COLLEGE OF SCIENCE AND TECHNOLOGY

LECTURE NOTES OF PHYSICS FOR ENGINEERS II PHY1264

Compiled for Year 1, Engineering Options (UR–CST)

2019–2020

Contents

Physical Constants

Name	Symbol	Value	Unit
Number π	π	3.14159265358979323846	
Number e	e	2.71828182845904523536	
Euler's constant	$\gamma = \lim_{n\to\infty} \left(\sum_{k=1}^{n} 1/k - \ln(n) \right) = 0.5772156649$		
Elementary charge	e	$1.60217733 \cdot 10^{-19}$	C
Gravitational constant	G, κ	$6.67259 \cdot 10^{-11}$	$m^3kg^{-1}s^{-2}$
Fine-structure constant	$\alpha = e^2/2hc\varepsilon_0$	$\approx 1/137$	
Speed of light in vacuum	c	$2.99792458 \cdot 10^8$	m/s (def)
Permittivity of the vacuum	ε_0	$8.854187 \cdot 10^{-12}$	F/m
Permeability of the vacuum	μ_0	$4\pi \cdot 10^{-7}$	H/m
Coulomb Constant $(4\pi\varepsilon_0)^{-1}$	k_e	$8.9876 \cdot 10^9$	Nm^2C^{-2}
Planck's constant	h	$6.6260755 \cdot 10^{-34}$	Js
Dirac's constant	$\hbar = h/2\pi$	$1.0545727 \cdot 10^{-34}$	Js
Bohr magneton	$\mu_B = e\hbar/2m_e$	$9.2741 \cdot 10^{-24}$	Am^2
Bohr radius	a_0	0.52918	Å
Rydberg's constant	Ry	13.595	eV
Electron Compton wavelength	$\lambda_{Ce} = h/m_e c$	$2.2463 \cdot 10^{-12}$	m
Proton Compton wavelength	$\lambda_{Cp} = h/m_p c$	$1.3214 \cdot 10^{-15}$	m
Reduced mass of the H-atom	μ_H	$9.1045755 \cdot 10^{-31}$	kg
Stefan-Boltzmann's constant	σ	$5.67032 \cdot 10^{-8}$	$Wm^{-2}K^{-4}$
Wien's constant	k_W	$2.8978 \cdot 10^{-3}$	mK
Molar gas constant	R	8.31441	$J \cdot mol^{-1} \cdot K^{-1}$
Avogadro's constant	N_A	$6.0221367 \cdot 10^{23}$	mol^{-1}
Boltzmann's constant	$k = R/N_A$	$1.380658 \cdot 10^{-23}$	J/K
Electron mass	m_e	$9.1093897 \cdot 10^{-31}$	kg
Proton mass	m_p	$1.6726231 \cdot 10^{-27}$	kg
Neutron mass	m_n	$1.674954 \cdot 10^{-27}$	kg
Elementary mass unit	$m_u = \frac{1}{12} m({}^{12}_{6}C)$	$1.6605656 \cdot 10^{-27}$	kg
Nuclear magneton	μ_N	$5.0508 \cdot 10^{-27}$	J/T
Diameter of the Sun	$D_\odot$	$1392 \cdot 10^6$	m
Mass of the Sun	$M_\odot$	$1.989 \cdot 10^{30}$	kg
Rotational period of the Sun	$T_\odot$	25.38	days
Radius of Earth	R_A	$6.378 \cdot 10^6$	m
Mass of Earth	M_A	$5.976 \cdot 10^{24}$	kg
Rotational period of Earth	T_A	23.96	hours
Earth orbital period	Tropical year	365.24219879	days
Astronomical unit	AU	$1.4959787066 \cdot 10^{11}$	m
Light year	lj	$9.4605 \cdot 10^{15}$	m
Parsec	pc	$3.0857 \cdot 10^{16}$	m
Hubble constant	H	$\approx (75 \pm 25)$	$km \cdot s^{-1} \cdot Mpc^{-1}$

Chapter 1

ELECTRICITY

1.1 ELECTRIC FORCES AND ELECTRIC FIELDS

Electricity is a general term encompassing a variety of phenomena resulting from the presence and flow of electric charge. These include many easily recognizable phenomena, such as lightning, static electricity, and the flow of electrical current in electrical wires. In addition, electricity encompasses less familiar concepts such as the electromagnetic field and electromagnetic induction.
The word electricity is from the New Latin electricus, "*amber–like*", coined in the year 1600 from the Greek electron meaning amber (hardened plant resin), because static electricity effects were produced classically by rubbing amber.
In this chapter we will study basic concepts and principles related to electricity, magnetism will be the subject of the next chapter.

1.1.1 Electric Charge

An electric charge is an intrinsic characteristics of the fundamental particles, such as a protons and electrons, which make up different objects. In facts, all atoms are composed of protons, electrons and neutrons. The first two of these particles have positive and negative charges, respectively.
Thus, there are two types of observed electric charge, which we designate as positive and negative; the convention was derived from Benjamin Franklin's experiments. He rubbed a glass rod with silk and called the charges on the glass rod positive. He rubbed sealing wax with fur and called the charge on the sealing wax negative.

The unit of charge is called the **Coulomb, (C)**.
The smallest unit of free charge known in nature is the charge of an electron or proton, which has a magnitude of $e = 1.60217733 \times 10^{-19}$ C.

Electrons and protons are not the only things that carry charges. Other particles (positrons, for example) also carry charge in multiples of the electronic charge. When we say that an object is charged, we mean that it contains an excess charge, either an excess of protons or an excess of electrons.
Charge comes in multiples of an indivisible unit of charge, represented by the letter **e**. In other words, charge comes in multiples of the charge on the electron or the proton. These things have the same size charge, but the sign is different. A proton has a charge of $+e$, while an electron has a charge of $-e$.

Properties of Electric Charge

Electric charge observes some properties.

Putting "charge is quantified" in terms of an equation, we say:

$$q = ne \tag{1.1}$$

where q is the symbol used to represent charge, while n is a positive or negative integer, and e is the electronic charge,$e = 1.60217733 \times 10^{-19}$ C. Meaning that electric charge is a quantized physical quantity.

Charge with opposite electrical sign (i.e. a positive charge and a negative charge) attract each other and charge with the same electrical sign (i.e. two positive charges or two negative charges)repeal each other.

The law of conservation of charge states that "*The net charge of an isolated system remains constant*". If a system starts out with an equal number of positive and negative charges, there is nothing we can do to create an excess of one kind of charge in that system unless we bring in charge from outside the system (or remove some charge from the system). Likewise, if something starts out with a certain net charge, say +100 e, it will always have +100 e unless it is allowed to interact with something external to it.
Charge can be created and destroyed, but only in positive-negative pairs. Within the system, charge may be transferred between bodies, either by direct contact, or by passing along a conducting material, such as a wire. The informal term static electricity refers to the net presence (or "*imbalance*") of charge on a body, usually caused when dissimilar materials are rubbed together, transferring charge from one to the other.

Electrostatic Charging

Forces between two electrically-charged objects can be extremely large. Most things are electrically neutral; they have equal amounts of positive and negative charge. If this was not the case, the world we live in would be a much stranger place. We also have a lot of control over how things get charged. This is because we can choose the appropriate material to use in a given situation.

Metals are good conductors of electric charge, while plastics, wood, and rubber are not. They are called insulators. Charge does not flow nearly as easily through insulators as it does through conductors, which is why wires you plug into a wall socket are covered with a protective rubber coating. Charge flows along the wire, but not through the coating to you.
Materials are divided into three categories, depending on how easily they will allow charge (i.e., electrons) to flow along them. These are:

- **Conductors**–metals, for example;
- **Semi-conductors**–silicon is a good example;
- **Insulators**–rubber, wood plastic are some examples

Most materials are either conductors or insulators. The difference between them is that in conductors, the outermost electrons in the atoms are so loosely bound to their atoms that they are free to travel around. In insulators, on the other hand, the electrons are much more tightly bound to the atoms, and are not free to flow. Semi-conductors are a very useful intermediate class, not as conductive as metals but considerably more conductive than insulators. By adding certain impurities to semi-conductors in the appropriate concentrations the conductivity can be well-controlled.
There are three ways that objects can be given a net charge. These are:

a) Charging by friction: This is useful for charging insulators. If you rub one material with another (say, a plastic ruler with a piece of paper towel), electrons have a tendency to be transferred from one material to the other. For example, rubbing glass with silk or saran wrap generally leaves the glass with a positive charge; rubbing PVC rod with fur generally gives the rod a negative charge.

b) Charging by contact: It is useful for charging metals and other conductors. If a charged object touches a conductor, some charge will be transferred between the object and the conductor, charging the conductor with the same sign as the charge on the object.

c) Charging by induction: This is also useful for charging metals and other conductors. Again, a charged object is used, but this time it is only brought close to the conductor, and does not touch it. If the conductor is connected to ground (ground is basically anything neutral that can give up electrons to, or take electrons from, an object), electrons will either flow on to it or away from it. When the ground connection is removed, the conductor will have a charge opposite in sign to that of the charged object.

