UNIVERSITY of RWANDA

COLLEGE OF SCIENCE AND TECHNOLOGY

School of Engineering

Civil, Environmental & Geomatics Engineering (CEGE)

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Course name: MECHANICS OF MATERIALS

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Lecture xii – *Simple stresses and strains*

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xii. SIMPLE STRESSES & STRAINS (1 of 58)
xii.1. Simple Stresses (1 of 14)

xii.1.1. Introduction

When an external force is applied on a body, internal resistance is developed within the body to balance the effect of externally applied force. The resistive force per unit area is called as ***stress***.

It is particularly significant to determine the intensity of these forces, called stress on any arbitrary selected section (oriented in a particular direction to fit the special requirement), as resistance to deformation and capacity of material to resist forces depend on these intensities.

xii.1.1. Introduction

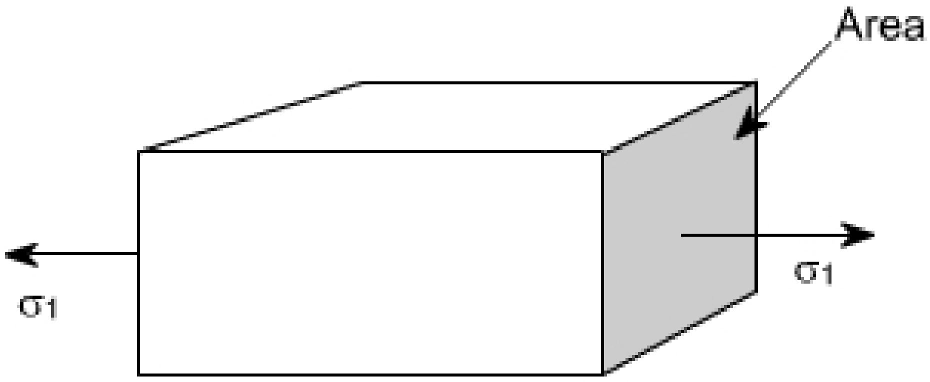
The external force acting on the body is called ***load***.

The load is applied on the body while the stress is induced in the material of the body. A loaded member remains in equilibrium when the resistance offered by the member against the deformation and the applied load are equal.

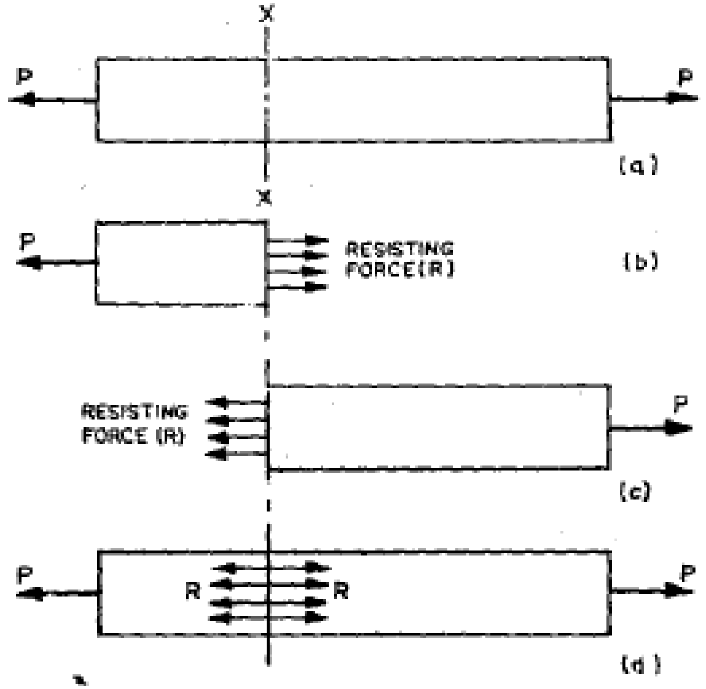
xii.1.2. Types of stresses

Only two basic stresses exists: **(1)** Normal or direct stress and **(2)** Shear stress. Other stresses either are similar of those basic stresses or a combination of theses **e.g.** bending stress is a combination of tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.

A. Normal stresses: We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter (σ).



- When a force is transmitted through a body, the body tends to change its shape or deform. The body is said to be strained.



- Direct Stress = $\frac{\text{Applied Force (F)}}{\text{Cross Sectional Area (A)}}$

- **Units:** Usually N/m^2 (Pa), N/mm^2, MN/m^2, GN/m^2 or N/cm^2

- **Note:** 1 N/mm^2 = 1 MN/m^2 = 1 MPa

$$\sigma = \frac{P}{A} \;or\; \sigma = \frac{F}{A} \;or\; \sigma = \frac{N}{A}$$

Where σ is the stress

P or F or N is the applied force (load)

A is the cross-sectional area

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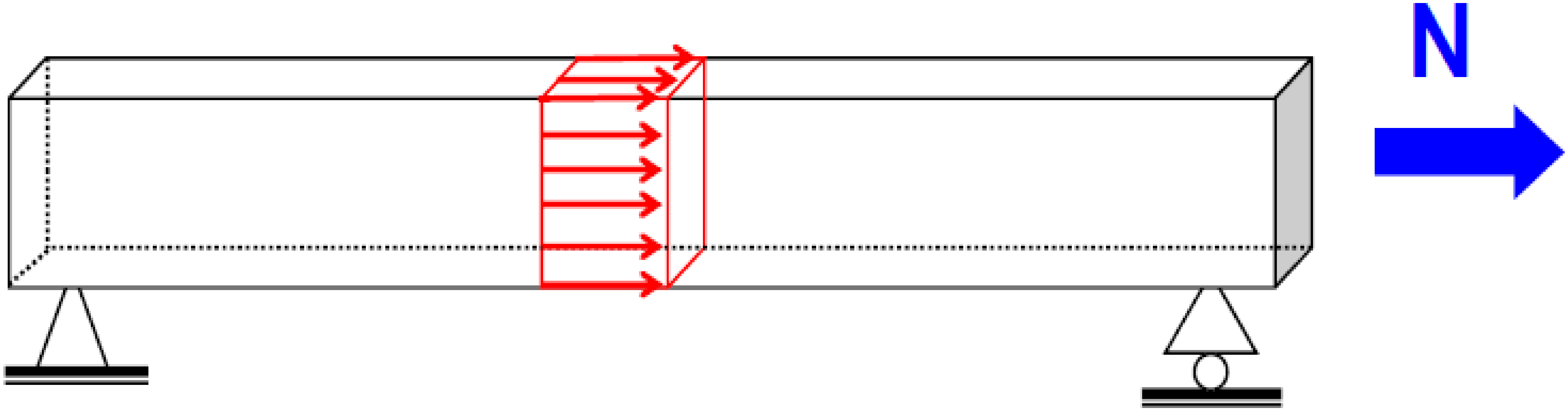
xii.1. Simple Stresses (5 of 14)

$$\sigma = N / A \ (N/mm^2)$$



xii. SIMPLE STRESSES & STRAINS (6 of 58)
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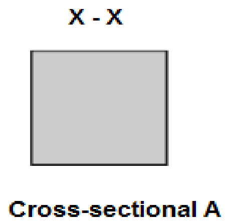
- Here we are using an assumption that the total force or total load carried by the rectangular bar is uniformly distributed over its cross-section.



But the stress distribution may be from uniform, with regions of high stress known as stress concentrations.

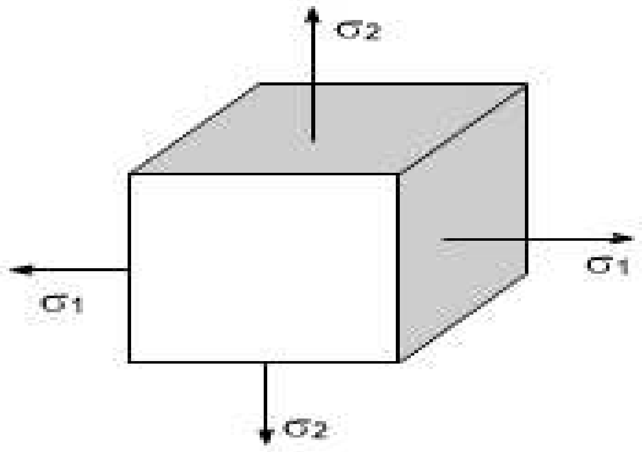
- If the force carried by a component is not uniformly distributed over its cross-sectional area, A, we must consider a small area δA which carries a small load δP, of the total force P. Then definition of stress

$$\sigma = \frac{\delta P}{\delta A}$$

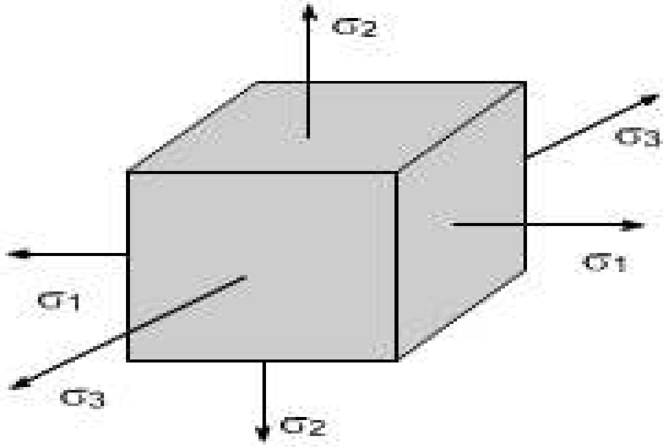
As a particular stress generally holds true only at a point, therefore it is defined mathematically as $\sigma = \lim_{\delta A \to 0} \left(\frac{\delta P}{\delta A} \right)$

xii. SIMPLE STRESSES & STRAINS (7 of 58)
xii.1. Simple Stresses (7 of 14)

This is also known as uniaxial state of stress, because the stress acts only in one direction however, such a state rarely exists, therefore we have biaxial and triaxial stresses where two mutually perpendicular normal stress acts or three mutually perpendicular normal stresses acts as shown in Figures below.



