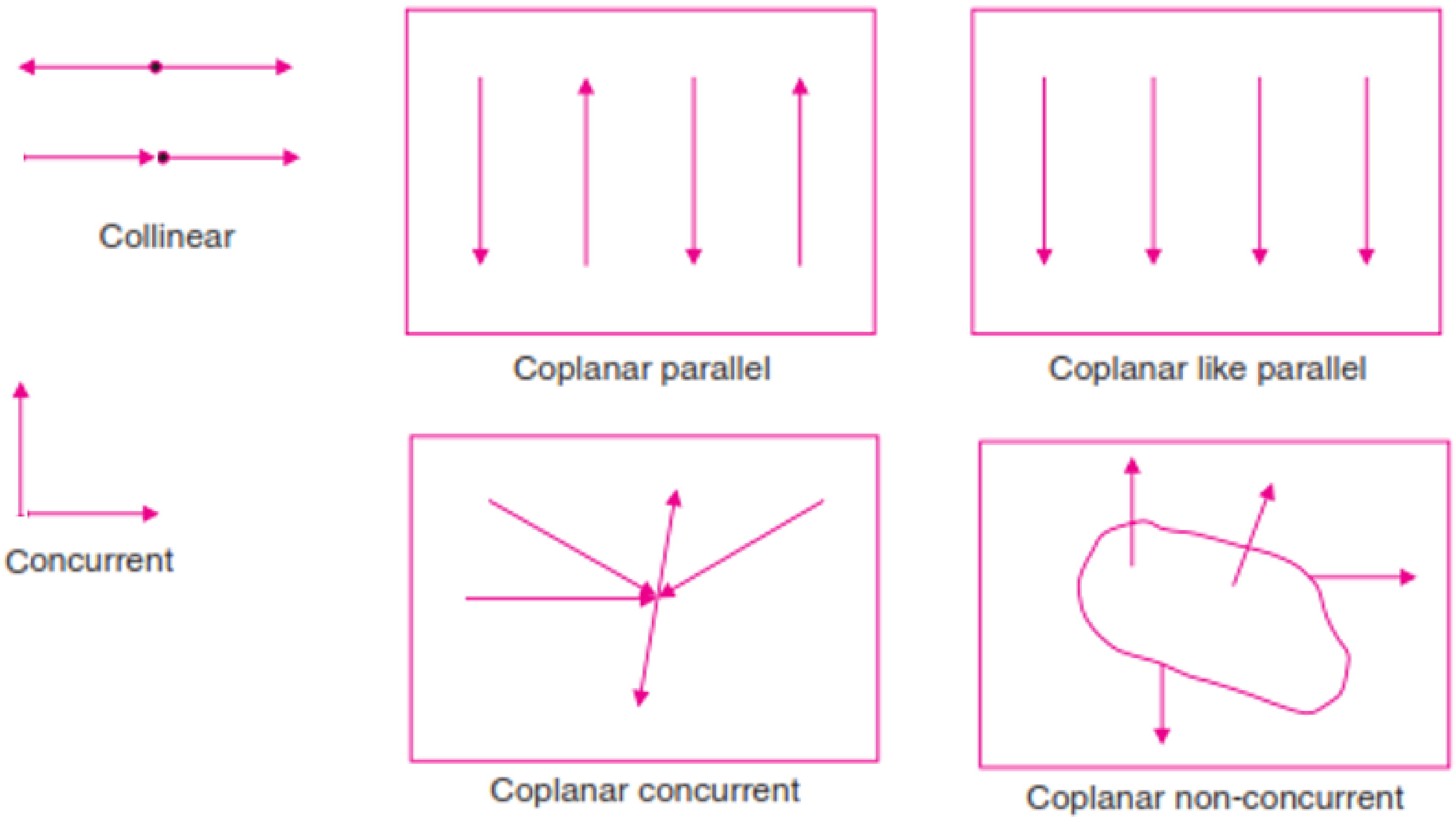
- Because weight is a force, it is also characterized by magnitude, direction, and point of application.
- The direction in which weight acts is always toward the center of the earth.
- The point at which weight is assumed to act on a body is the body's **center of gravity**.

System of forces

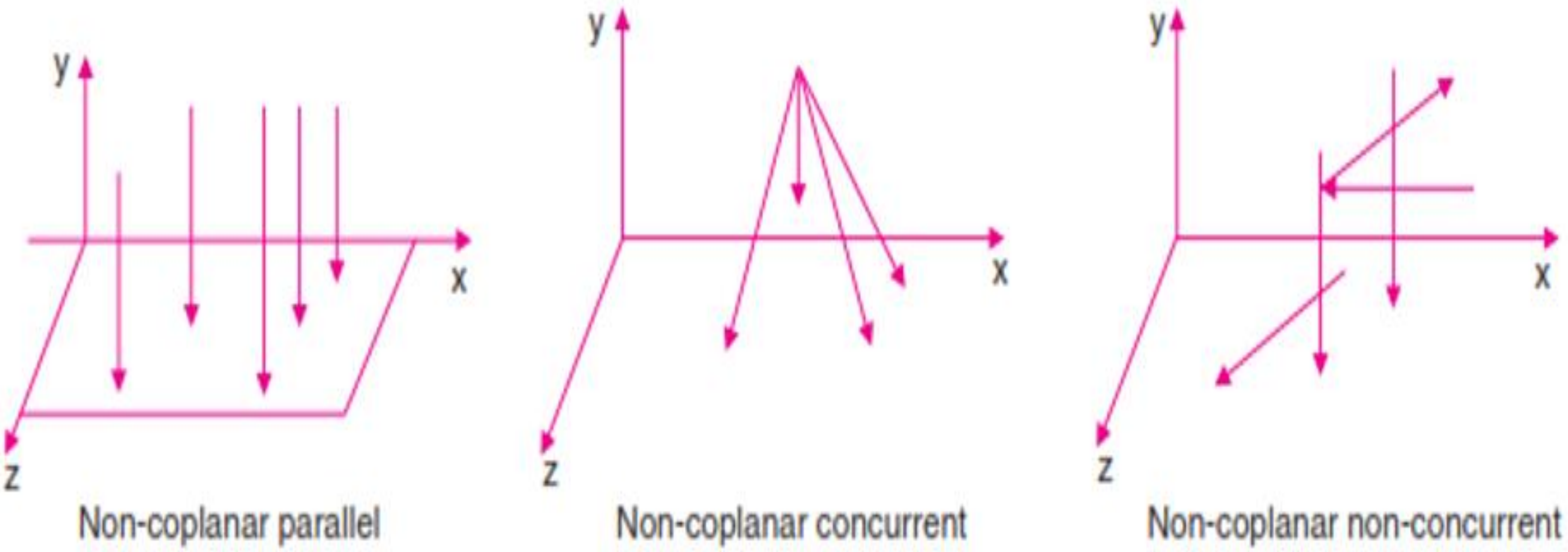
- When several forces act simultaneously on a body, they constitute a **system of forces**.