Coulomb's Law

As we have mentioned, two electric charges of like sign repel and two electric charges of opposite sign attract each other with a force that is proportional to the product of their charges and inversely proportional to the square of the distance between them.
Consider a system of two point charges, q_1 and q_2, separated by a distance r in vacuum. The force exerted by, q_1 on q_2 is given by Coulomb's law:

$$\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{r} \tag{1.2}$$

Where k_e is the coulomb constant, and $\hat{r} = \frac{\vec{r}}{r}$ is a unit vector directed from q_1 to q_2 as illustrated in the Figure 1.1a.

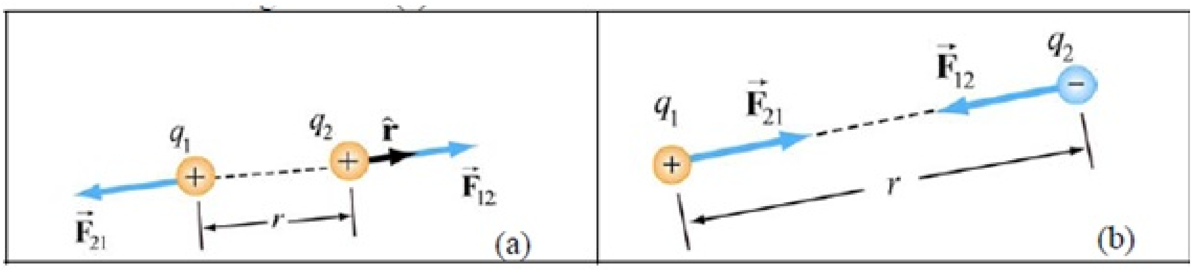

Figure 1.1: *Interaction between two charges*

Note that electric force is a vector which has both magnitude and direction.
Similarly, the force on q_1 due to q_2 is given by $\vec{F}_{12} = \vec{F}_{21}$, as illustrated in Figure 1.1b. This is consistent with Newton's third law. If the medium in which charges are placed is different from the free space the Coulomb's law is given by:

$$\vec{F}_e = \frac{1}{4\pi\varepsilon} \frac{qq'}{r^2} \hat{r} \tag{1.3}$$

where ε is the permittivity of the medium in which charges are situated.

Coulomb's law applies to any pair of point charges. When more than two charges are present, the net force on any one charge is simply the vector sum of the forces exerted on it by the other charges. For example, if three charges are present, the resultant force experienced by q_3 due to q_1 and q_2 will be

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} \tag{1.4}$$

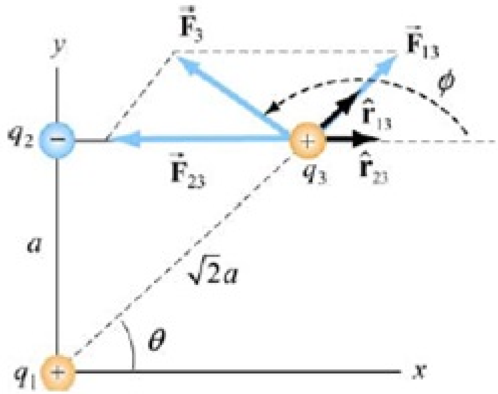

Figure 1.2: *Superposition principle applied on a system of threes charges*

For a system of N charges, the net force experienced by the j^{th} particle would be

$$\vec{F}_j = \sum_{i=1, i\neq j}^{N} \vec{F}_{ij} \tag{1.5}$$

The superposition principle implies that the net force between any two charges is independent of the presence of other charges. This is true if the charges are in fixed positions.

1.1.2 Electric Field

Definition

The region around a charged body within which it can exert its electrostatic influence may be called an electric field. In principle, it extends to infinity, but in practice it falls off more or less rapidly with distance. We can define the *intensity* or *strength* **E** of an electric field as follows: Suppose that we place a small test charge **q** in an electric field. This charge will then experience a force. The ratio of the force to the charge is called the *intensity of the electric field*, or, more usually, simply the *electric field*. Thus, we have used the words "electric field" to mean either the region of space around a charged body, or, quantitatively, to mean its intensity.
Usually it is clear from the context which is meant, but, if you wish, you may elect to use the longer phrase intensity of the electric field if you want to remove all doubt. The field and the force are in the same direction, and the electric field is a vector quantity, so the definition of the electric field can be written as

$$\vec{E} = \lim_{q_o \to 0} \frac{\vec{F}}{q_o} \tag{1.6}$$

The limiting process is included in the definition of $\vec{E}$ to ensure that the test charge does not affect the charge distribution that produces $\vec{E}$. To determine the electric field $\vec{E}$ due to a point charge q at any point P a distance r from the point charge, we put a test charge q_o at the point P, and the electric field is given by:

$$\vec{E} = \frac{\vec{F}}{q_o} \tag{1.7}$$

We take q_0 to be infinitesimally small so that the field generates does not disturb the source charges. The analogy between the electric field and the gravitational field is depicted in the Figure 1.3. From the field theory point of view, we say that the charge q creates an

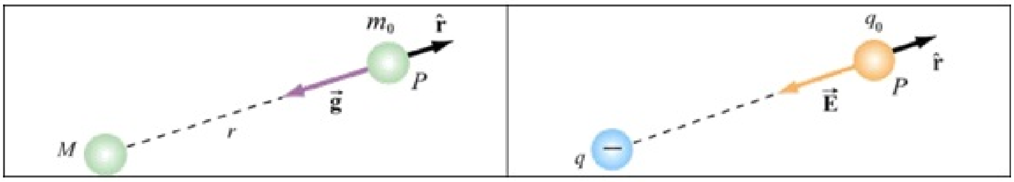

Figure 1.3: *Analogy between the gravitational field $\vec{g}$ and the electric field $\vec{E}$*

electric field $\vec{E}$ which exerts a force $\vec{F_e} = q_o \times \vec{F_e}$ on a test charge q_o.
Using the definition of electric field given in the Equation 1.7, and the Coulomb's law, the electric field at a distance r from a point charge q is given by

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} \tag{1.8}$$

For the continuous charge distribution, the electric field $\vec{E}(\vec{r})$ at a point P (P is distant to r from the charged elements) due to all charged elements in the charge distribution is given by

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\hat{r}}{r^2}dq \tag{1.9}$$

where the integration is over entire charge distribution. This integration is a vector operation and must be treated appropriately. When performing such calculation, it is convenient to use the concept of *charge density* along with the following notation:

- $dq = \rho dV$, where ρ is the volume charge density; dV is small volume element;
- $dq = \sigma dA$, where σ is the surface charge density; dA is small area element;
- $dq = \lambda dl$, where λ is the linear charge density; dl is small length element.

The electric field $\vec{E}$ obey the superposition principle, that is the total electric field due to a group of charges is equal to the vector sum of the electric fields of individual charges:

$$\vec{E} = \sum_i \vec{E_i} \tag{1.10}$$

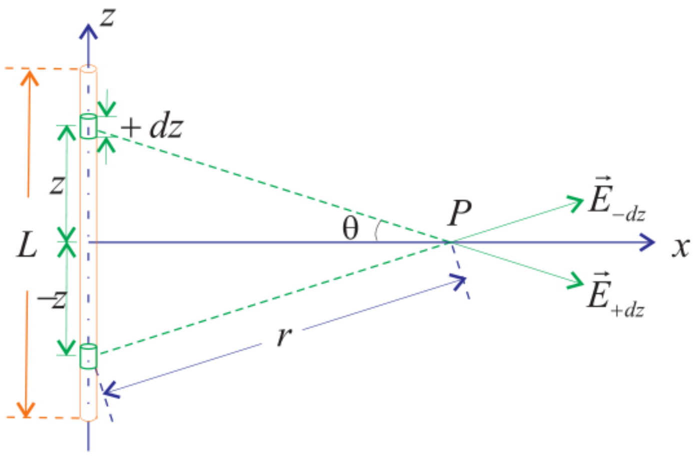

Figure 1.4: *Electric field of a uniform line of charge*

Example 1: Uniform line of charge

A non-conducting rod of length L with a uniform charge density λ and a total charge Q is lying along the z -axis, as illustrated in Figure 1.4. Compute the electric field at a point P, located at a distance x from the center of the rod along its perpendicular bisector.

- *Symmetry considered*: The E–field from $+z$ and $-z$ directions cancel along z–direction. Only horizontal E–field components need to be considered.

- For each element of length dz, charge $dq = \lambda dz$; Horizontal E-field at point P due to element dz

$$|d\vec{E}|cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda dz}{r^2}cos\theta$$

E-field due to entire line charge at point P is

$$E = \int_{-L/2}^{L/2} \frac{1}{4\pi\epsilon_0}\frac{\lambda dz}{r^2}cos\theta = 2\int_0^{L/2} \frac{1}{4\pi\epsilon_0}\frac{\lambda dz}{r^2}cos\theta$$

To calculate this integral:

First, notice that x is fixed, but z, r, θ all varies.