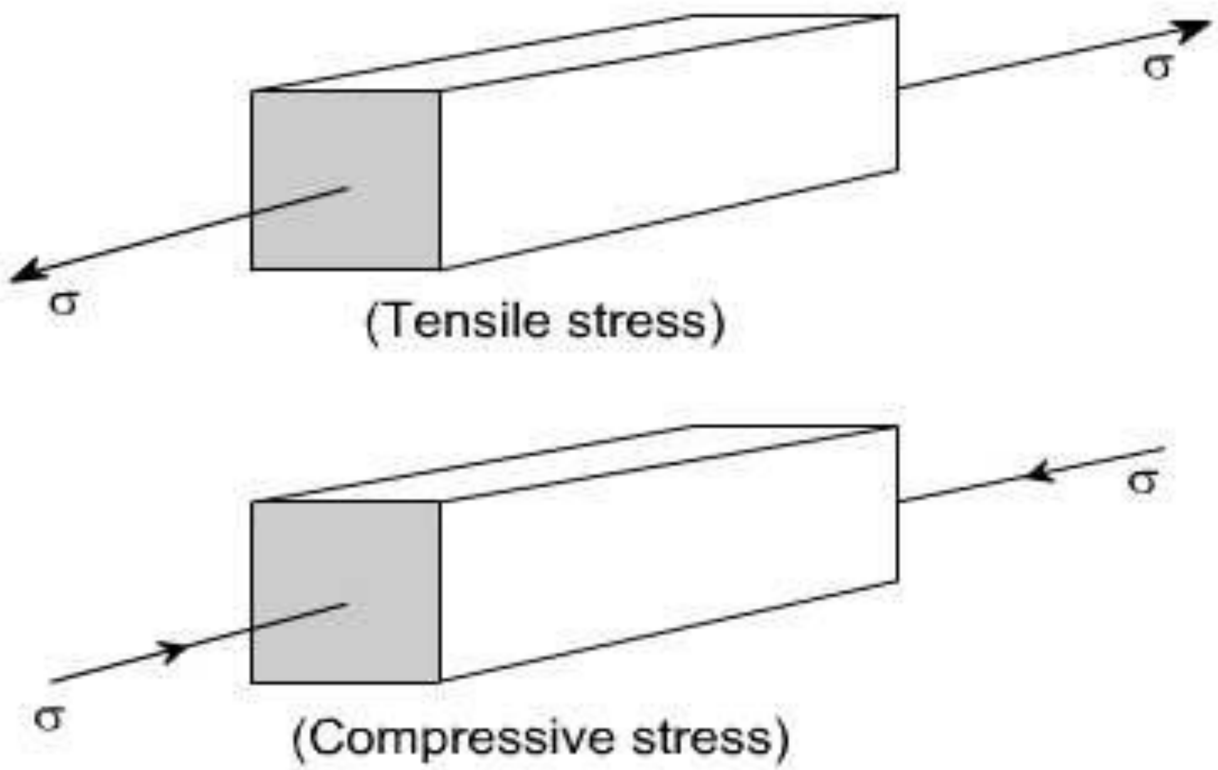
(Biaxial state of stress)



(Triaxial state of stress)

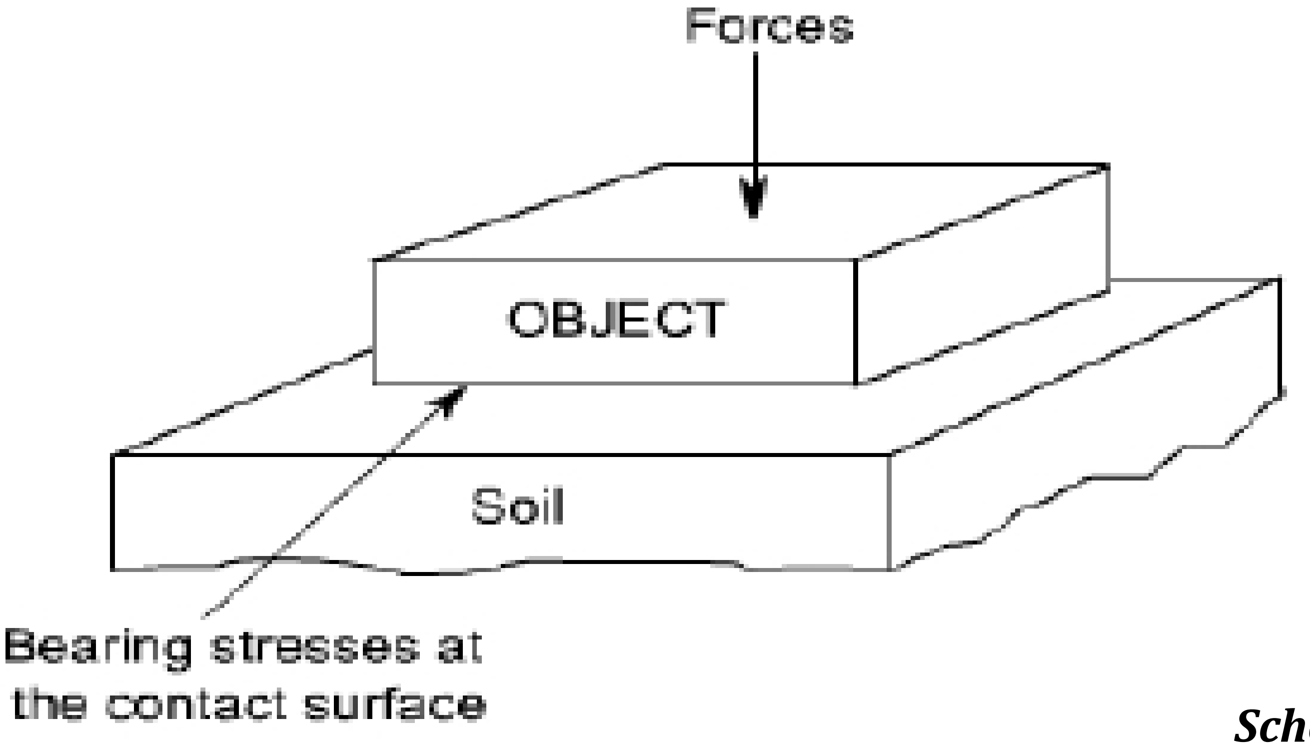
Tensile and compressive stresses:

The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area.



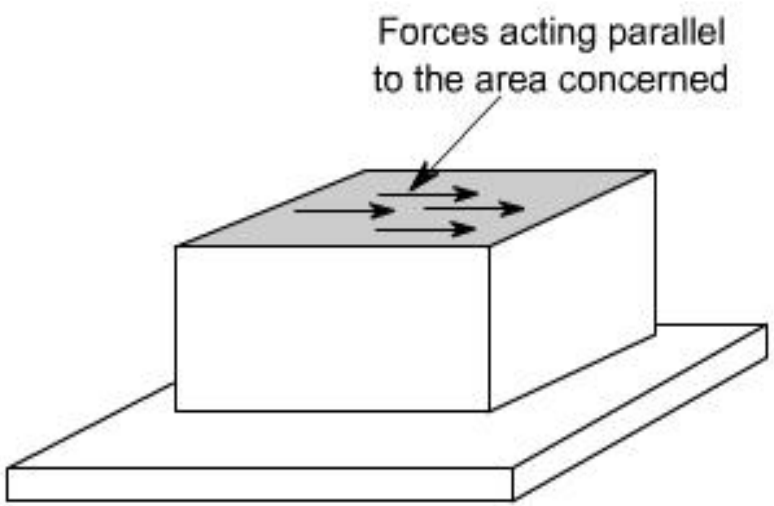
Bearing stress:

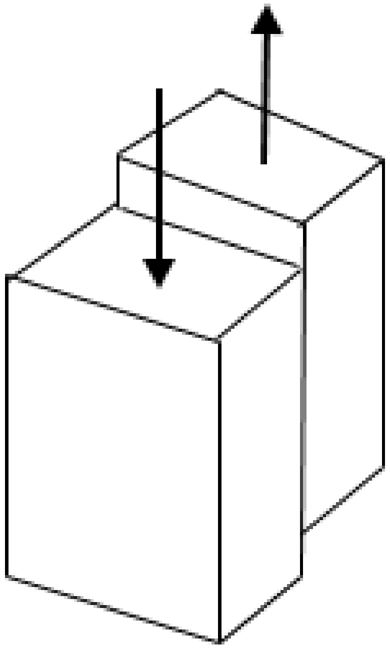
When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses).



B. Shear Stresses:

Considering the situation on figure below, where the cross-sectional area of a block of material is subjected to a distribution of forces which are parallel , rather than normal, to the area of concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force intensities are known as shear stresses.