- A **group** or **set** of **forces** is called **system of forces.**

Classifications of System of forces



Classifications of System of forces



Classifications of System of forces

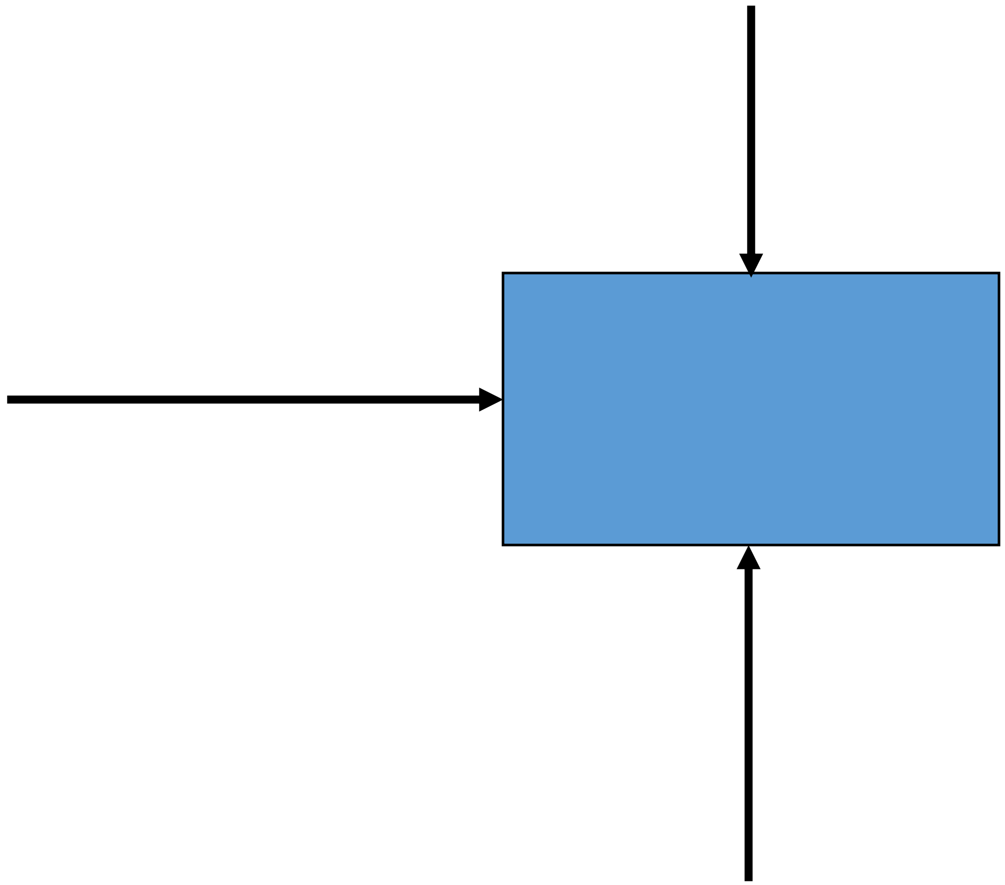
Force System	*Characteristics*
Collinear forces	Line of action of all the forces act along the same line.
Concurrent forces	All forces do not act on the same line of action, but do act through the same point.
Coplanar parallel forces	All forces are parallel to each other and lie in a single plane.
Coplanar like parallel forces	All forces are parallel to each other, lie in a single plane and are acting in the same direction.
Coplanar concurrent forces	Line of action of all forces pass through a single point and forces lie in the same plane.

Classifications of System of forces

Force System	*Characteristics*
Coplanar non-concurrent forces	All forces do not meet at a point, but lie in a single plane.
Non-coplanar parallel forces	All the forces are parallel to each other, but not in same plane.
Non-coplanar concurrent forces	All forces do not lie in the same plane, but their lines of action pass through a single point.
Non-coplanar non-concurrent forces	All forces do not lie in the same plane and their lines of action do not pass through a single point.

Recall

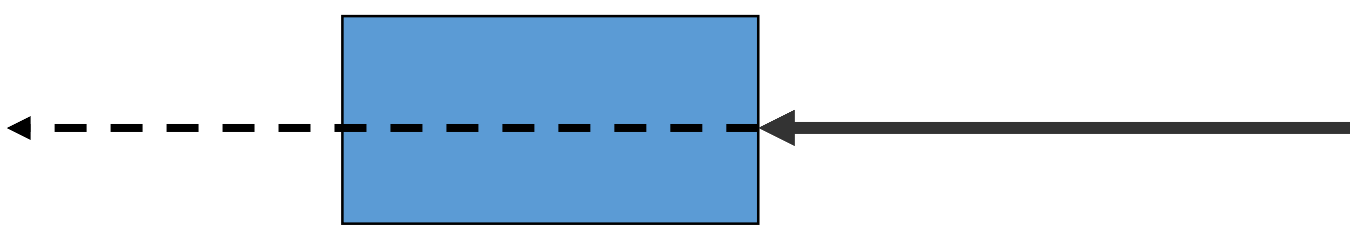
- Concept of *Net External Force*



Must add all forces acting on an object together.

Free body diagram

- Free body diagram - sketch that shows a defined system in isolation with all the force *vectors* acting on the system
  - defined system: the body of interest
  - vector: arrow to represent a force
    - length: size of the force
    - tip: indicates direction
    - location: point of application



Free body diagram

- ✓at rest in front of class
- What are the names of the forces?
- How big are the forces?
- What direction are the forces?
- Where are the forces applied?

Addition of Forces

(calculating the net [resultant] force)

- Net force = *vector* sum of all external forces acting on the object (body)
  - account for magnitude and *direction*
  - e.g., Add force of 100N and 200N

Addition of Forces

(calculating the net [resultant] force)

- Net force = *vector* sum of all external forces acting on the object (body)
  - account for magnitude and *direction*
  - e.g., Add force of 100N and 200N
    - act in same direction???
    - Act in opposite direction???
    - Act orthogonal to each other???
    - Act at angles to each other???