Change of variable (from z to θ):

$z = x\ tan\theta \quad \Rightarrow \quad dz = x\ sec^2\theta\ d\theta$

$x = r\ cos\theta \quad \Rightarrow \quad r^2 = x^2\ sec^2\theta$

When $z = 0\ ;\ \theta = 0^0 \quad and\ when \quad z = L/2\ ;\ \ \theta = \theta_0\ , \quad where \ \ tan\theta_0 = \frac{L/2}{x}$

$$E = 2\frac{\lambda}{4\pi\epsilon_0}\int_0^{\theta_0} \frac{x\ sec^2\theta}{x^2\ sec^2\theta}d\theta\ cos\theta = 2\frac{\lambda}{4\pi\epsilon_0}\int_0^{\theta_0}\frac{1}{x}\ cos\theta\ d\theta = 2\frac{\lambda}{4\pi\epsilon_0}\ \frac{1}{x}\ (sin\theta)|_0^{\theta_0}$$

$$= 2\frac{\lambda}{4\pi\epsilon_0}\ \frac{1}{x}\ sin\theta_0 = 2\frac{\lambda}{4\pi\epsilon_0}\ \frac{1}{x}\ \frac{L/2}{\sqrt{x^2 + (L/2)^2}}$$

$$E = \frac{1}{4\pi\epsilon_0}\ \frac{\lambda L}{x\ \sqrt{x^2 + (L/2)^2}} \tag{1.11}$$

Important limiting cases:

- $x \gg L$; λL = Total charge on rod; therefore the system behave like a point charge.

$$E = \frac{1}{4\pi\epsilon_0}\frac{\lambda L}{x^2}. \tag{1.12}$$

- $L \gg x$; Electric field due to infinitely long line of charges:

$$E = \frac{1}{4\pi\epsilon_0}\frac{\lambda L}{x\ L/2} = \frac{\lambda}{2\pi\epsilon_0\ x} \tag{1.13}$$

Example 2: Electric field on the axis of a ring

A non-conducting ring of radius R with a uniform charge density λ and a total charge Q is lying in the xy - plane, as shown in Figure 1.5. Compute the electric field at a point P, located at a distance z from the center of the ring along its axis of symmetry.

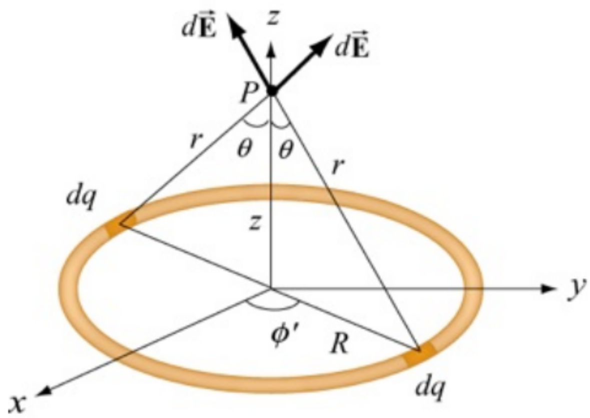

Figure 1.5: *Electric field of a uniform ring of charge*

Consider a small length element dl' on the ring. The amount of charge contained within this element is $dq = \lambda dl' = \lambda R d\phi'$. Its contribution to the electric field at P is:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\,\widehat{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda R}{r^2}\,d\phi'\,\widehat{r}$$

Using the symmetry argument illustrated in Figure 1.5, we see that the electric field at P must point in the $+z$ -direction. The z -component of the electric field is given by:

$$dE_z = dE\ cos\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda R d\phi'}{R^2+z^2}\frac{z}{\sqrt{R^2+z^2}} = \frac{\lambda}{4\pi\epsilon_0}\frac{Rz\ d\phi'}{(R^2+z^2)^{3/2}}$$

Upon integrating over the entire ring, we obtain:

$$E_z = \frac{\lambda}{4\pi\epsilon_0}\frac{Rz}{(R^2+z^2)^{3/2}}\int_0^{2\pi} d\phi' = \frac{\lambda}{4\pi\epsilon_0}\frac{2\pi Rz}{(R^2+z^2)^{3/2}} = \frac{1}{4\pi\epsilon_0}\frac{Qz}{(R^2+z^2)^{3/2}} \tag{1.14}$$

where the total charge is $Q = \lambda 2\pi R$. Notice that the electric field at the center of the ring vanishes. This is to be expected from symmetry arguments.

Example 3: Electric field due to a uniformly charged disk

A uniformly charged disk of radius R with a total charge Q lies in the xy -plane. Find the electric field at a point P, along the z -axis that passes through the center of the disk perpendicular to its plane.

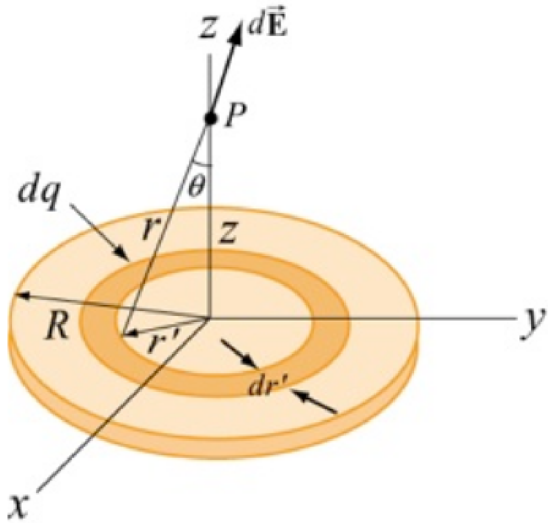

Figure 1.6: *A uniformly charged disk of radius R*

We find the E-field of a disk by integrating concentric rings of charges.

By symmetry arguments, the electric field at P points in the + z -direction. Since the ring has a charge $dq = \sigma(2\pi r' dr')$;from Eq. 1.14, we see that the ring gives a contribution to the z -component of the electric field

$$dE_z = \frac{1}{4\pi\epsilon_0}\frac{zdq}{(r'^2+z^2)^{3/2}} = \frac{1}{4\pi\epsilon_0}\frac{z\sigma(2\pi r'dr')}{(r'^2+z^2)^{3/2}}$$

Integrating from $r' = 0$ to $r' = R$, the z -component of the electric field at P becomes:

$$E_z = \int dE_z = \frac{\sigma z}{2\epsilon_0}\int_0^R \frac{r'dr'}{(r'^2+z^2)^{3/2}}$$

Change of variable:

$u = z^2 + r'^2 \quad \Rightarrow \quad (z^2+r'^2)^{3/2} = u^{3/2} \Rightarrow \quad du = 2rdr \quad \Rightarrow \quad rdr = \frac{1}{2}du$

Change of integration limit: $r' = 0, u = z^2$ and $r' = R, u = z^2 + R^2$

$$E_z = \frac{\sigma z}{4\epsilon_0}\int_{Z^2}^{R^2+z^2}\frac{du}{u^{3/2}} = \frac{\sigma z}{4\epsilon_0}\frac{u^{-1/2}}{-1/2}|_{R^2}^{R^2+z^2} = -\frac{\sigma z}{2\epsilon_0}[\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{z^2}] = \frac{\sigma}{2\epsilon_0}(\frac{z}{|z|} - \frac{z}{\sqrt{R^2+z^2}})$$

The above equation may be rewritten as:

$$E_z = \frac{\sigma}{2\epsilon_0}(1 - \frac{z}{\sqrt{R^2+z^2}}) \;\; for \;\; z > 0; \;\; and \;\; E_z = \frac{\sigma}{2\epsilon_0}(-1 - \frac{z}{\sqrt{R^2+z^2}}) \;\; for \;\; z < 0. \tag{1.15}$$

To show that the *point-charge* limit is recovered for $z \gg R$, we make use of the Taylor-series expansion:

$$1 - \frac{z}{\sqrt{R^2 + z^2}} = 1 - (1 + \frac{R^2}{z^2})^{-1/2} = 1 - (1 - \frac{1}{2}\frac{R^2}{z^2} + ...) \approx \frac{1}{2}\frac{R^2}{z^2}$$

The z -component of the electric field is then

$$E_z = \frac{\sigma}{2\epsilon_0}\frac{R^2}{2z^2} = \frac{\sigma}{4\pi\epsilon_0}\frac{\pi R^2}{z^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{z^2} \tag{1.16}$$

which is indeed the expected *point-charge* result. On the other hand, we may also consider the limit where $R \gg z$. Physically this means that the plane is very large, or the field point P is extremely close to the surface of the plane. The electric field in this limit becomes:

$$E_z = \frac{\sigma}{2\epsilon_0}(1 - \frac{z}{\sqrt{R^2 + z^2}}) \simeq \frac{\sigma}{2\epsilon_0}(1 - \frac{z}{R}) \approx \frac{\sigma}{2\epsilon_0} \tag{1.17}$$

$\Rightarrow$ E-field due to an infinite sheet of charge, charge density $= \sigma$.

Electric Field Lines

An electric field line is an imaginary line (or curve) drawn through a region of space so that its tangent at any point is in the direction of the electric field at that point. Electric field lines provide a convenient graphical representation of the electric field in space, the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the magnitude of $\vec{E}$.