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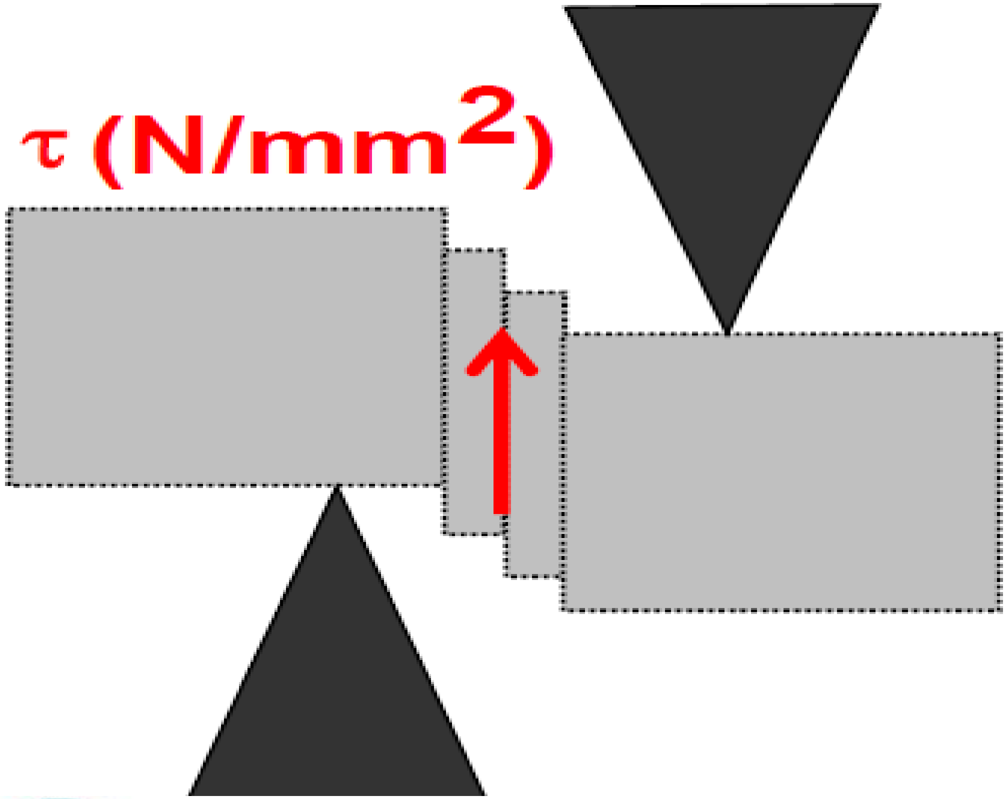
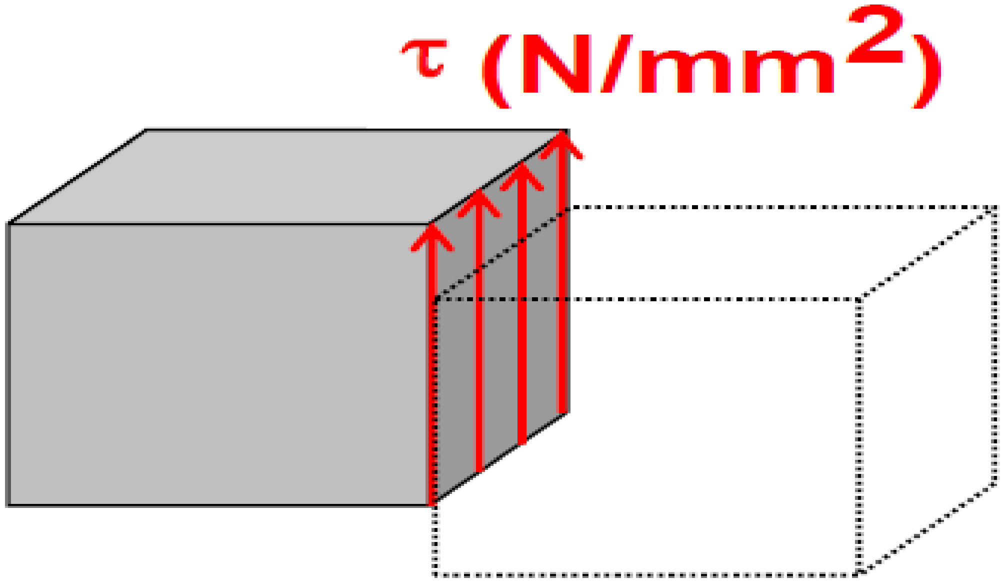
xii.1. Simple Stresses (11 of 14)





- Shear stresses are produced by equal and opposite parallel forces not in line.
- The forces tend to make one part of the material slide over the other part.
- Shear stress is tangential to the area over which it acts.

The resulting force intensities are known as shear stress, the mean shear stress being equal to $\tau = \frac{P}{A}$

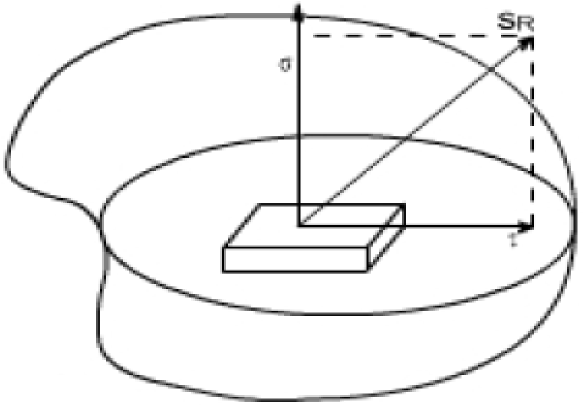
Where τ is the stress
P is total applied force (load)
A is the area over which it (P) acts

It is known that the particular stress generally holds good only at a point therefore we can define shear stress at a point as $\tau = \lim_{\delta A \to 0} \left(\frac{\delta P}{\delta A} \right)$

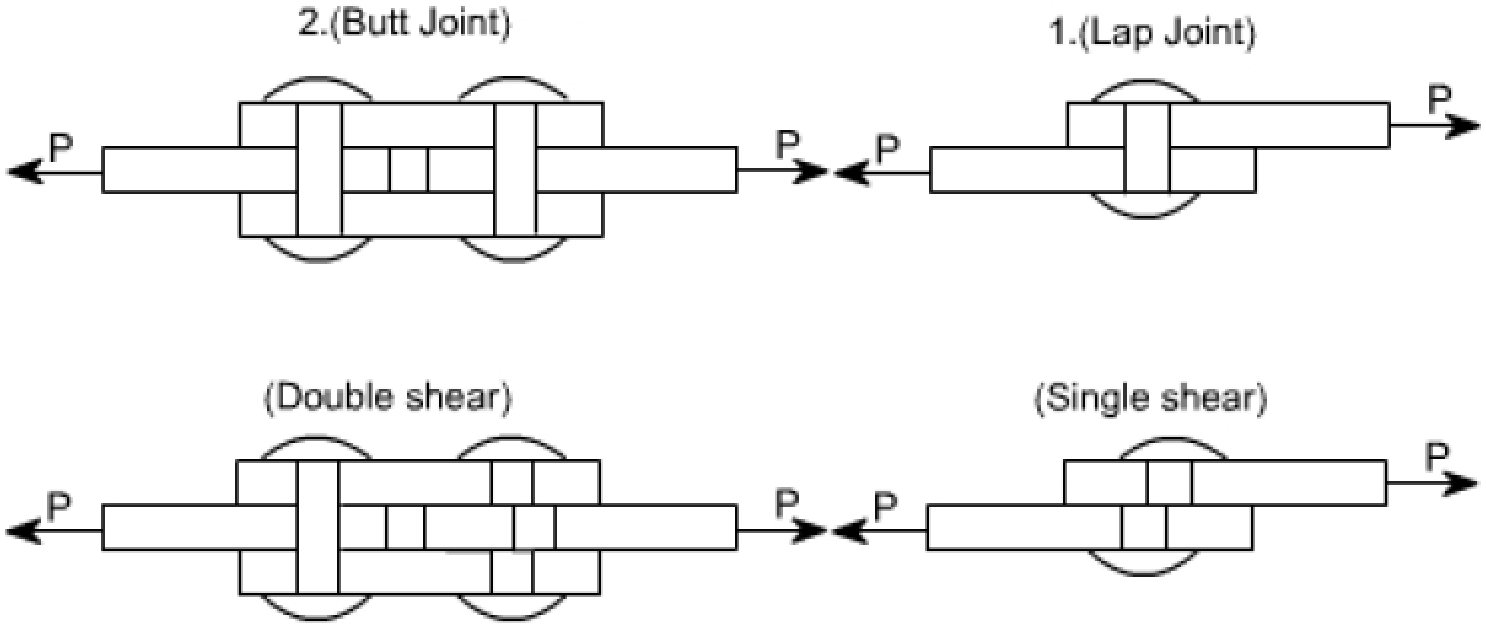
The Greek symbol τ (tau) (suggesting tangential) is used to denote shear stress.

However it should be borne in mind that the stress (resultant stress) at any point in a body is basically resolved into two components σ and τ one acts perpendicular and other parallel to the area concerned, as it is clearly defined on the following figure.

Shear failure of Rivetts



xii. SIMPLE STRESSES & STRAINS (14 of 58)
xii.1. Simple Stresses (14 of 14)

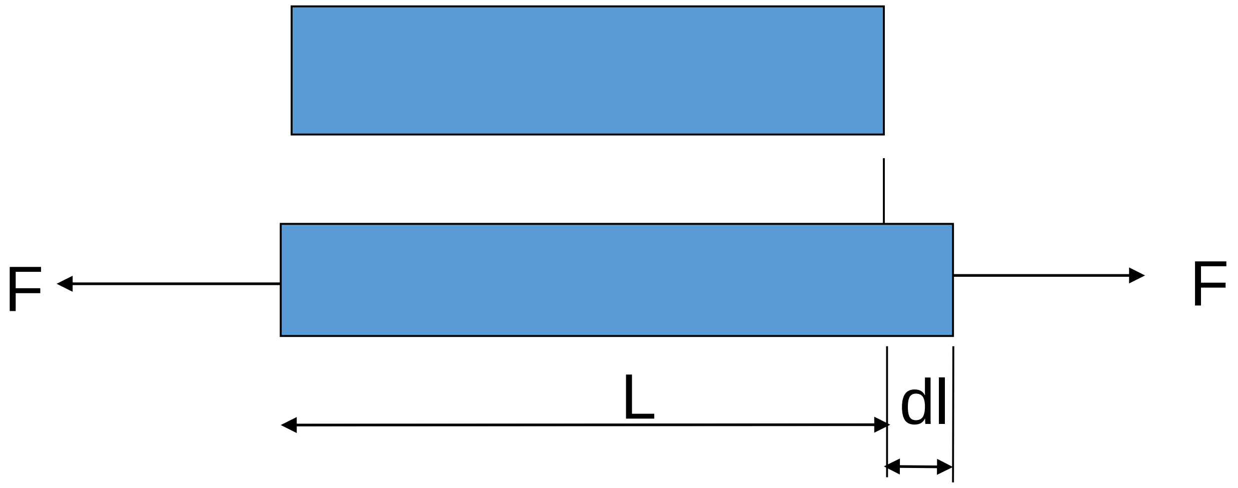


The single shear takes place on the single plane and the shear area is the cross-sectional of the rivet, whereas the double shear takes place in the case of Butt joints of rivets and the shear area is the twice the cross-sectional area of the rivet.

xii.2. Simple Strains (1 of 5)

When loads are applied to a body, some deformation will occur resulting to a change in dimension.