Typical collinear forces

- Forces have the same line of action
- May act in same or different directions
  - i.e., tug of war teammates: 100N, 200N, 400N
    - show force on rope graphically

Typical collinear forces

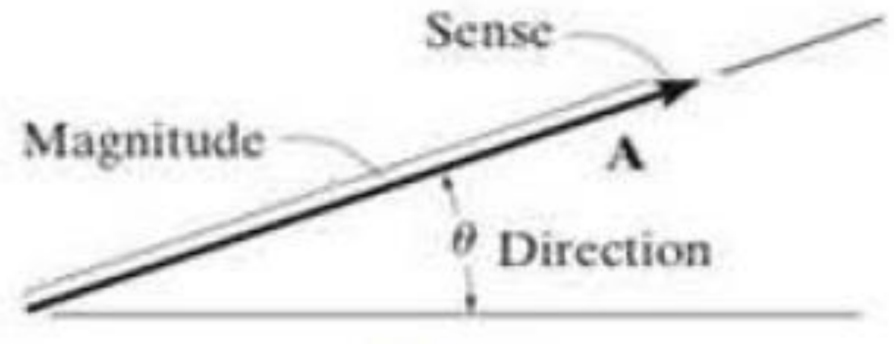
- Forces have the same line of action
- May act in same or different directions
  - ie tug of war teammates: 100N, 200N, 400N
  - tug of war opponents: 200N, 200N, 200N
  - calculate resultant of the two teams
    - show force on rope graphically
    - calculate algebraically

Scalars and Vectors

All physical quantities in engineering mechanics are measured using either scalars or vectors.

Scalar. A scalar is any positive and negative physical quantity that can be completely specify by its magnitude. *Examples of scalar quantities* include length, mass, and time.

Vector. A vector is any physical quantity that requires both a magnitude *and a direction for its complete description.* Examples of vectors encountered in statics arc force. position. and moment.



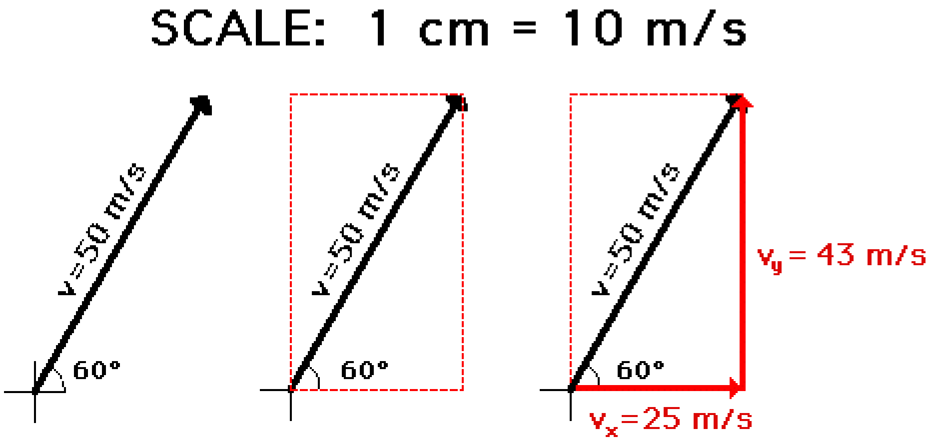
Quantifying vectors

- Vector composition - process of determining a single vector from 2 or more vectors through vector addition.

- Resultant - single vector that results from vector composition.

Quantifying vectors

- Vector resolution - breaking down a resultant vector into its horizontal and vertical components.
  - Graphic method.
  - Trigonometric method.

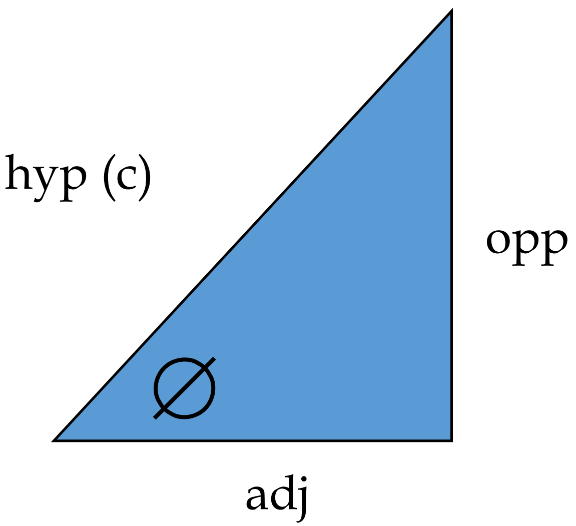


Graphic and Trigonometric Methods

Recall: Trigonometry

Key Equations:

- sin ∅ = opp/hyp
- cos ∅ = adj/hyp
- tan ∅ = opp/adj



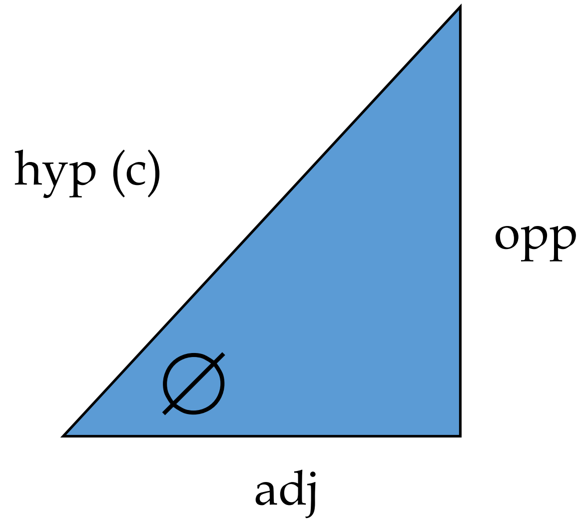
- Pythagoras theorem
- $a^2 + b^2 = c^2$

Vector Composition

Sample:

∅ = 30 degrees

- adj = 100 N
- Find opp and hyp

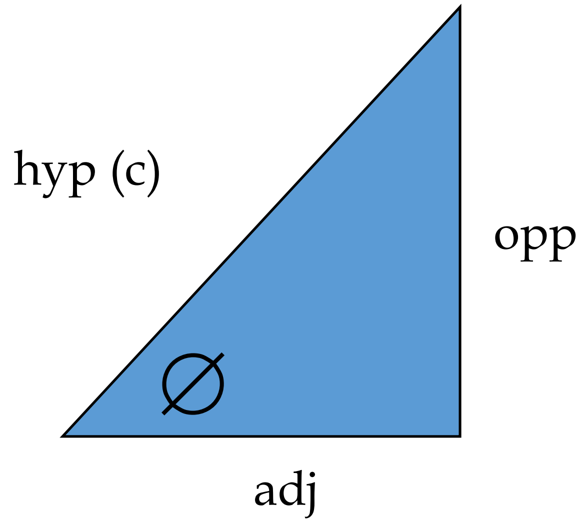


- use
- cos ∅ = adj/hyp
- tan ∅ = opp/adj

Vector Composition

- cos ∅ = adj/hyp
- cos 30 = 100/hyp
- hyp = 100/.866
- hyp = 115.47 N

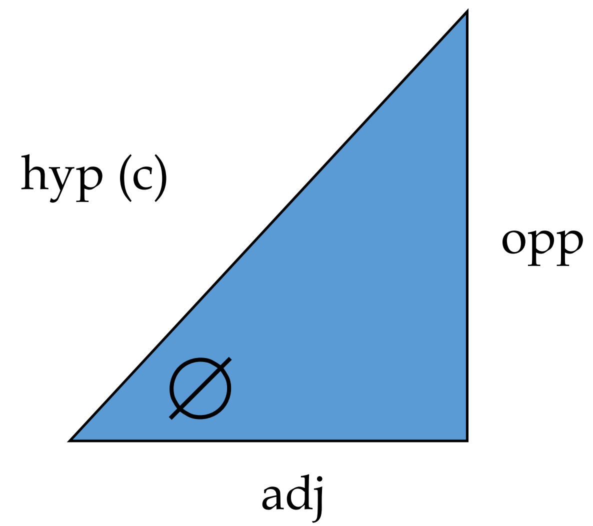
- tan ∅ = opp/adj
- tan 30 = opp/100
- opp = .5774 x 100
- opp = 57.74 N



Vector Resolution

Sample

- ∅ = 35 degrees
- hyp = 120 N
- Find opp and adj



Trigonometric Calculations

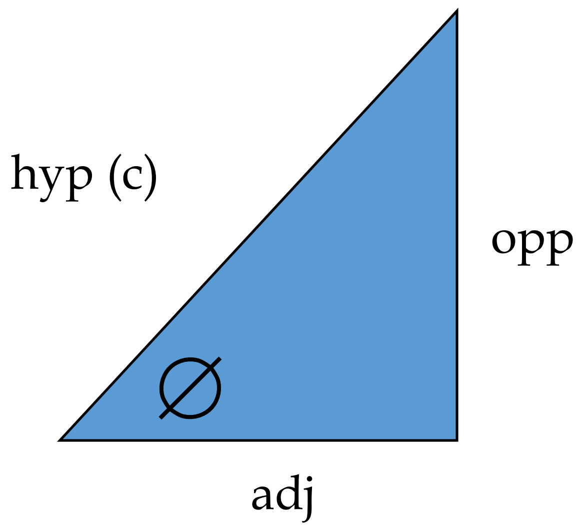
Use

- sin ∅ = opp/hyp
- cos ∅ = adj/hyp

Vector Resolution

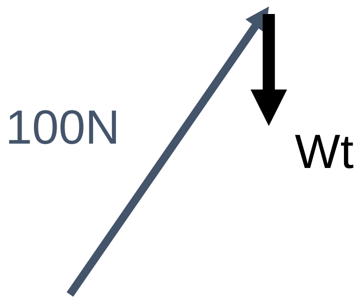
- sin ∅ = opp/hyp
- sin 35 = opp/120
- opp = 120 X .5736
- opp = 68.83 N

- cos ∅ = adj/hyp
- cos 35 = adj/120
- adj = 120 X .8192
- adj = 98.30 N



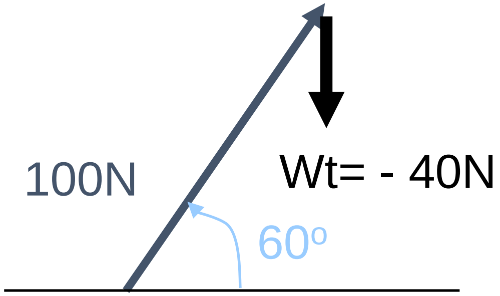
Resolution of Forces

- Forces are *not* collinear and *not* Horizontal & Vertical?
  - i.e., figure 1.1: forces on shot
    - 100 N from shot-putters hand
    - mass of shot = 4 kg
  - What is the resultant force on shot?
    - Draw *Components* of 100N force
      - solve graphically: tedious & imprecise
      - *trigonometric* technique



Resolution of Forces

- Forces are *not* collinear and *not* concurrent
  - Forces on shot
    - 100 N from shot-putters hand
    - mass of shot = 4 kg
    - W = mg = (4kg)(-10m/s/s)= -40 N
  - What is the resultant force on shot?
    - Draw *Components* of 100N force
      - solve graphically: tedious & imprecise
      - *trigonometric* technique



Solution

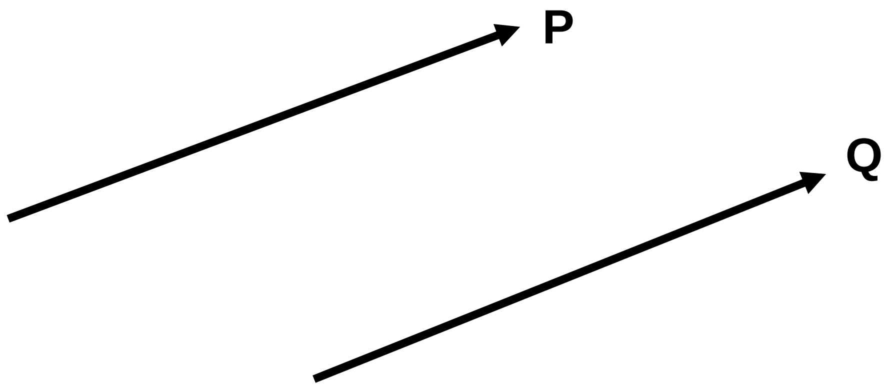
- Given
- Hyp = 100N
- Angle = 60 degrees
- Sin angle = opp/hyp
- Sin 60 = opp/hyp
- Sin 60*hyp = opp
- (0.866)*(100N)= 86.6N

- Given
- Hyp = 100N
- Angle = 60 deg
- Cos angle = adj/hyp
- Cos 60 = adj/hyp
- Cos 60*hyp = adj
- (0.500)*(100N) = 50N