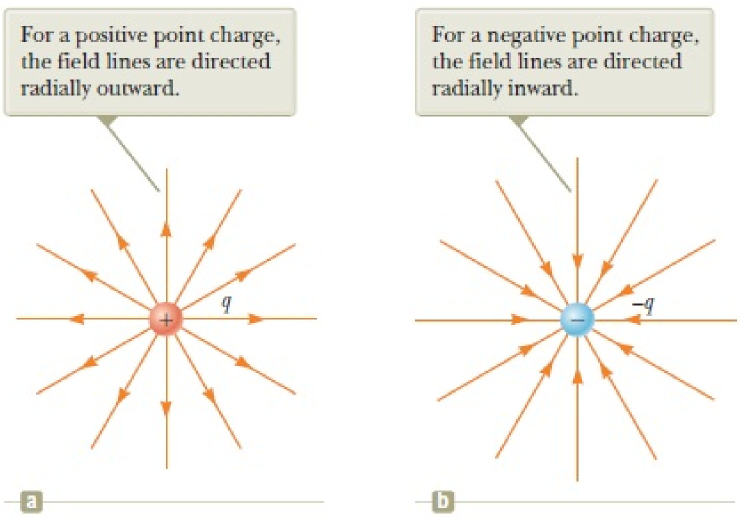

Figure 1.7: *Electric field lines for (a) positive charge and (b) negative charge*

Notice that the direction of field lines is radially outward for a positive charge and radially inward for a negative charge.
The pattern of electric field lines can be obtained by considering the following:

- **Symmetry**: For every point above the line joining the two charges there is an equivalent point below it. Therefore, the pattern must be symmetrical about the line joining the two charges;

- **Near field**: Very close to a charge, the field due to that charge predominates. Therefore, the lines are radial and spherically symmetric;

- **Far field**: Far from the system of charges, the pattern should look like that of a single point charge of value $Q = \sum_i Q_i$. Thus, the lines should be radially outward, unless $Q = 0$;

- **Null point**: This is a point at which $\vec{E} = 0$, and no field lines should pass through it.

The properties of electric field lines may be summarized as follows:

1. The direction of the electric field vector $\vec{E}$ at a point is tangent to the field lines;

2. The direction of the electric field vector $\vec{E}$ at a point is tangent to the field lines;

3. The number of lines per unit area through a surface perpendicular to the line is devised to be proportional to the magnitude of the electric field in a given region;

4. The field lines must begin on positive charges (or at infinity) and then terminate on negative charges (or at infinity);

5. The number of lines that originate from a positive charge or terminating on a negative charge must be proportional to the magnitude of the charge;

6. No two field lines can cross each other; otherwise the field would be pointing in two different directions at the same point.

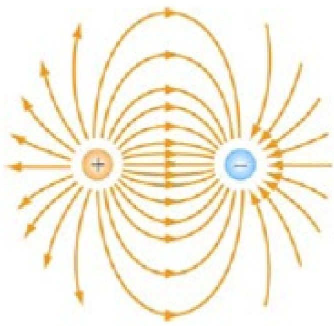

Figure 1.8: *Electric field lines for an electric dipole*

Motion of a Charged Particle in a Uniform Electric Field

When a particle of charge q and mass m is placed in an electric field $\vec{E}$, the electric force exerted on the charge is

$$\vec{F_e} = q\vec{E} \tag{1.18}$$

according to Coulomb's law and the definition of the electric field. If that is the only force exerted on the particle, it must be the net force, and it causes the particle to accelerate according to the Newton's second law. Therefore,

$$\vec{F_e} = q\vec{E} = m\vec{a} \tag{1.19}$$

and the acceleration of the particle is

$$\vec{a} = \frac{q\vec{E}}{m} \tag{1.20}$$

If $\vec{E}$ is uniform (that is, constant in magnitude and direction), and the particle is free to move, the electric force on the particle is constant and we ca apply the particle under constant acceleration law to the motion of the particle. If the particle has a positive charge, its acceleration is in the direction of the electric field, but if the particle has a negative charge, its acceleration is in the direction opposite to the electric field.

1.1.3 Electric Dipole

An electric dipole consists of two equal but opposite charges, $+q$ and $-q$, separated by a distance d (here $d = 2a$), as shown in Figure 1.9.

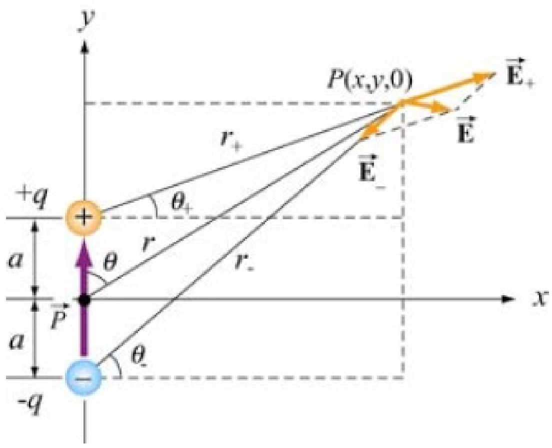

Figure 1.9: *An electric dipole*

An electric dipole is characterised by a *"dipole moment vector"*.
The dipole moment vector $\vec{p}$ which points from -q to +q (in the +y-direction, as in the Figure 1.9) is given by

$$\vec{p} = 2pa\vec{j} \tag{1.21}$$

The magnitude of the electric dipole is $p = 2qa$, where $q > 0$. For an overall charge-neutral system having N charges, the electric dipole vector $\vec{p}$ is defined as

$$\vec{p} = \sum_{i=1}^{N} q_i\vec{r_i} \tag{1.22}$$

where $\vec{r_i}$ is the position vector of the charge q_i .

Examples of dipoles include HCl, CO, H_2O molecules and other polar molecules. In principle, any molecule in which the centers of the positive and negative charges do not coincide may be approximated as a dipole.

Electric Field of an Electric Dipole

Referring to Figure 1.9, we see that the x-component of the electric field strength at the point P is

$$E_x = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta_+}{r_+^2} - \frac{\cos\theta_-}{r_-^2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{x}{[x^2+(y-a)^2]^{3/2}} - \frac{x}{[x^2+(y+a)^2]^{3/2}}\right) \quad (1.23)$$

Where $r_\pm = \sqrt{r^2+a^2\mp 2ra\cos\theta} = \sqrt{x^2+(y\mp a)^2}$,
Similarly, y-component is

$$E_y = \frac{q}{4\pi\varepsilon_0}\left(\frac{\cos\theta_+}{r_+^2} - \frac{\cos\theta_-}{r_-^2}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{y-a}{[x^2+(y-a)^2]^{3/2}} - \frac{y+a}{[x^2+(y+a)^2]^{3/2}}\right) \quad (1.24)$$

In the "point-dipole" limit, when $r \gg a$, the above expressions reduce to

$$E_x = \frac{3p}{4\pi\varepsilon_0 r^2}\sin\theta\cos\theta \quad (1.25a)$$

$$E_y = \frac{p}{4\pi\varepsilon_0 r^3}\left(3\cos^2\theta - 1\right) \quad (1.25b)$$

where $\sin\theta = \frac{x}{r}$, and $\cos\theta = \frac{y}{r}$

With $3pr\cos\theta = 3\vec{p}\cdot\vec{r}$ and some algebra, the electric field due to the electric dipole may be written as

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(-\frac{\vec{p}}{r^3} + \frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5}\right) \quad (1.26)$$

Note that the above equation is valid also in three dimensions where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$r. The equation indicates that the electric field $\vec{E}$ due to a dipole decreases with r as $1/r^3$, unlike the $1/r^2$ behavior for a point charge. This is to be expected since the net charge of a dipole is zero and therefore must fall off more rapidly than $1/r^2$ at large distance.

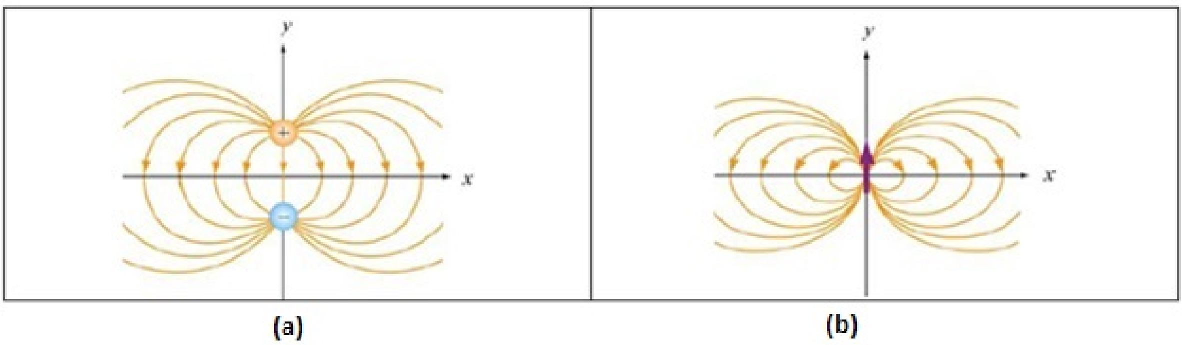

Figure 1.10: *Electric field lines for (a) a finite dipole and (b) a point dipole*

The electric field lines due to a finite electric dipole and a point dipole are shown in Figure 1.10 .