Consider a bar, subjected to axial tensile loading force, F. If the bar extension is dl and its original length (before loading) is L, then tensile strain is:



$$\text{Direct Strain } (\varepsilon) = \frac{\text{Change in Length}}{\text{Original Length}}$$

i.e. $\varepsilon = dl/L$

xii. SIMPLE STRESSES & STRAINS (16 of 58)
xii.2. Simple Strains (2 of 5)

- As strain is a ratio of lengths, it is dimensionless.
- Similarly, for compression by amount,
  dl: Compressive strain = - dl/L
- **Note:** Strain is ***positive*** for an increase in dimension and ***negative*** for a reduction in dimension.

- Similarly the ratio of change of volume of the body to the original volume is known as ***volumetric strain***. The strain produced by shear stress is known as ***shear strain***.

xii.2. Simple Strains (3 of 5)

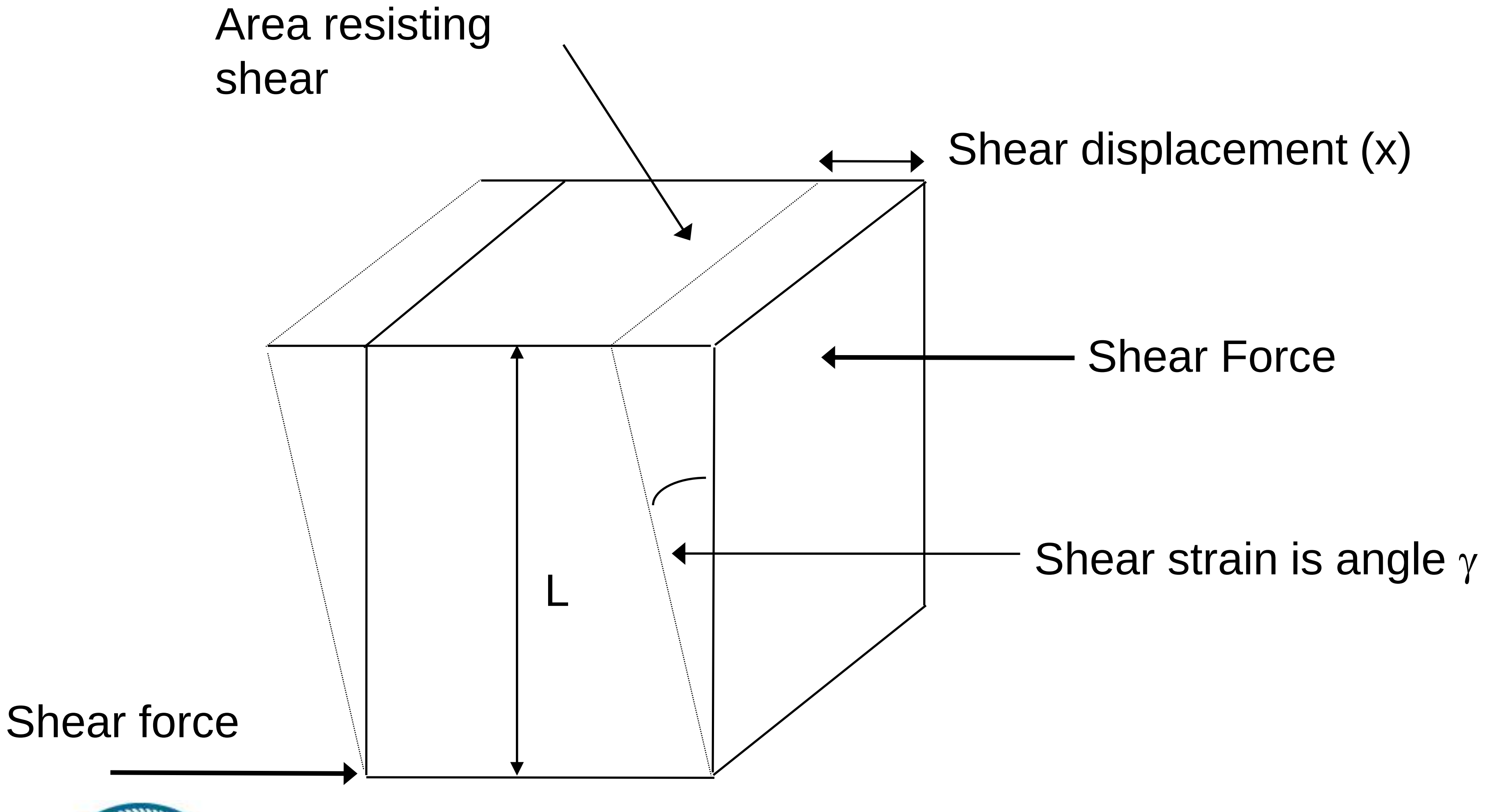
Strain is dimensionless, i.e. it is not measured in metres, kilogrammes etc.

$$\text{shear strain} \qquad \gamma \approx \frac{\text{shear displacement } x}{\text{width } L}$$

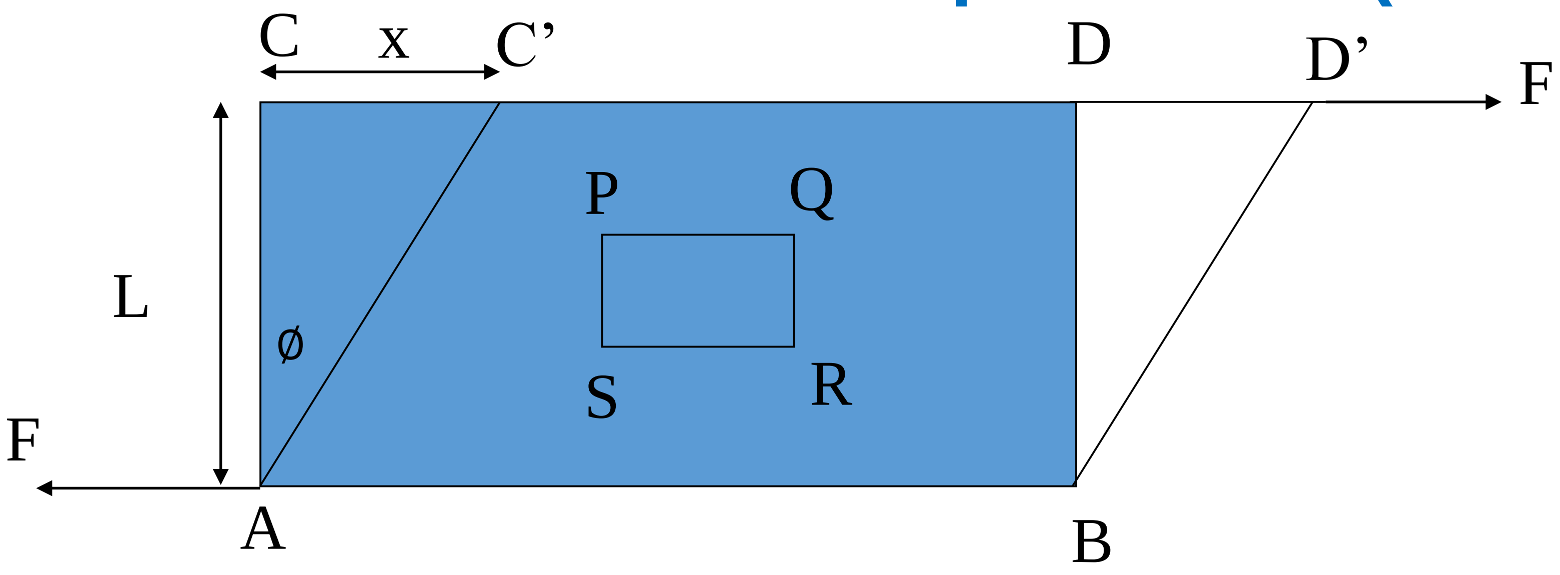
For shear loads the strain is defined as the angle γ This is measured in radians.

xii. SIMPLE STRESSES & STRAINS (18 of 58)

xii.2. Simple Strains (4 of 5)



xii.2. Simple Strains (5 of 5)