VECTOR OPERATIONS

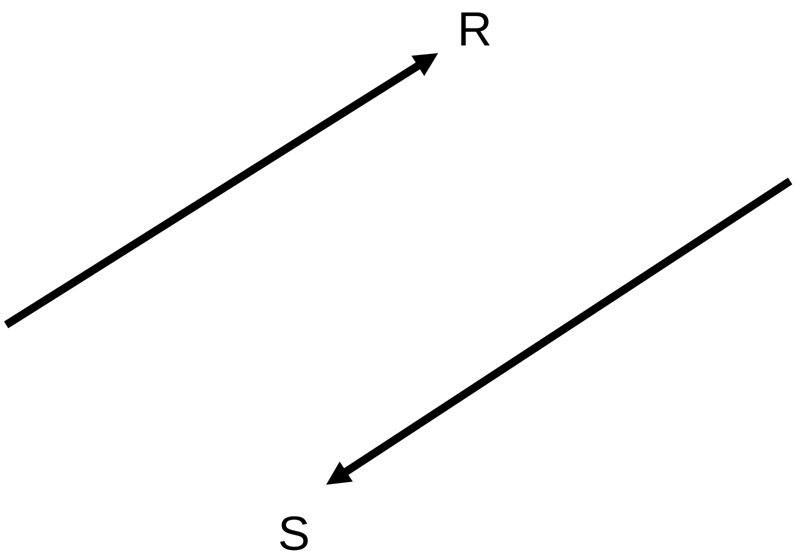
- **EQUAL VECTORS**

Two *vectors* are *equal* if they are equal in *magnitude* and act in the *same* direction.



Equal Vectors Contd.

- Forces equal in Magnitude can act in opposite Directions



Fundamental laws in Engineering Mechanics

- Following are considered as the fundamental laws in Engineering Mechanics.

1) Newton’s Three Fundamental Laws.
2) Principle or Law of transmissibility of forces.
3) Parallelogram law of forces.

Fundamental laws in Engineering Mechanics

- Newton's Three Fundamental Laws.

1st law Every object remains at rest or moves with constant velocity unless an external force is applied on it.

2nd law F = m x a Net force = mass x acceleration.

3rd law For every action there is an equal and opposite reaction.

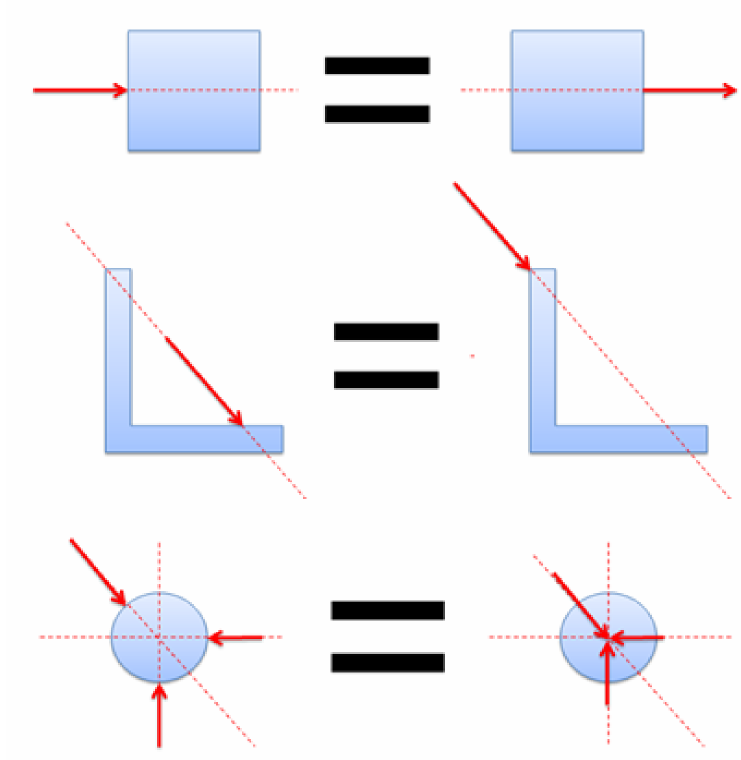
Fundamental laws in Engineering Mechanics

- Principle or law of transmissibility of forces

The principle of transmissibility states that: the point of application of a force can be moved anywhere along its line of action without changing the external reaction forces on a rigid body.

Fundamental laws in Engineering Mechanics

- Principle or law of transmissibility of forces



Fundamental laws in Engineering Mechanics

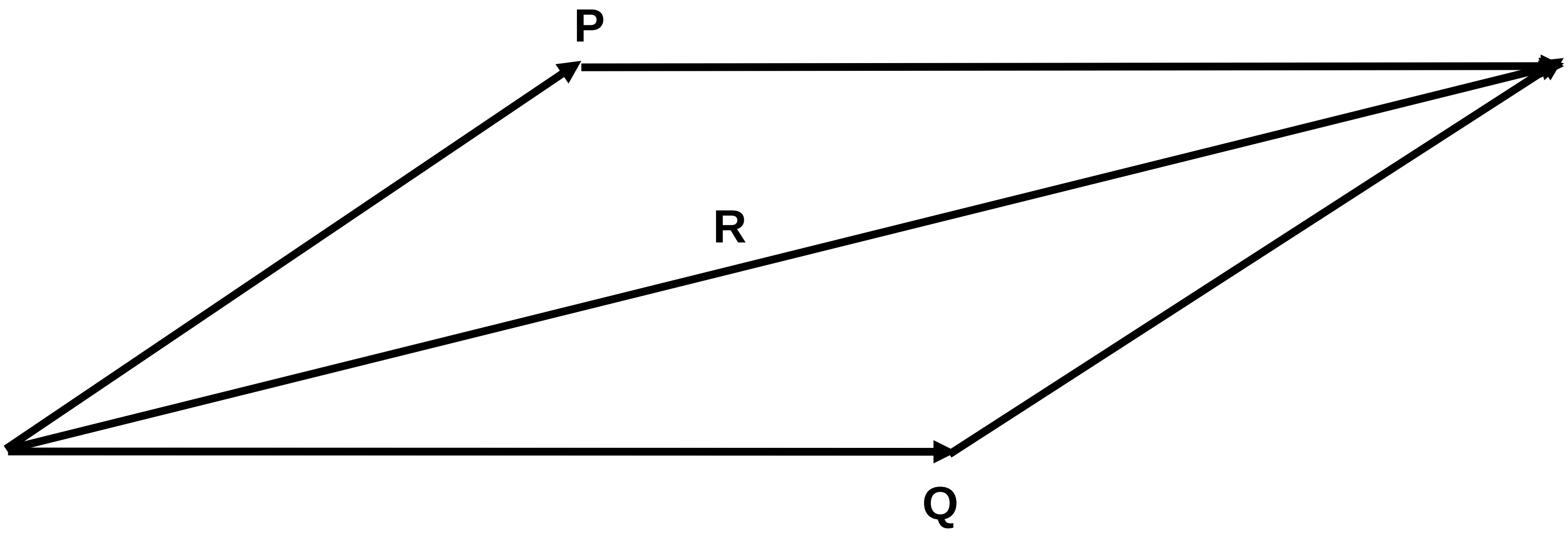
❑Parallelogram law of forces

This law states that: if two forces acting at a point represent both in magnitude and direction by two adjacent sides of a parallelogram, then the resultant of the two forces is represented both in magnitude and direction by the diagonal of the parallelogram passing through the same point.