Electric Dipole in Electric Field

What happens when we place an electric dipole in a uniform field, $E = E\vec{i}$, with the dipole moment vector $\vec{p}$ making an angle with the x-axis? From Figure 1.11, we see that the unit vector which points in the direction of $\vec{p}$ is $\cos\theta\vec{i} + \sin\theta\vec{j}$. Thus, we have $\vec{p} = 2qa(\cos\theta\vec{i} + \sin\theta\vec{j})$ As we see in the Figure 1.11, since each charge experiences an

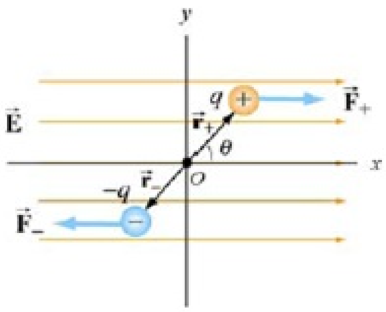

Figure 1.11: Electric dipole placed in a uniform electric field

equal but opposite force due to the field, the net force on the dipole is $\vec{F}_{net} = \vec{F}_+ + \vec{F}_- = 0$. Even though the net force vanishes, the field exerts a torque on the dipole. The torque about the midpoint O of the dipole is

$$\begin{aligned}
\vec{\tau} &= \vec{r}_+ \times \vec{F}_+ + \vec{r}_- \times \vec{F}_- \\
&= \left(a\cos\theta\vec{i} + a\sin\theta\vec{j}\right) \times \left(F_+(\vec{i})\right) + \left(-a\cos\theta\vec{i} - a\sin\theta\hat{j}\right) \times \left(F_-(\vec{-i})\right) \\
&= a\sin\theta F_+(-\vec{k}) + a\sin\theta F_-(-\vec{k}) \\
&= 2aF\sin\theta(-\vec{k})
\end{aligned}$$

Thus,

$$\vec{\tau} = 2aF\sin\theta\,(-\vec{k}) \tag{1.28}$$

Where we have used $F_- = F_+ = F$. The direction of the torque is $-\vec{k}$, or into the page. The effect of the torque $\vec{\tau}$ is to rotate the dipole clockwise so that the dipole moment $\vec{p}$ becomes aligned with the electric field $\vec{E}$.
With $\vec{F} = q\vec{E}$, the magnitude of the torque can be rewritten as

$$\tau = (2aq)E\sin\theta = pE\sin\theta \tag{1.29}$$

And so, the general expression for toque becomes

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{1.30}$$

1.1.4 Electric Flux and Gauss' Law

Electric Flux

Electric flux is a measure of the number of electric field lines passing through a given surface. To calculate the flux through a particular surface, multiply the surface area by the component of the electric field perpendicular to the surface. If the electric field is parallel to the surface, no field lines pass through the surface and the flux will be zero. The maximum flux occurs when the field is perpendicular to the surface.

- For a plane surface, the surface vector $\vec{A}$ is defined as $\vec{A} = A \cdot \hat{e}$. Where $\hat{e}$ is the normal vector to the surface;
- When the electric field $\vec{E}$ is not uniform , i.e. $\vec{E} = E(x, y, z)$; or if the surface is not a plane, then

$$\Phi_E = \sum_i \vec{E} \cdot \left(\Delta \vec{A}_i\right) \tag{1.31}$$

or in the integral form

$$\Phi_E = \int \vec{E} \cdot d\vec{A} \tag{1.32}$$

- If the surface through which the flux is calculated is closed, the sign of the flux is positive for field lines that leave the enclosed volume of the surface; and it is negative for field lines that enter the enclosed volume of the surface.

Gauss's Law

The Gauss's law relates the net flux of the Φ_E of electric field though a closed surface (known as Gaussian surface) to the net charge q enclosed by the surface and, it states that:

> *"The net electric flux Φ_E of electric field though any closed surface is equal to $1/\varepsilon_0$ times the net charge* q *enclosed by the surface and is independent of the shape of the surface".*

or in the integral form:

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \tag{1.33}$$

The Gauss's law is valid for any Gaussian surface. However, it is easy to use Gauss's law to find electric field due to some symmetrical charge distributions. When we want to apply Gauss's law to find electric field, Gaussian surface should be selected carefully so that it will be easy to find electric flux through it. Therefore, the suitable Gaussian surface should have the following characteristics:

1. The surface is closed (volume is contained inside a closed surface), but it can be made up of several surface elements,
2. At each point of the surface, $\vec{E}$ is either tangential or normal to the surface,
3. E is constant over that part of the surface where $\vec{E}$ is normal to the surface (that is $\vec{E}$ is parallel to the surface vector, $d\vec{A}$).

When we want to apply Gauss's law to find electric field at a point P due to a charge distribution, the following steps are useful:

Step 1: Write Gauss's law

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

It is assumed that you can interpret it.

Step 2: Construct a suitable Gaussian surface passing though point P, where you want to find electric field. At point P, $\vec{E}$ should be parallel to the surface element vector $\vec{A}$. The suitable Gaussian surface should have the characteristics as discussed above.

Note that ti is a very important step as it clear from the Gauss's law that if you want to find electric flux Φ through a Gaussian surface, first you need to think about a suitable closed surface.

Step 3: Determine electric flux Φ through the Gaussian surface.

Step 4: Determine charge q enclosed by the Gaussian Surface.

Step 5: Put Φ and q, determined in steps 3 and 4 respectively, in Gauss's law (step 1) and solve for E.

Example 1: Infinitely long rod of uniform charge density

An infinitely long rod of negligible radius has a uniform charge density λ. Calculate the electric field at a distance r from the wire. The charge density is uniformly distributed

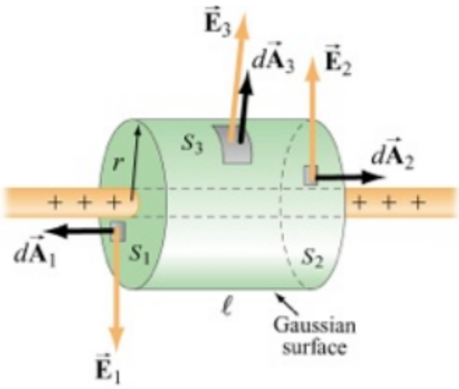

Figure 1.12: *Gaussian surface for a uniformly charged rod*

throughout the length, and the electric field $\vec{E}$ must be point radially away from the symmetry axis of the rod. The magnitude of the electric field is constant on cylindrical surfaces of radius r. Therefore, we choose a coaxial cylinder as our Gaussian surface.

- The amount of charge enclosed by the Gaussian surface, a cylinder of radius r and length l (Figure 1.12), is $q_enc = \lambda l$.

- As indicated in Figure 1.12, the Gaussian surface consists of three parts: two end-cap surfaces S_1 and S_2 plus the cylindrical sidewall S_3. The flux through the Gaussian surface is

$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = \iint_{S_1} \vec{E}_1\cdot d\vec{A}_1 + \iint_{S_2} \vec{E}_2\cdot d\vec{A}_2 + \iint_{S_3} \vec{E}_3\cdot d\vec{A}_3 = 0+0+E_3 A_3 = E(2\pi r l);$$

  where we have set $E_3 = E$. As can be seen from the figure, no flux passes through the ends since the area vectors $d\vec{A}_1$ and $d\vec{A}_2$ are perpendicular to the electric field which points in the radial direction.

- Applying Gauss's law gives $E(2\pi rl) = \frac{\lambda l}{\epsilon_0}$, or
$$E = \frac{\lambda}{2\pi\epsilon_0 r} \tag{1.34}$$
The result is in complete agreement with that obtained using Coulomb's law. Notice that the result is independent of the length l of the cylinder, and only depends on the inverse of the distance r from the symmetry axis.

Example 2: Infinite plane of charge

Consider an infinitely large non-conducting plane in the xy-plane with uniform surface charge density σ. Determine the electric field everywhere in space. We choose our Gaussian

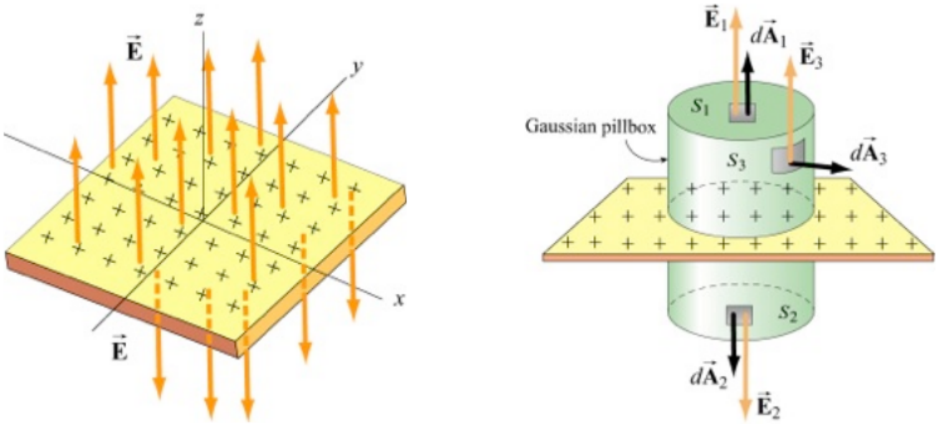

Figure 1.13: *Electric field for uniform plane of charge and Gaussian surface for calculating the electric field due to a large plane.*

surface to be a cylinder, which is often referred to as a pillbox (Figure 1.13). The pillbox also consists of three parts: two end-caps S_1 and S_2, and a curved side S_3.