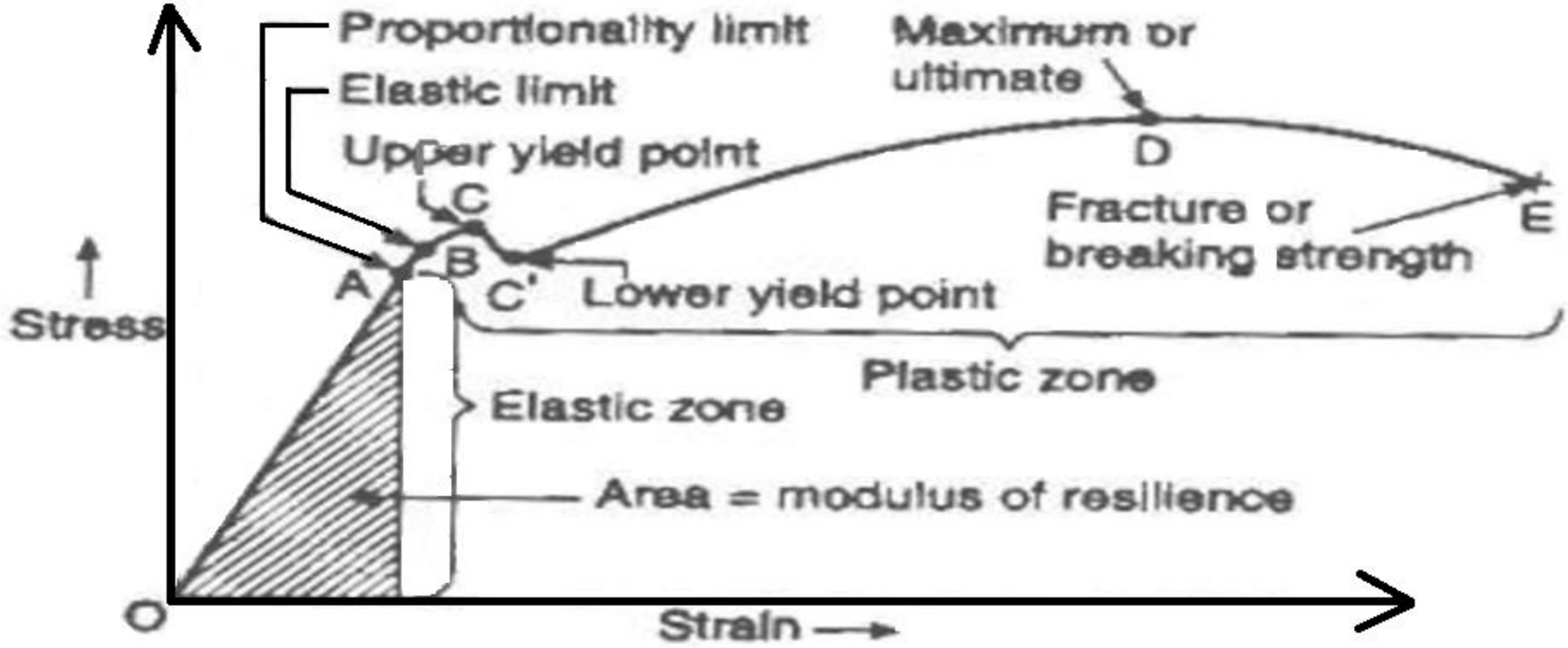
Shear strain is the distortion produced by shear stress on an element or rectangular block as above. The shear strain, γ (gamma) is given as:

$= x/L = \tan \emptyset$

For small , $\emptyset$ $\gamma = \emptyset$

Shear strain then becomes the change in the right angle.

xii. SIMPLE STRESSES & STRAINS (20 of 58)
xii.3. Stress-strain diagram (1 of 3)



Features of stress-Strain Curve (for ductile materials)

xii. SIMPLE STRESSES & STRAINS (21 of 58)
xii.3. Stress-strain diagram (2 of 3)

OA = Proportionality limit
OB = Elastic limit

- Beyond proportionality limit ‘A’ stress is no longer proportional to strain.
- The stress-strain diagram will not be straight line after this point.
- The elastic limit ‘B’ is the maximum stress that may be developed such that there is no ***permanent deformation*** after the removal of load.
- It is also stress beyond which the material will not return to its ***original shape*** when ***unloaded*** but will retain a ***permanent deformation***.
- Beyond ‘B’ ***elongation*** is more rapid and diagram becomes curved.

xii. SIMPLE STRESSES & STRAINS (22 of 58)
xii.3. Stress-strain diagram (3 of 3)

- At ‘C’ sudden elongation of the bar takes place without appreciable increase in stress (or Tensile force). This phenomenon is called ***yielding of material*** and the corresponding stress is called ***yield point***.
- Beyond ‘C’ material recovers resistance for deformation and tensile force required increases with elongation, up to point ‘D’ where the force attains a maximum value. The corresponding stress is called the ***ultimate strength of material.***
- Beyond ‘D’ , the elongation continues even on reduction of load and fracture finally occurs at a load corresponding to point ‘E’ of the diagram and corresponding stress is called ***rupture strength***.

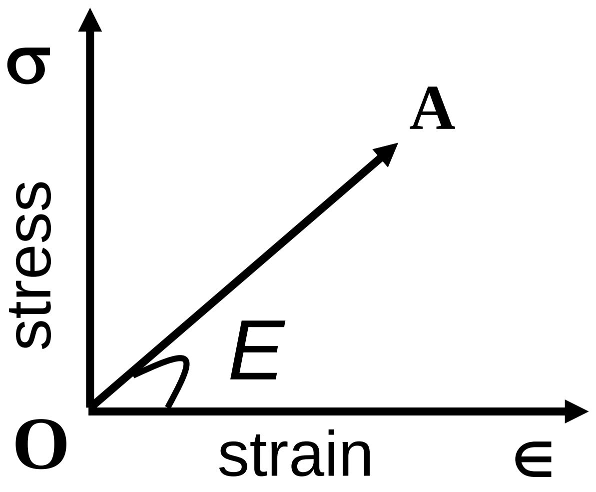
xii.4. Elastic constant (1 of 12)

xii.4.1. Modulus of elasticity

- The slope (gradient) of the stress-strain-graph represents the correlation between stress and strain. It is a specific property (constant) of a material, indicating its elastic behaviour. It is called the Modulus of Elasticity or the Young's Modulus of a material denoted by ***E***.

$$\sigma = E\varepsilon$$

$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\delta L/L} = \frac{PL}{A\delta L}$$



Value of *E* is same in Tension & Compression.

- A high modulus of elasticity therefore represents a hard, rigid material like steel, a low modulus of elasticity a soft, easily deformable material like rubber.

xii.4.2. Hooke's law

It states that a material is loaded within elastic limit; the stress is proportional to the strain produced by the stress.

This means that the ratio of the stress to the corresponding strain is a ***constant*** within the ***elastic limit.***

This constant is known as ***Modulus of Elasticity*** or ***Young's Modulus*****.**

xii.4.3. Modulus of rigidity

- It is the coefficient of elasticity for shearing force. It is defined as the ratio of shear stress to the displacement per unit sample length (shear strain).

$$G = \frac{\tau}{\gamma}$$

$$G = \frac{\tau}{\emptyset}$$

Where G Shear modulus or Modulus of rigidity
τ is the shear stress
γ is the shear strain
$\emptyset$ is the shear strain when the angle is **too small**

- Similar to the Young's modulus, shear modulus is also a specific property (constant) of a material to characterise its behaviour.

xii. SIMPLE STRESSES & STRAINS (26 of 58)
xii.4. Elastic constant (4 of 12)

E . g:

Material	**Shear Modulus (GPa)**
Carbon Steel	77
Cast Iron	41
Concrete	21
Iron, Ductile	63-66
Iron, Malleable	64
Lead	13.1
Structural Steel	79.3
Stainless Steel	77.2
Steel , Cold-rolled	75
Wood, Douglas Fir	13

xii.4. Elastic constant (5 of 12)

Table . Elastic constants of commonly used metallic materials

Metal	*E*	*G*	*K*	*μ*
	Modulus of Elasticity (Young's modulus) MN/mm^2	*Modulus of Rigidity (Shear modulus) MN/mm^2*	*Bulk modulus MN/mm^2*	*Poisson's Ratio*
Cast steel	0.196	0.078	0.139	0.265
Cold Rolled steel	0.203	0.079	0.159	0.287
Stainless steel	0.190	0.073	0.163	0.305
High carbon steel	0.197—0.207	0.076—0.082	0.156—0.166	0.283—0.292
Cast Iron	0.093—0.145	0.036—0.057	0.058—0.107	0.211—0.299
Copper	0.108	0.040	0.123	0.355
Brass	0.109	0.041	0.116	0.331
Aluminium Alloys	0.068—0.071	0.026—0.027	0.068—0.070	0.330—0.334

xii.4.4. Bulk modulus

- When a body is subjected to normal stress of some nature along 3 mutually perpendicular directions.

- The ratio of normal stress to the corresponding volumetric strain is called *Bulk Modulus of Elasticity*, K which is constant for a given material.

K = Normal stress/Volumetric strain

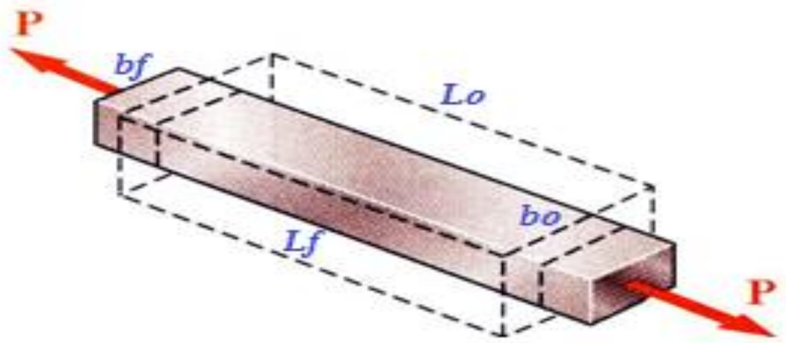
$$K = \frac{\sigma}{\delta v / V}$$

Where K is Bulk modulus
σ is the normal stress
δv is the change in volume
V is the initial (original) volume

xii.4.5. Poisson's ratio

- When a bar is loaded in tension its length increases but the other two dimensions (Width and Thickness) decrease. For compressive loads, the length decreases and the other dimensions increase.
- Lateral strain thus developed has a definite relationship to the longitudinal strain depending on the material.
- The ratio between transverse (lateral strain) and longitudinal strain, is known as ***Poisson's ratio*** an it denoted by μ , ϑ or $1/m$.
- The value of the Poisson's ratio depends on the material and for most materials lies between 0.25 and 0.33.
- Poisson's ratio is a dimensionless material property that never exceeds 0.5.

$$\mu = \frac{\varepsilon_t}{\varepsilon_l} \text{ or } \mu = \left|\frac{\varepsilon_t}{\varepsilon_l}\right|$$



Where μ is the Poisson's ratio
ε_t is the lateral strain
ε_l is the longitudinal strain

xii.4. Elastic constant (8 of 12)

- Longitudinal strain $\varepsilon_l = \frac{\delta l}{l}$ and Lateral strain $\varepsilon_t = \frac{\delta d}{d}$
- Proportional change in area, ${}^{\delta A}/_{A}$ is the sum of proportional changes l in length and width b.

$$\frac{\delta A}{A} = \frac{\delta l}{l} + \frac{\delta b}{b}$$

- For a circular section, proportional change in area is twice the proportional change in diameter (or radius) as $l = b = d$.

$$\frac{\delta A}{A} = 2\frac{\delta d}{d} = 2\frac{\delta r}{r}$$

- Similarly, proportional change in volume of a rectangular block is the sum of proportional changes in l , b and t.

$$\frac{\delta v}{V} = \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta t}{t}$$

xii.4. Elastic constant (9 of 12)

- There three moduli

- Elastic modulus or Young’s modulus ***E***
- Modulus of rigidity or Shear modulus G , it also called Modulus of transverse elasticity
- Bulk modulus of elasticity ***K***

These three moduli and Poisson’s ratio μ are known as *elastic constants*.