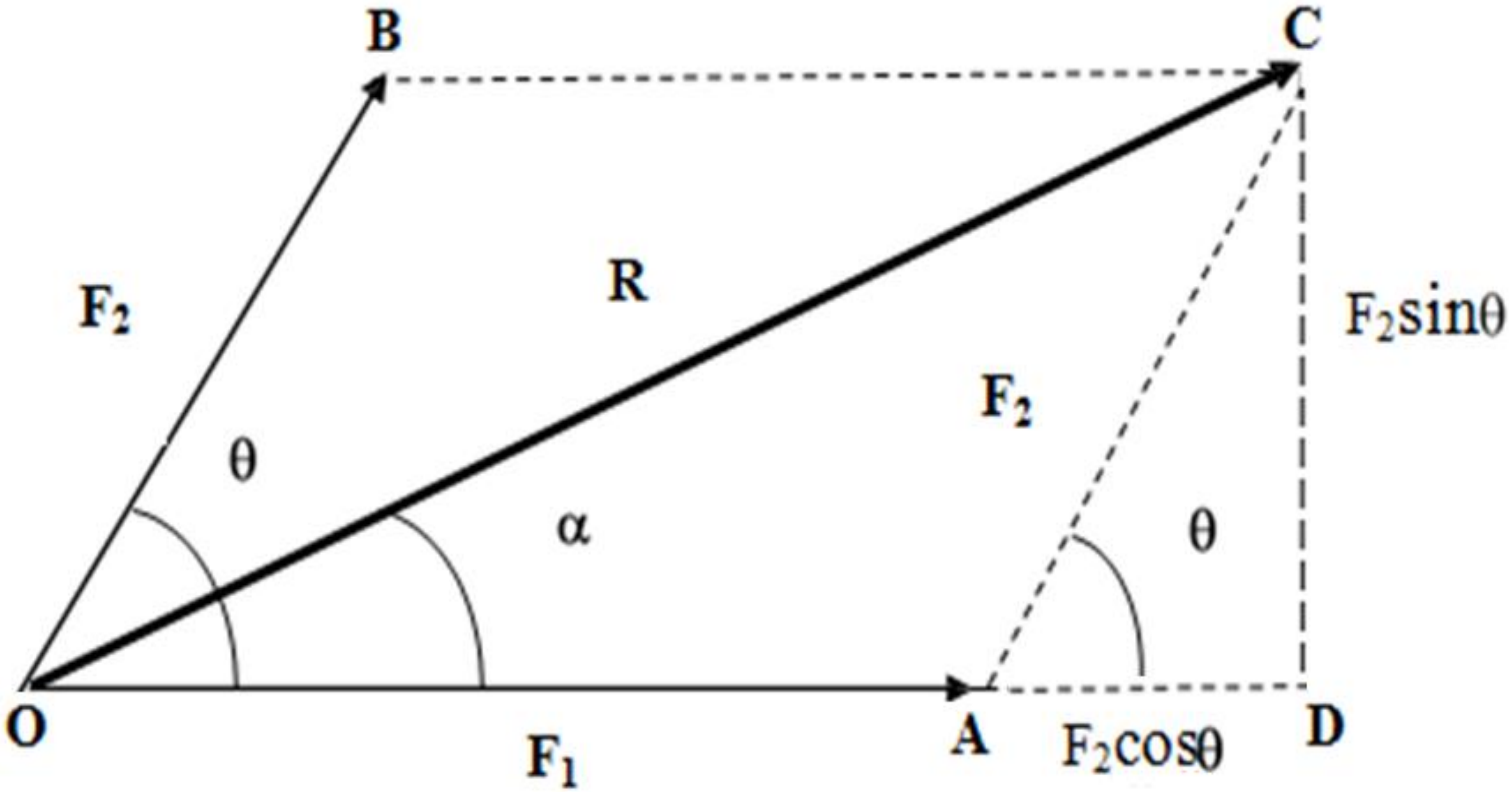
Note: This law is applicable to determine the resultant of two coplanar concurrent forces only.

Parallelogram law of forces

E.g.: Construct a Palm with two Forces as Parts.
The resultant of the forces is the diagonal.



Parallelogram law of forces



Parallelogram law of forces

From the figure $OC = \sqrt{OD^2 + CD^2}$

$$= \sqrt{(OA + AD)^2 + CD^2}$$

$$= \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

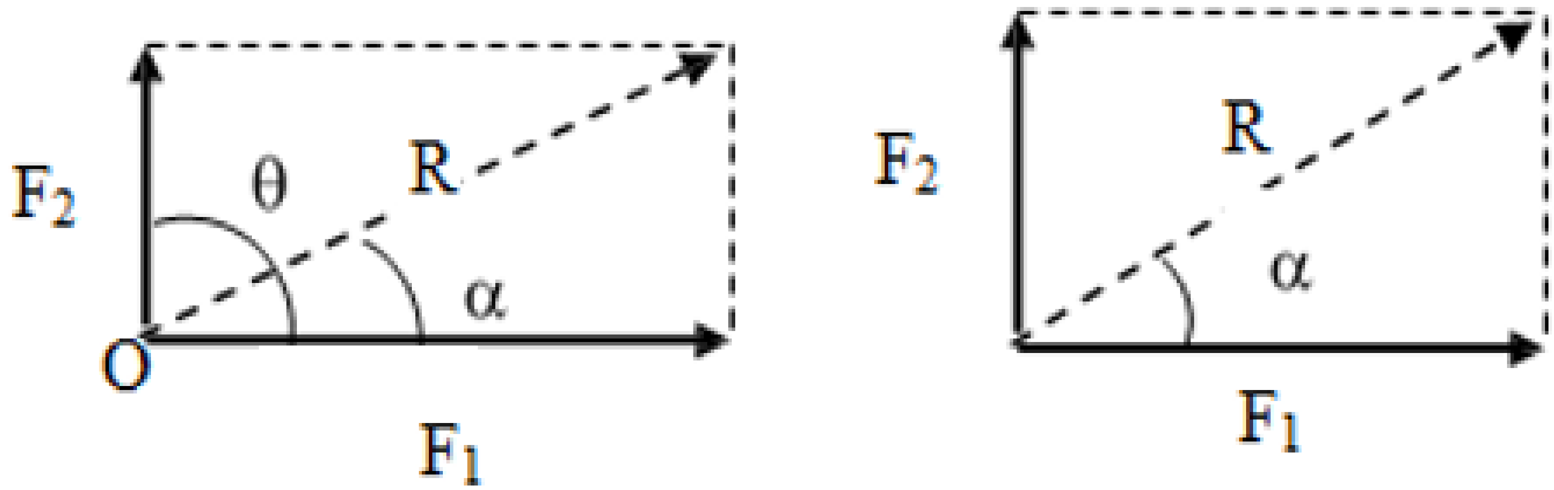
$$\text{i.e } R = \sqrt{F_1^2 + F_2^2 + 2.F_1.F_2.\cos\theta} \quad \text{--------}> 1$$