- Since the surface charge distribution on is uniform, the charge enclosed by the Gaussian pillbox is $q_{enc} = \sigma A$, where $A = A_1 = A_2$ is the area of the end-caps.
- The total flux through the Gaussian pillbox flux is
$$\Phi_E = \oiint \vec{E}\cdot d\vec{A} = \iint_{S_1} \vec{E_1}\cdot d\vec{A_1} + \iint_{S_2} \vec{E_2}\cdot d\vec{A_2} + \iint_{S_3} \vec{E_3}\cdot d\vec{A_3} = E_1A_1 + E_2A_2 + 0 = (E_1 + E_2)A$$
Because the two ends are at the same distance from the plane, by symmetry, the magnitude of the electric field must be the same: $E_1 = E_2 = E$. Hence, the total flux can be rewritten as
$$\Phi_E = 2EA$$
- By applying Gauss's law, we obtain
$$2EA = \frac{q_{enc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$
Therefore the magnitude of the electric field is
$$E = \frac{\sigma}{2\epsilon_0} \tag{1.35}$$
Thus, we see that the electric field due to an infinite large non-conducting plane is uniform in space.

Example 3: Spherical shell

A thin spherical shell of radius a has a charge $+Q$ evenly distributed over its surface. Find the electric field both inside and outside the shell. The charge distribution is spherically

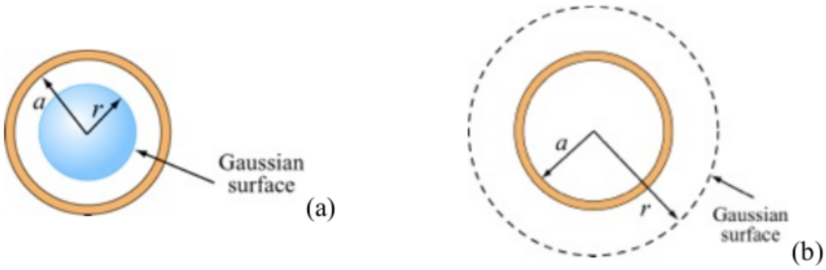

Figure 1.14: *Gaussian surface for uniformly charged spherical shell for (a) $r \leq a$, and (b) $r \geq a$.*

symmetric, with a surface charge density $\sigma = \frac{Q}{A_s} = \frac{Q}{4\pi a^2}$, where $A_s = 4\pi a^2$ is the surface area of the sphere. The electric field $\vec{E}$ must be radially symmetric and directed outward. We treat the regions $r \leq a$ and $r \geq a$ separately.
Case 1: $r \leq a$: We choose our Gaussian surface to be a sphere of radius $r \leq a$, as shown in Figure 1.14(a).

- The charge enclosed by the Gaussian surface is $q_{enc} = 0$ since all the charge is located on the surface of the shell. Thus, from Gauss' law, $\Phi_E = \frac{q_{enc}}{\epsilon_0}$, we conclude

$$E = 0 \; ; \quad r < a$$

Case 2: $r \geq a$. In this case, the Gaussian surface is a sphere of radius $r \geq a$, as shown in Figure 1.14(b).

- Since the radius of the "Gaussian sphere" is greater than the radius of the spherical shell, all the charge is enclosed $q_{enc} = Q$. Because the flux through the Gaussian surface is

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = EA = E(4\pi r^2)$$

- By applying Gauss's law, we obtain

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \; ; \quad r > a \tag{1.36}$$

Note that the field outside the sphere is the same as if all the charges were concentrated at the center of the sphere.

Example 4: Non-conducting solid sphere

An electric charge $+Q$ is uniformly distributed throughout a non-conducting solid sphere of radius a. The charge distribution is spherically symmetric with the charge density given by

$$\rho = \frac{Q}{V} = \frac{Q}{(4/3)\pi a^2}$$

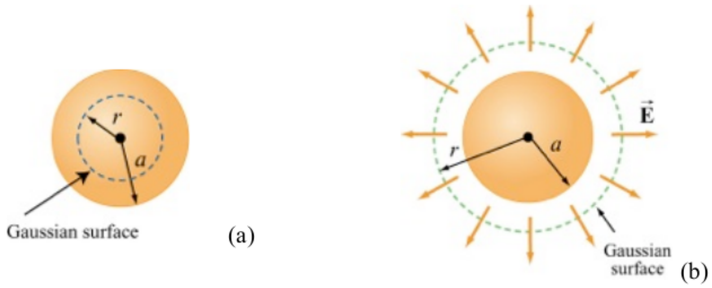

Figure 1.15: *Gaussian surface for uniformly charged solid sphere, for (a) $r \leq a$, and (b) $r \geq a$.*

where V is the volume of the sphere. In this case, the electric field $\vec{E}$ is radially symmetric and directed outward. The magnitude of the electric field is constant on spherical surfaces of radius r.

Case 1: $r \leq a$: We choose our Gaussian surface to be a sphere of radius $r \leq a$, as shown in Figure 1.15(a).

- The flux through the Gaussian surface is

$$\Phi_E = \oiint \vec{E} \cdot d\vec{A} = EA = E(4\pi r^2).$$

- With uniform charge distribution, the charge enclosed is

$$q_{enc} = \int \rho dV = \rho V = \rho(\frac{4}{3}\pi r^3) = q\frac{r^3}{a^3}$$

which is proportional to the volume enclosed by the Gaussian surface. Applying Gauss's law, we obtain

$$E(4\pi r^2) = \frac{\rho}{\epsilon_0}(\frac{4}{3}\pi r^3)$$

The magnitude of the electric field is therefore

$$E = \frac{\rho r}{3\epsilon_0} = \frac{Qr}{4\pi\epsilon_0 a^3} \quad ; \quad r \leq a \tag{1.37}$$

Case 2: $r \geq a$: We choose our Gaussian surface to be a sphere of radius $r \geq a$, as shown in Figure 1.15(b).

Since the radius of the Gaussian surface is greater than the radius of the sphere all the charge is enclosed in our Gaussian surface: $q_{enc} = Q$. With the electric flux through the Gaussian surface given by $\Phi_E = E(4\pi r^2)$, upon applying Gauss's law, we obtain $E(4\pi r^2) = \frac{Q}{\epsilon_0}$. The magnitude of the electric field is therefore

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad ; \quad r > a. \tag{1.38}$$

The field outside the sphere is the same as if all the charges were concentrated at the center of the sphere.

Gauss' Law and Conductors

An insulator such as glass or paper is a material in which electrons are attached to some particular atoms and cannot move freely. On the other hand, inside a conductor, electrons are free to move around. The basic properties of a conductor in electrostatic equilibrium are as follows.

(1) The electric field is zero inside a conductor.

(2) Any net charge must reside on the surface.

(3) The tangential component of $\vec{E}$ is zero on the surface of a conductor.

(4) $\vec{E}$ is normal to the surface just outside the conductor.

Consider Gaussian surface S of shape of cylinder:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{\sigma A}{\epsilon_0}$$

Therefore the normal component of the electric field is proportional to the surface charge density

$$E = \frac{\sigma}{\epsilon_0}. \tag{1.39}$$

The above result holds for a conductor of arbitrary shape.

Conductor with charge inside a cavity

Consider a hollow conductor shown in Figure 1.16. Suppose the net charge carried by the conductor is $+Q$. In addition, there is a charge q inside the cavity. What is the charge on the outer surface of the conductor? Since the electric field inside a conductor must

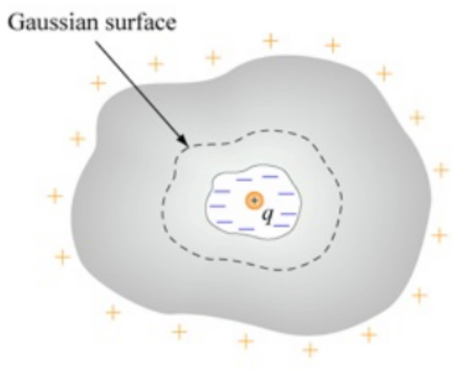

Figure 1.16: *Conductor with a cavity.*

be zero, the net charge enclosed by the Gaussian surface shown in Figure 1.16 must be zero. This implies that a charge $-q$ must have been induced on the cavity surface. Since the conductor itself has a charge $+Q$, the amount of charge on the outer surface of the conductor must be $Q + q$.