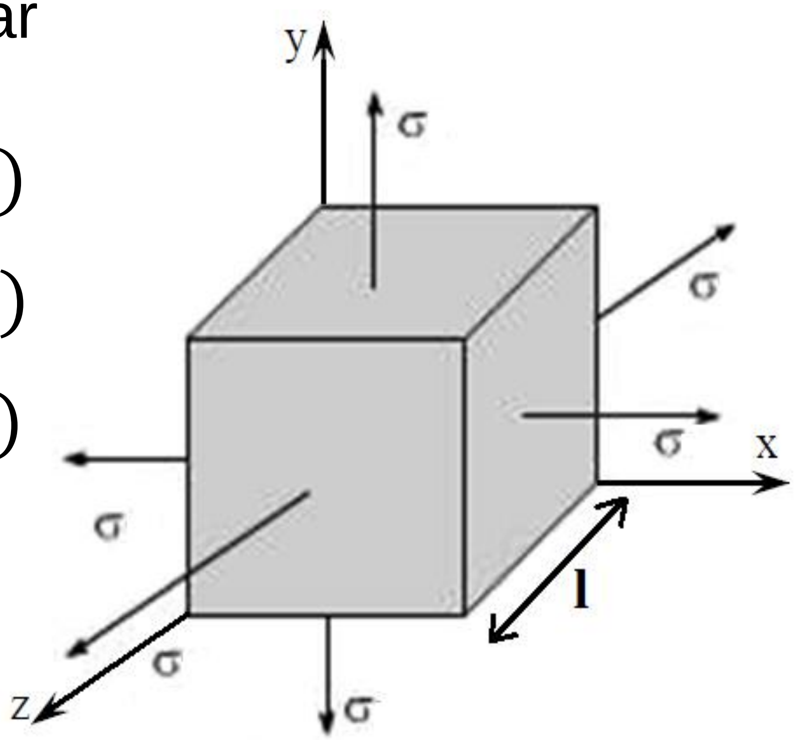
xii.4.6. Relation between *K*, *E* and μ

Let us consider a cube of dimension l, which is subjected to normal stress σ along a three mutually perpendicular axes, x, y and Z

ε_x , strain in x direction $= \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = \frac{\sigma}{E}(1-2\mu)$

ε_y , strain in y direction $= \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = \frac{\sigma}{E}(1-2\mu)$

ε_z , strain in z direction $= \frac{\sigma}{E} - \mu\frac{\sigma}{E} - \mu\frac{\sigma}{E} = \frac{\sigma}{E}(1-2\mu)$



Volumetric strain $\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{3\sigma}{E}(1-2\mu)$

Bulk modulus, $K = \frac{\sigma}{\varepsilon_v} = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)} = \frac{E}{3(1-2\mu)}$

$$K = \frac{E}{3(1-2\mu)} \quad \text{or} \quad E = 3K(1-2\mu)$$

xii.5.7. Relation between G and *E*

$$E = 2G(1+\mu)$$

xii.5.8. Relation between *K*, *G* and *E*

From the above two relations *K* and *E* and between *E* and G in terms of Poisson's ratio μ.

$$E = 3K(1-2\mu) = 2G(1+\mu)$$

By eliminating μ from the two equations of *E* given above, we can get a relationship between *E*, G and *K* free from terms μ.

xii.4. Elastic constant (12 of 12)

$$\mu = \frac{E}{2G} - 1 \;, \quad E = 3K\left[1 - \left(\frac{E}{G} - 2\right)\right]$$

or $$E = 3K\left(3 - \frac{E}{G}\right) = 9K - \frac{3KE}{G}$$

So $$E\left(1 + \frac{3K}{G}\right) = 9K$$ or $$E\left(\frac{G + 3K}{G}\right) = 9K$$

Or $$\boldsymbol{E = \frac{9KG}{G + 3K}}$$

So $$E = 3K(1 - 2\mu) = 2G(1 + \mu) = \frac{9KG}{G + 3K}$$

xii.5. Thermal Stresses (1 of 7)

xii.5.1. Introduction

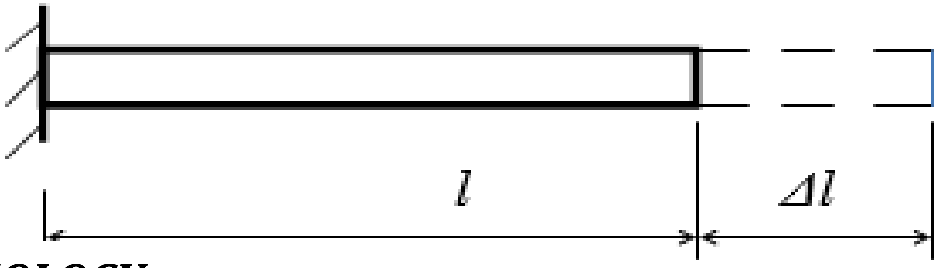
If a member is subjected to temperature variation, its dimension get change; increase in dimensions i.e. expansion, if temperature increases and contraction, if the temperature decreases. Increase or decrease in length/ unit due to unit rise in temperature is known as *coefficient of thermal expansion*, α.

This coefficient α is different for different materials and is expressed in mm/mm/°C

If the change in length is allowed to take place freely, no stress will be developed in the material and it expands longitudinally causing strain (or deformation).

Change in length, $\Delta l = \alpha t l$; t = change in temperature ; l = original length

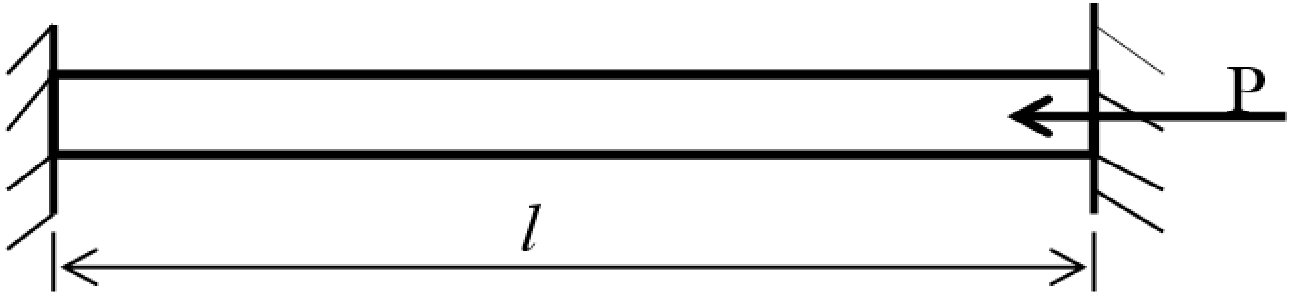
If t_1 is initial temperature and t_2 is final temperature



xii.5. Thermal Stresses (2 of 7)

$\Delta l = \alpha(t_2 - t_1)l$ and thermal strain $\varepsilon_t = \frac{\alpha(t_2 - t_1)l}{l} = \alpha(t_2 - t_1)$.

If the expansion is restricted, fully or partially by fixtures, then the compressive stresses are developed in the material and these stresses are called *thermal stresses*, σ_t given by $\sigma_t = E\alpha t$



as $\Delta l = \frac{Pl}{AE} = \alpha t l$ or $\sigma_t = E\alpha t$

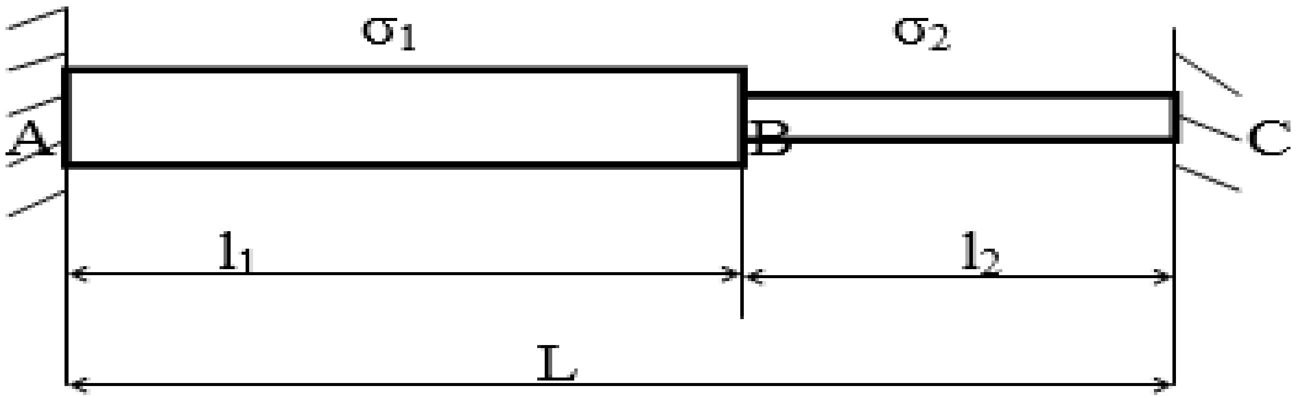
xii. SIMPLE STRESSES & STRAINS (37 of 58)
xii.5. Thermal Stresses (3 of 7)

If there is some yielding of the supports (partial change in length δl), thermal stresses will reduce.