Let α be the inclination of the resultant with the direction of the F1, then

$$\alpha = \tan^{-1}\left[\frac{F_2\sin\theta}{F_1 + F_2.\cos\theta}\right] \quad \text{-------}> 2$$

Parallelogram law of forces

<u>Case 1: When θ = 90</u>^{0}:



$$R = \sqrt{F_1^2 + F_2^2}$$

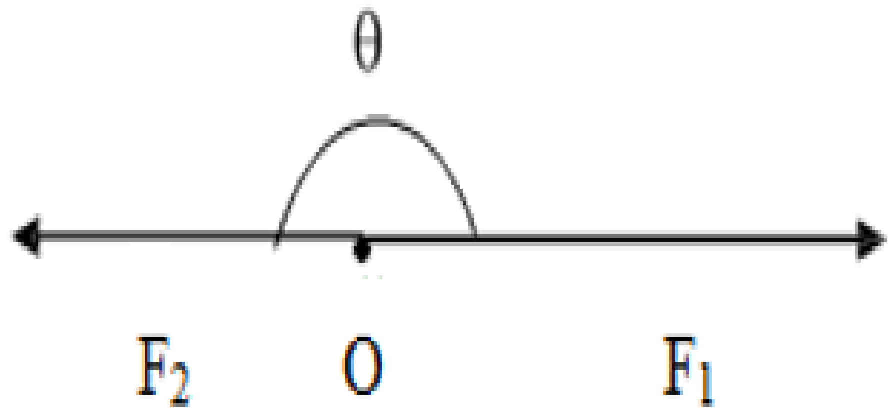
$$\alpha = \tan^{-1}\left[\frac{F_2}{F_1}\right]$$

Parallelogram law of forces

Case 2: When $\theta = 180^0$:

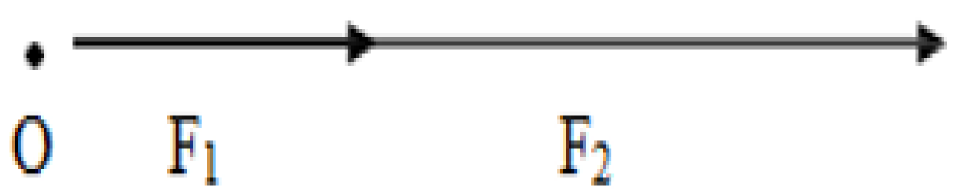
$R = [F_1 - F_2]$

$\alpha = 0^0$



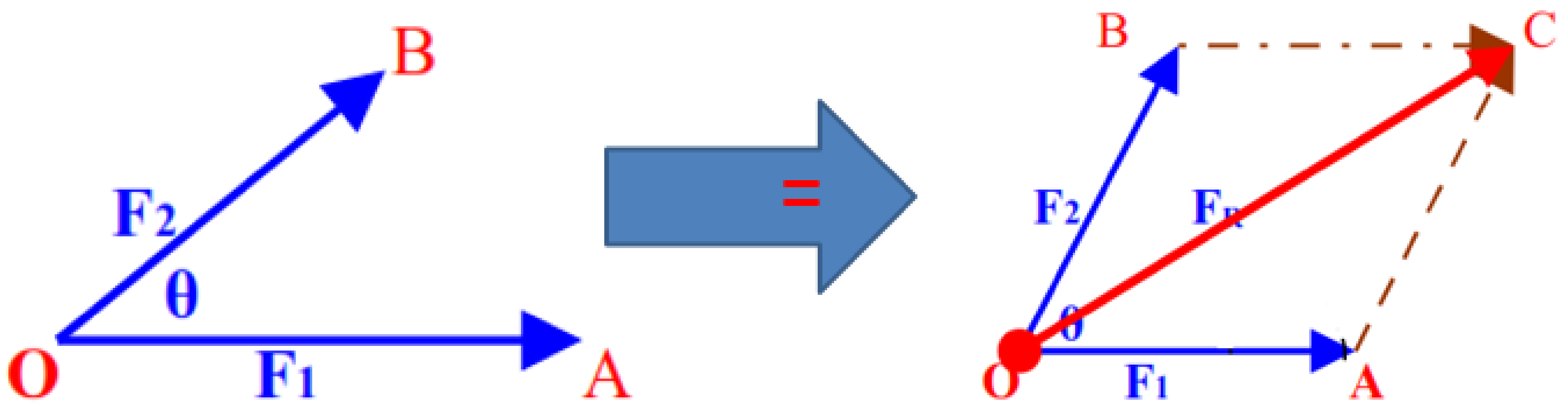
Case 3: When $\theta = 0^0$:

$R = [F_1 + F_2]$



Method of problem solutions

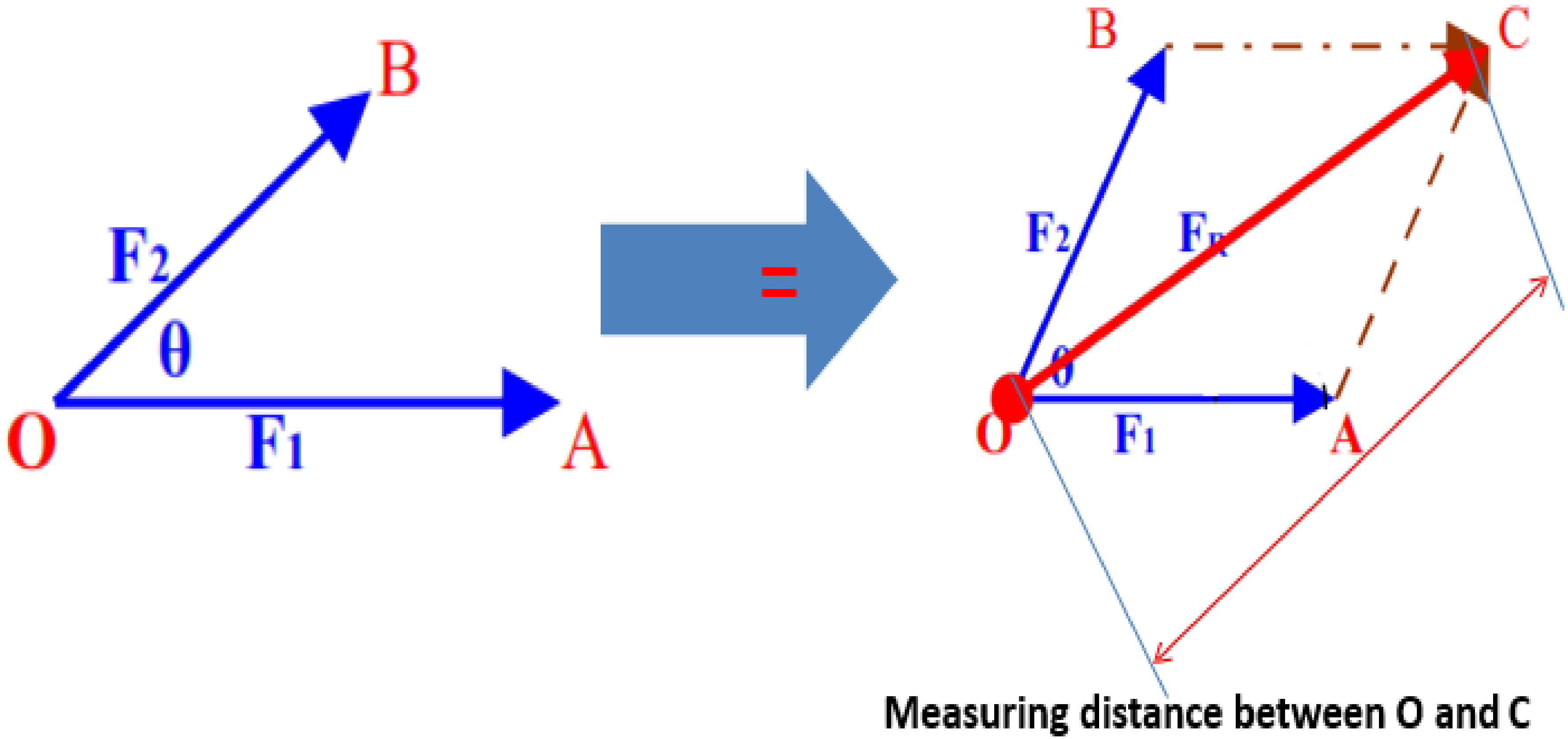
- Algebraic methods



$$F_R = \sqrt{F_1^2 + F_2^2 + 2\, F_1\, F_2 \text{ Cosine } \theta}$$

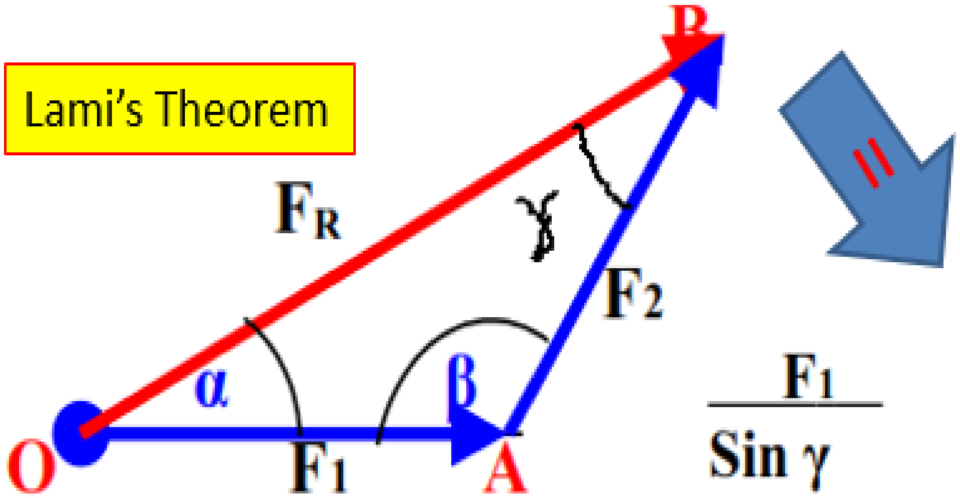
Method of problem solutions

- Graphical methods

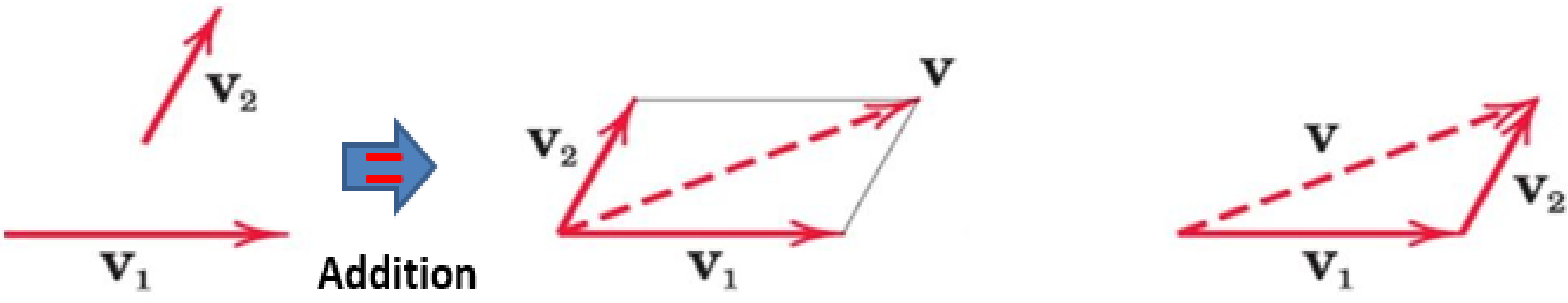


Method of problem solutions

- Graphical methods



$$\frac{F_1}{\sin\gamma} = \frac{F_2}{\sin\alpha} = \frac{F_R}{\sin\beta}$$



Method of problem solutions

- Polygon method