1.2 ELECTRIC ENERGY AND CAPACITANCE

1.2.1 Electric Potential Energy and Electric Potential

When a charge q is placed in an electric field $\vec{E}$ created by some source charge distribution, we saw that there is an electric force $q\vec{E}$ acting on the charge. This force is conservative because the force between charges described by Coulomb's law is conservative. If the charge is free to move, it will do so in response to the electric force.
Therefore, the electric field will be doing work on the charge. This work is internal to the system. For the an infinitesimal displacement $d\vec{l}$ (a vector that is oriented tangent to a path through the space) of a point charge q immersed in an electric field, the work done within the charge–field system by the electric field on the charge is

$$W_{int} = \vec{F}_e \cdot d\vec{l} \tag{1.40}$$

The internal work done in a system is equal to the negative of the change in the potential energy of the system: $W_{int} = -\Delta U$
Therefore, as the charge q is displaced, the electric potential energy of the charge–field system is changed by an amount $dU = -W_{int} = -q\vec{E} \cdot d\vec{l}$. For a finite displacement of the charge from some point A in space to some other point B, the change in electric potential energy of the system is

$$\Delta U = -q \int_A^B \vec{E} \cdot d\vec{l} \tag{1.41}$$

The integration is performed along the path that q follows as it moves from A to B. Because the force $q\vec{E}$ is conservative, this line integral does not depend on the path taken from A to B.

For a given position of the charge in the field, the charge–field system has a potential energy U relative to the configuration of the system that is defined as $U = 0$.
Dividing the potential energy by the charge gives a physical quantity that depends only on the source charge distribution and has a value at every point in an electric field. This quantity is called the *electric potential* (or simply the *potential*) V:

$$V = \frac{U}{q} \tag{1.42}$$

The *potential difference* $\Delta V = V_B - V_A$ between two points A and B in an electric field is defined as the change in electric potential energy of the system when a charge q is moved between the points divided the charge:

$$\Delta V = \frac{\Delta U}{q} = -\int_A^B \vec{E} \cdot d\vec{l} \tag{1.43}$$

In this definition (Equation 1.43), the infinitesimal displacement $d\vec{l}$ is interpreted as the displacement between two points in space rather than the displacement of a point charge as in the definition of the electric potential energy.

1.2.2 Potential Difference in a Uniform Electric Field

The Equation 1.43 and Equation 1.41 hold in all electric fields. whether uniform or varying, but it can be simplified for the special case of uniform field. The potential difference between two points A and B separated by a distance d, where the displacement $\vec{dl}$ points from A toward B and parallel to the field lines. We have

$$V_B - V_A = \Delta V = -\int_A^B \vec{E} \cdot \vec{dl} = -\int_A^B Edl(\cos 0^o) = -\int_A^B Edl \tag{1.44}$$

Because E is constant, it can be removed from the integral sign, which gives

$$\Delta V = -E\int_A^B dl = -Ed \tag{1.45}$$

Thus,

$$\Delta V = -Ed \tag{1.46}$$

The negative sign in the Equation 1.46 indicates that the electric potential at point B is lower than at point A, this is $V_B < V_A$. Electric field lines always point in the direction of decreasing electric potential.

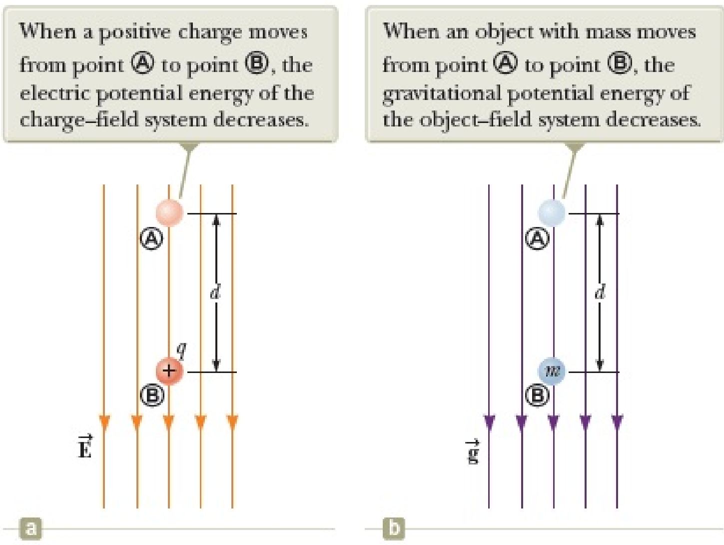

Figure 1.17: *(a) When the electric field is directed downward, point B is at lower electric potential than point A. (b) A gravitational analog to situation in (a)*

.

Now suppose a charge q moves from A to$B$ as in the Figure 1.17. The change in the potential energy of the charge–field system is given by

$$\Delta U = q\Delta V = -qEd \tag{1.47}$$

The Equation 1.47 shows that if q is negative, then ΔU is positive. Therefore, in a system consisting of a positive charge and an electric field, the electric potential energy of the system decreases when the charge moves in the direction of the field.

Applying Equation 1.43 to the field of a point charge, q, and choosing the convention that the electric potential is zero at infinity, point $A \to \infty$, the electric potential due to a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r} \tag{1.48}$$

If the electric charge is continuously distributed, the electric potential due to it will be given by

$$V = k_e \int \frac{dq}{r} \tag{1.49}$$

In the Equation 1.49, the electric potential is taken to be zero when point P (the point at which we are calculating the potential) is infinitely far from the charge distribution.
The electric potential resulting from two or more point charges is obtained by applying the supposition principle. That is, the total electric potential at some point P due to several point charges is the sum of the potentials due to the individual charges. For a group of point charges, we can write the total electric potential at P as

$$V = k_e \sum_i \frac{q_i}{r_i} \tag{1.50}$$

The Figure 1.18 shows a charge q_1, which sets up an electric field throughout space. The charge also establishes an electric potential at all points, including point P, where the electric potential is V_1. The work that must be done by an external agent to bring a charge q_2 from infinity to point P is given by $W = q_2 \Delta V$. This work represents a transfer of energy across the boundary of the two-charge system, and the energy appears in the system as potential energy U when the particles are separated by a distance r_{12}. Thus, $W = \Delta U$. Therefore, the *electric potential energy* of a pair of point charges can be found as follows:

$$\Delta U = W = q_2 \Delta V \to U - 0 = q_2 \left(k_e \frac{q_1}{r_{12}} - 0 \right) \tag{1.51}$$

That is,

$$U = k_e \frac{q_2 q_1}{r_{12}} \tag{1.52}$$

If the charge are of the same sign, then U is positive. Positive work must be done by an external agent on the system to bring the two charges near each other. If the charges are of opposite sign, then U is negative. Negative work is done by an external agent against the attractive force between the charges of opposite sign as they are brought near each other other; a force must be applied opposite the displacement to prevent q_2 from accelerating toward q_1.
If the system consists of more than two charged particles, we can obtain the total potential energy of the system by calculating U for every *pair* of charges and summing the terms algebraically.

$$U = k_e \sum_{i \neq j} \frac{q_i q_j}{r_{ij}} \tag{1.53}$$

For example, the total potential energy of the system of three charges as shown in the Figure 1.19 is given by.

$$U = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right) \tag{1.54}$$

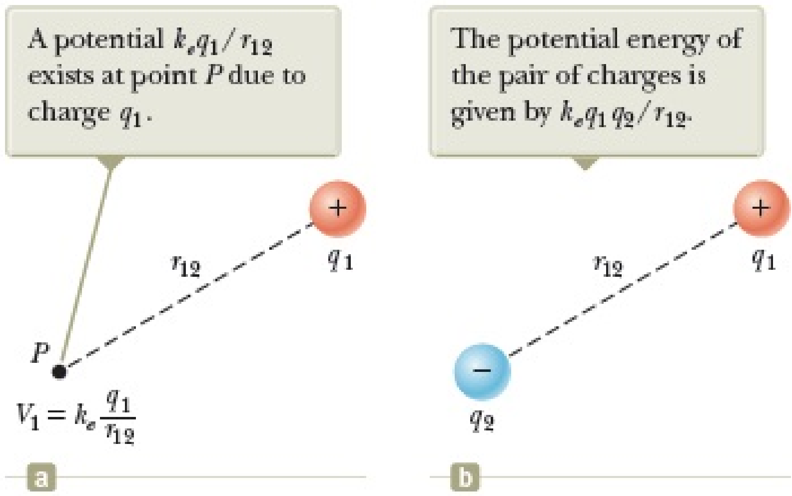

Figure 1.18: *(a) Charge q_1 establishes an electric potential v_1 at point P. (b) Charge q_2 is brought from infinity to point P*

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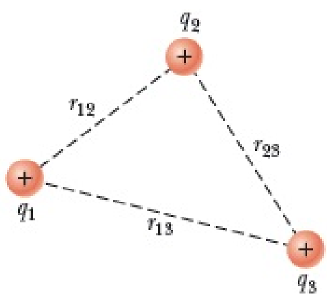


Figure 1.19: *Three point charges are fixed at the positions shown*

1.2.3 Electric potential of continuous charge distribution

For any charge distribution, we write the electrical potential dV due to infinitesimal charge dq by

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = k_e\frac{dq}{r} \implies V = \int k_e\frac{dq}{r}. \tag{1.55}$$

- **Uniformly charged rod**

  Consider a non-conducting rod of length l having a uniform charge density λ. Find the electric potential at P, a perpendicular distance y above the midpoint of the rod.

  $$Answer: \quad V = k_e\lambda\; ln[\frac{(l/2) + \sqrt{(l/2)^2 + y^2}}{(-l/2) + \sqrt{(l/2)^2 + y^2}}] \tag{1.56}$$