Net elongation $= \alpha t l - \delta l$ and $\sigma_t = \left(\frac{\alpha t l - \delta l}{l}\right) E$

xii.5.2. Thermal stresses in bars of varying section

Let us consider a bar of varying section. A1 and A2 are cross-sections of two parts. If ***E*** is the modulus of elasticity and α is the coefficient of thermal expansion and T is the change in temperature.



Total pull in bar $= \sigma_1 A_1 = \sigma_2 A_2$, if the bar is prevented from expending.

xii.5. Thermal Stresses (5 of 7)

Total deformation, if bar is free, $\delta l = \delta l_1 + \delta l_2$

$$\delta l = \frac{\sigma_1 l_1}{E} + \frac{\sigma_2 l_2}{E} = \frac{1}{E}(\sigma_1 l_1 + \sigma_2 l_2)$$

If two portions are different materials having moduli of elasticity $E_1\ and\ E_2$,

$$\delta l = \frac{\sigma_1 l_1}{E_1} + \frac{\sigma_2 l_2}{E_2}$$

When partial change in length δ is permitted

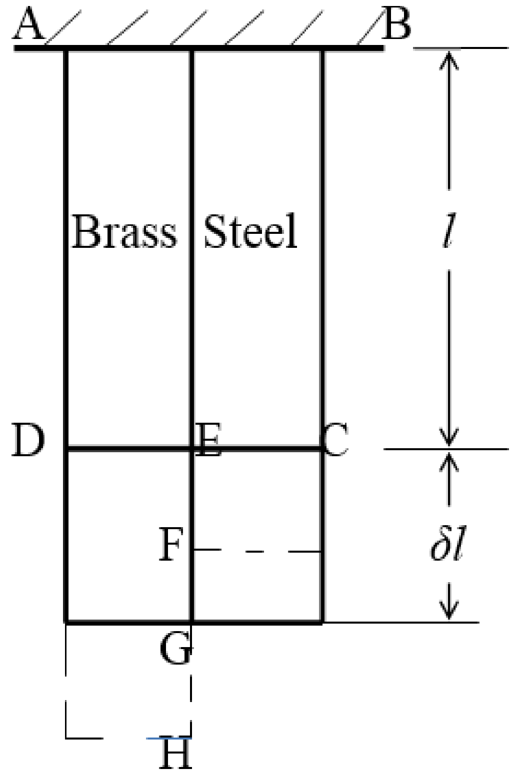
Actual strain, $\varepsilon_t = \dfrac{actual\ expansion}{Original\ length} = \dfrac{\propto TL - \delta}{L}$

and thermal stress, $\sigma_t = actual\ strainx\ E = \left(\dfrac{\alpha TL - \delta}{L}\right) E$

xii.5.3. Thermal stresses in composite bars

Let us consider a compound bar having brass and steel bars. If the ends of the two bars are not joined rigidly, the two bars will expand by different amounts and there will be no stresses induced in two bars.

If the two ends are rigidly fixed, the composite section will expand as a whole unit. The coefficients of thermal expansion of two materials will be different, α_b for brass and α_s for steel. As $\alpha_b > \alpha_s$ brass will expand more. The expansion of composite unit due to a temperature rise of T ^{o}C is δl. σ_b and σ_s are stresses in brass and steel and ε_b and ε_s are temperature strains in brass and steel. A_b and A_s are areas of cross-section of brass and steel. E_b and E_s are moduli of elasticity for brass and steel. Let ε be the actual strain in composite bar.



Compressive force exerted in brass = tensile force exerted in steel

$$\sigma_b \; x \; A_b = \sigma_s \; x \; A_s \; , \qquad \sigma_b = \sigma_s \left(\frac{A_s}{A_b}\right)$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} \quad , \quad \varepsilon_s = \frac{\sigma_s}{E_s}$$

Reduction in elongation of brass bar $= GH = \alpha_b Tl - \delta l$

So, $\varepsilon_b = \dfrac{\alpha_b Tl - \delta l}{l} = \alpha_b T - \varepsilon$

Extra elongation of steel bar $= FG = \delta l - \alpha_s Tl$

$$\varepsilon_s = \frac{\delta l - \alpha_s Tl}{l} = \varepsilon - \alpha_s T$$

Adding , and $\varepsilon_b + \varepsilon_s = (\propto_b T - \varepsilon) + (\varepsilon - \propto_s T) = (\alpha_b - \alpha_s)T$

xii.6. Load and stress limit (1 of 3)

xii.6.1. INTRODUCTION

❖ DESIGN CONSIDERATION

- Will help engineers with their important task in designing a structural member/machine that is ***safe*** and ***economically*** performing for a specified function/purpose.

❖ DETERMINATION OF ULTIMATE STRENGTH

- An important element to be considered by a designer is how the material that has been selected will behave under a certain load
- This is determined by performing specific test (e.g. Tensile test)
- ***Ultimate force (P_u)=*** The largest force that may be applied to the specimen so that if it is reached the specimen either breaks or begins to carry less load.
- ***Ultimate normal stress*** (σ_u) = Ultimate force(P_u) /Area (A)

xii.6.2. ALLOWABLE LOAD / ALLOWABLE STRESS

- Max load that a structural member/machine component will be allowed to carry under normal conditions of utilization (use) is considerably smaller than the ultimate load.
- This smaller load = ***Allowable load / Working load / Design load***
- Only a fraction of ultimate load capacity of the member is utilised when allowable load is applied.
- The remaining portion of the load-carrying capacity of the member is kept in reserve to assure its safe performance.

- The ***strength*** of a material is a measure of the stress that it can take when in use.
- The ***ultimate strength*** is the measured stress at failure but this is not normally used for design because safety factors are required. Therefore one of the ways to define a ***safety factor*** or ***factor of safety*** is :

$$\text{safety factor} = \frac{\text{stress at failure}}{\text{stress when loaded}} = \frac{\text{Ultimate stress}}{\text{Permissible stress}} = \frac{\text{Ultimate load}}{\text{Allowable load}}$$

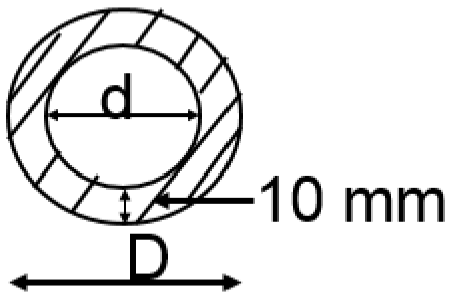
xii.6. Load and stress limit (3 of 3)

xii.6.3. SELECTION OF F.S.

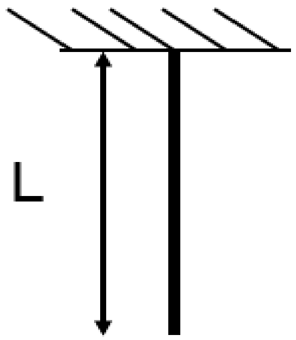
1. Variations that may occur in the properties of the member under considerations
2. The number of loading that may be expected during the life of the structural member/machine
3. The type of loading that are planned for in the design, or that may occur in the future
4. The type of failure that may occur
5. Uncertainty due to the methods of analysis
6. Deterioration that may occur in the future because of poor maintenance /because of unpreventable natural causes (earthquake, tornado, etc.)
7. The importance of a given member to the integrity of the whole structure

GIVEN EXAMPLES

1. A short hollow, cast iron cylinder with wall thickness of 10 mm is to carry compressive load of 100 kN. Compute the required outside diameter 'D' , if the working stress in compression is 80 N/mm^2.



2. A Steel wire hangs vertically under its weight. What is the greatest length it can have if the allowable tensile stress σ_t =200 MPa? Density of steel γ = 80 kN/m^3.

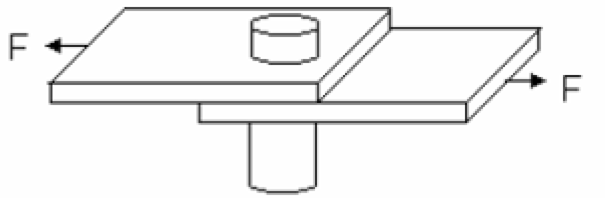


GIVEN EXAMPLES

3. A steel tensile test specimen has a cross sectional area of 100 mm^2 and a gauge length of 50 mm, the gradient of the elastic section is $410x10^3$ N/mm. Determine the modulus of elasticity.
4. A steel column is 3m long and 0.4m diameter. It carries a load of 50MN. Given that the modulus of elasticity is 200GPa. Calculate the compressive stress and strain and determine how much the column is compressed.
5. Calculate the force needed to punch a hole 30mm diameter in a sheet metal 3 mm thick given that the ultimate shear stress is 60 MPa.

GIVEN EXAMPLES

6. Two strips of metal are pinned together as shown with a rod 10 mm diameter. The ultimate shear stress for the rod is 60MPa. Determine the maximum force required to break the pin.



7. A metal wire is 2.5mm diameter and 2 m long. A force of 12N is applied to it and it stretches 0.3 mm. Assume the material is elastic.

   Determine the following: i. The stress in the wire
   ii. The strain in the wire

Solutions

1. $\sigma = 80 N/mm^2$; $t = 10$ mm

$P = 100$ kN $= 100 \times 10^3$ N

as $\sigma = P/A$, therefore $A = P/\sigma$

$A = (100 \times 10^3)/80 = 1250 mm^2$ **(i)**

$A = (\pi/4) \times \{D^2 - d^2\}$

$A = (\pi/4) \times \{D^2 - (D - 2t)^2\}$

$A = (\pi/4) \times \{D^2 - (D - 20)^2\}$ **(ii)**

By considering **(i)** and **(ii)** simultaneously

$1250 = (\pi/4) \times \{D^2 - (D - 20)^2\}$

$1250 = (\pi/4) \times \{D^2 - (D^2 - 40D + 400)\}$

$1250 = (\pi/4) \times \{D^2 - D^2 + 40D - 400)\}$

$1250 = (\pi/4) \times \{40D - 400)\}$

$1250 = (10D - 100)\ \pi$

$D = (1250 + 100\ \pi)/10\ \pi$

$D = 49.8$ mm

xii. SIMPLE STRESSES & STRAINS (49 of 58)

2. σ_t = 200 MPa = 200x10^3 kN/m^2 ;

   γ = 80 kN/m^3.

   Weight of wire P = (π/4) x D^2 x L x γ

   c/s area of wire A = (π/4) x D^2

   σ_t = P/A

   σ_t = ((π/4) x D^2 x L x γ) / ((π/4) x D^2)

   σ_t = L x γ

   L = σ_t/γ

   L = 200 x 10^3 /80 = 2500 m

3. A = 100 mm^2 ; L = 50 mm

   The gradient gives the ratio P/δ. This ratio as stiffness can also be used to find *E*.

$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\delta l/L} = \frac{P}{\delta l} \text{x} \frac{L}{A}$$

$$= 410 \text{ x } 10^3 \text{x} \frac{50}{100}$$

$$E = 205000 N/mm^2 = 205 GPa$$

4. $$A = \frac{\pi d^2}{4} = \frac{\pi \text{ x } 0.4^2}{4} = 0.126m^2$$

$$\sigma = \frac{P}{A} = \frac{50 \text{ x } 10^6}{0.126} = 397.9 \text{ x } 10^6 Pa$$

$$E = \frac{\sigma}{\varepsilon} \quad therefore \quad \varepsilon = \frac{\sigma}{E} = \frac{397.9 \text{ x } 10^6}{200 \text{ x } 10^9} = 0.001989$$

$$\varepsilon = \frac{dl}{L} \quad therefore \quad dl = \varepsilon L = 0.001989 \text{ x } 3000 = 5.97mm$$

5. The area to be cut is the circumference x thickness $= \pi\, d \times t$

$$A = \pi \times 30 \times 3 = 282.7mm^2$$

The ultimate shear stress is $60MPa = 60N/mm^2$

$\tau = \frac{P}{A}$ therefore P$= \tau \times A = 60 \times 282.7 = 16965N \; or \; 16.965kN$

6. The area of the rod to be broken $= \pi \frac{d^2}{4} = \pi \frac{100}{4} = 25\pi = 78.54mm^2$

The ultimate shear stress is $60MPa = 60N/mm^2$

$\tau = \frac{P}{A}$ therefore P $= \tau \times A = 60 \times 78.54 = 4712.4N \; or \; 4.712kN$

7. $A = \frac{\pi d^2}{4} = \frac{\pi \text{ x } 2.5^2}{4} 4.909mm^2$

$$\sigma = \frac{P}{A} = \frac{12}{4.909} = 2.44N/mm^2 = 2.44MPa$$

$$\varepsilon = \frac{dl}{L} = \frac{0.3}{2000} = 0.00015 = 150microstrains = 150\mu\varepsilon$$

GIVEN EXAMPLES

1. A concrete cylinder of diameter 150 mm and length 300 mm when subjected to an axial compressive load of 240 kN resulted in an increase in of diameter by 0.127 mm and a decrease in length of 2.08 mm. Compute the value of Poisson's ratio μ $(= \frac{1}{m})$ and Modulus of Elasticity E.

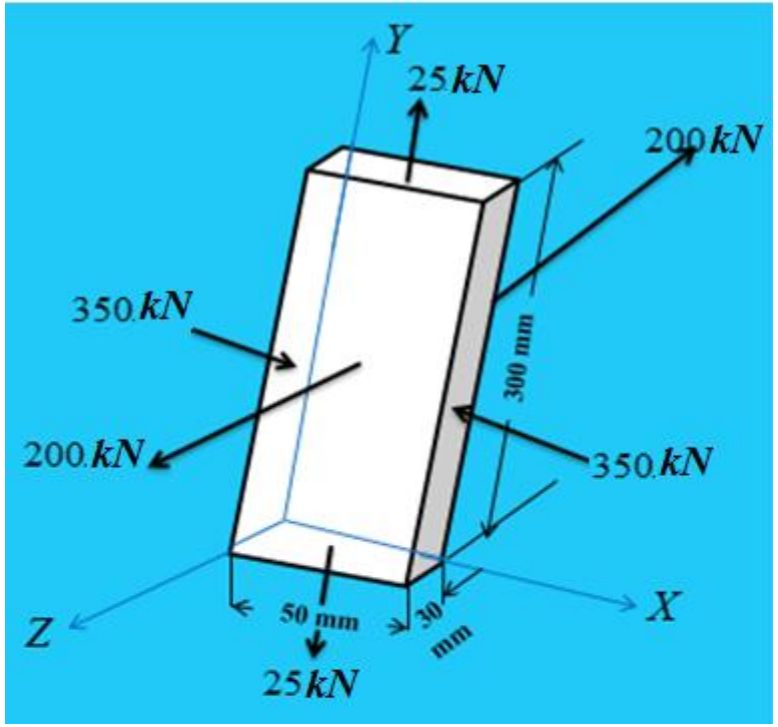
2. For a given material: Young's modulus , E =110GN/m^2;
   diameter of round bar, d = 37.5 mm = 0.0375 m;
   length of the bar, L = 2.4 m;
   extension of the bar, δL = 2.5 mm = 0.0025 m; and
   Poisson's ratio, μ = 0.31

   Find the bulk Modulus and lateral contraction of that round bar.

3. A flat rectangular, 300 mm long and of 30 mm x 50 mm uniform section, is acted upon by the following forces uniformly distributed over the respective cross-section; 25 kN in the direction of length (tensile); 350 kN in the direction of the width (compressive); and 200 kN in the direction of thickness (tensile).

Determine the change in volume of the flat.

Take $E = 140$ GN/m^2, and $m = 4$.



Solutions

1. Poisson's ratio, μ:

Longitudinal strain, $\varepsilon_l = \frac{\delta L}{L} = \frac{2.08}{300} = 0.006933$

Lateral strain, $\varepsilon_t = \frac{\delta d}{d} = \frac{0.127}{150} = 0.000846$

Poisson's ratio, $\mu = \frac{Lateral\ strain}{Longitudinal\ strain} = \frac{\varepsilon_t}{\varepsilon_l} = \frac{0.000846}{0.00693} = 0.122$

Modulus of Elasticity, E:

Using the relation, $E = \frac{Stress}{Strain(Longitudinal)} = \frac{\sigma}{\varepsilon_l} = \frac{P/A}{\delta L/L}$

$E = \frac{240/(\frac{\pi}{4}\times 0.15^2)}{(\frac{0.00028}{0.3})} = \frac{240\times 4\times 0.3}{\pi\times 0.15^2\times 0.00028} = 14.55 \times 10^6 \text{kN}/m^2$

Young's Modulus, $E = 14.55\ \text{GN}/m^2$

2. If Poisson's ratio, $\frac{1}{m}$ = 0.31, Then $m = \frac{1}{0.31}$ = 3.226

Substituting the value of *m* in the equation, we get

$$K = \frac{mE}{3(m-2)} = \frac{3.226 \times 110 \times 10^9}{3(3.226-2)} = 96.48 \text{ GN}/m^2$$

Longitudinal strain, $\varepsilon_l = \frac{\delta L}{L} = \frac{0.0025}{2.4}$ = 0.00104

and

Lateral strain, ε_t = Longitudinal strain x $\frac{1}{m}$ = ε_l x $\frac{1}{m}$

= 0.00104 x $\frac{1}{m}$ = 0.00104 x $\frac{1}{3.22}$ = 0.000323

Lateral contraction, δd = Lateral strain x d

= ε_t x d

= 0.000323 x 37.5 = 0.0121 mm

3. The stresses in the direction of the axes (X, Y, Z) are:

$$\sigma_x = \frac{P_x}{A_x} = \frac{350000}{0.003 \times 0.3} = 38.8 \times 10^6 N/m^2 \text{ (compressive)}$$

$$\sigma_y = \frac{P_y}{A_y} = \frac{25000}{0.05 \times 0.03} = 16.67 \times 10^6 N/m^2 \text{ (tensile)}$$

$$\sigma_z = \frac{P_z}{A_z} = \frac{200000}{0.3 \times 0.05} = 13.33 \times 10^6 N/m^2$$

The strains along the three principal directions are:

$$\varepsilon_x = -\frac{38.8 \text{ x } 10^6}{E} - \frac{16.67 \text{ x } 10^6}{mE} - \frac{13.33 \text{ x } 10^6}{mE} = -\frac{10^6}{E}\left(38.8 + \frac{30}{m}\right)$$

$$\varepsilon_y = \frac{16.67 \text{ x } 10^6}{E} + \frac{38.8 \text{ x } 10^6}{mE} - \frac{13.33 \text{ x } 10^6}{mE} = \frac{10^6}{E}\left(16.67 + \frac{25.47}{m}\right)$$

$$\varepsilon_z = \frac{13.33 \text{ x } 10^6}{E} - \frac{16.67 \text{ x } 10^6}{mE} + \frac{38.8 \text{ x } 10^6}{mE} = \frac{10^6}{E}\left(13.33 + \frac{22.13}{m}\right)$$

Volumetric Strain $\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$

$$\varepsilon_V = -\frac{10^6}{E}\left(38.8 + \frac{30}{m}\right) + \frac{10^6}{E}\left(16.67 + \frac{25.47}{m}\right) + \frac{10^6}{E}\left(13.33 + \frac{22.13}{m}\right)$$

$$= \frac{10^6}{140 \text{ x } 10^9}(-4.0) = -0.0000286 = 0.0000286 \text{ (comp.)}$$

Change in volume (decrease in Volume)

$\delta V = \varepsilon_V \text{ x } V = 0.0000286 \text{ x } (0.3 \text{ x } 0.03 \text{ x } 0.05)\ mm^3 = 12.87 \text{ x } 10^{-9}\ mm^3$