- **Uniformly-charged ring**

  Consider a uniformly charged ring of radius R and charge density λ. What is the electric potential at a distance z from the central axis?

  $$Answer: \quad V = k_e\frac{2\pi\lambda R}{\sqrt{R^2+z^2}} = k_e\frac{Q}{\sqrt{R^2+z^2}} \tag{1.57}$$

- **Uniformly charged disk**

Consider a uniformly charged disk of radius R and charge density σ lying in the xy-plane. What is the electric potential at a distance z from the central axis?

$$Answer: \quad V = \frac{\sigma}{2\epsilon_0}[\sqrt{R^2 + z^2} - |z|] \tag{1.58}$$

1.2.4 Relationship between electric field and electric potential

In Equation (1.43), we established the relation between $\vec{E}$ and V. If we consider two points that are separated by a small distance $d\vec{l}$, the following differential form is obtained:

$$dV = -\vec{E} \cdot d\vec{l}$$

In Cartesian coordinates, $\vec{E} = E_x\vec{i} + E_y\vec{j} + E_z\vec{k}$ and $d\vec{l} = dx\vec{i} + dy\vec{j} + dz\vec{k}$, and therefore $dV = (E_x\vec{i} + E_y\vec{j} + E_z\vec{k}) \cdot (dx\vec{i} + dy\vec{j} + dz\vec{k}) = E_x dx + E_y dy + E_z dz$
We define directional derivatives $\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial y}$ and $\frac{\partial V}{\partial z}$ such that

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz.$$

Therefore

$$E_x = -\frac{\partial V}{\partial x} \quad , \quad E_y = -\frac{\partial V}{\partial y} \quad , \quad and \quad E_z = -\frac{\partial V}{\partial z} \tag{1.59}$$

1.2.5 Equipotential Surfaces

Consider a set of charges producing a certain electrostatic field in a certain region. Alternatively, we could think of the potential at any point due to these charges. All the points having the same value of potential form a three dimensional surface in space, called an *equipotential surface*. An equipotential surface is a surface on which the potential, or voltage, is constant.

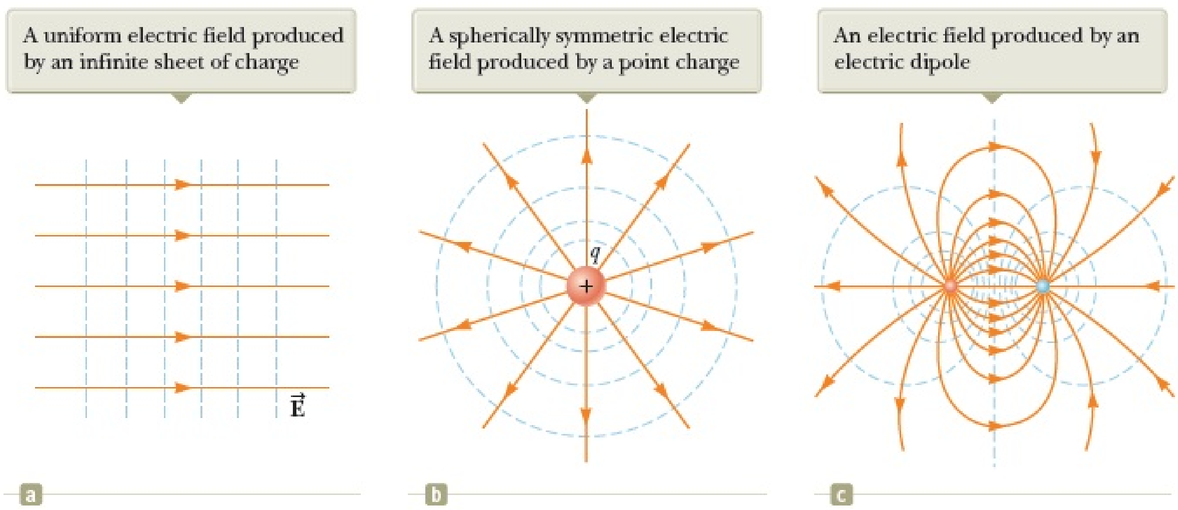


Figure 1.20: *Equipotential surfaces (the dashed lines are intersections of these surfaces with the page) and electric field lines. In all cases, the equipotential surfaces are perpendicular to the electric field lines at every point.*

Electric field lines are always perpendicular to these surfaces, and the electric field points from surfaces of high potential to surfaces of low potential. Usually the equipotential surfaces are plotted spaced in equal steps. That is the potential difference between any two neighboring equipotential is the same.

Lets take an example of a single point charge q at a point. The potential due to that charge is given by $V(\vec{r}) = k_e \dfrac{q}{r}$ (Equation 1.48). Imagine a sphere of radius R about this charge. Since all points on this sphere have $r = R$, they all have the same value of the potential, $k_e \frac{q}{R}$, so this sphere is an equipotential surface. It is clear that any other concentric sphere is also another equipotential. In two dimensions, equipotential surfaces become equipotential lines.
Note that, Every point on the surface of a charged conductor in electrostatic equilibrium is at the same electric potential. The potential is constant everywhere inside the conductor and equal to its value at the surface.

Example 1: Isolated spherical charged conductors or a Spherical shell

Consider a metallic spherical shell of radius R and charge Q. Recall that the charge is distributed on the outside of conductors.

Inside conductor:

E-field inside, $E = 0 \implies \Delta V = 0$ everywhere in conductor

$\implies V = k_e \frac{Q}{R} = constant$ everywhere in conductor

$\implies$ The entire conductor is at the same potential

Outside conductor:

For $r > R$, we have $V = k_e \frac{Q}{r}$

Example 2: Conducting spheres connected by a wire

Why does lightning strike the tip of a lightning rod?
Suppose two metal spheres with radii r_1 and r_2 are connected by a thin conducting wire, as shown in Figure 1.21. Charge will continue to flow until equilibrium is established such

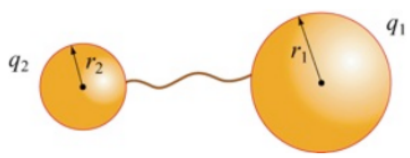


Figure 1.21: *Two conducting spheres connected by a wire.*

that both spheres are at the same potential $V_1 = V_2 = V$. Suppose the charges on the spheres at equilibrium are q_1 and q_2. Neglecting the effect of the wire that connects the two spheres, the equipotential condition implies

$$V = k_e \frac{q_1}{r_1} = k_e \frac{q_2}{r_2} \implies \frac{q_1}{r_1} = \frac{q_2}{r_2}$$

The electric fields can be expressed as

$$E_1 = k_e \frac{q_1}{r_1^2} = \frac{\sigma_1}{\epsilon_0} \ ; \ E_2 = k_e \frac{q_2}{r_2^2} = \frac{\sigma_2}{\epsilon_0}$$

where σ_1 and σ_2 are the surface charge densities on spheres 1 and 2, respectively. Divided the magnitudes of the electric fields yields

$$\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1} = \frac{q_2}{q_1}$$

With the surface charge density being inversely proportional to the radius, we conclude that the regions with the smallest radii of curvature have the greatest σ. Thus, the electric field strength on the surface of a conductor is greatest at the sharpest point. The design of a lightning rod is based on this principle. Lighting strikes the tip.

1.2.6 Capacitors and Dielectrics

Capacitors

A capacitor is an arrangement of conductors that is used to store electric charge. A very simple capacitor is an isolated metallic sphere. The potential difference of a sphere with radius R and charge Q is equal to

$$\Delta V = k_e \frac{Q}{R} \tag{1.60}$$

The Equation 1.60 shows that the potential of the sphere is proportional to the charge Q on the conductor. This is true in general for any configuration of conductors. This relationship can be written as

$$Q = C\Delta V \tag{1.61}$$

where C is called the *"capacitance"* of the system of conductors. In general, the capacitance C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors and it is always a positive quantity by definition. The unit of capacitance is the *farad* (F).

The capacitance of the metallic sphere is equal to

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e \frac{Q}{R}} = \frac{1}{k_e} R \tag{1.62}$$

This is

$$C = 4\pi\varepsilon_o R \tag{1.63}$$

This formula (Equation 1.63) shows that *the capacitance of a metallic sphere depends only on the geometry of the sphere not on the electric charges that it carries.* This is true in general for any configuration of conductors.

Parallel Plate Capacitors

Two parallel, metallic plates of equal area A are separated by a distance d. One plate carries a charge $+Q$, and the other carries a charge $-Q$. The surface charge density on each plate is $\sigma = Q/A$. If the plates are very close together (in comparison with their length and width), we can assume the electric field is uniform between the plates and zero elsewhere. The value of the electric field between the plates is

$$E = \frac{\sigma}{\varepsilon_o} = \frac{Q}{\varepsilon_0 A} \tag{1.64}$$

Because the field between the plates is uniform, the magnitude of the potential difference between the plates equals Ed, therefore

$$\Delta V = Ed = \frac{Qd}{\varepsilon_0 A} \tag{1.65}$$

Combining the Equation 1.65 with the Equation 1.61, we find that the capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\varepsilon_0 A} \tag{1.66}$$

or

$$C = \frac{\varepsilon_0 A}{d} \tag{1.67}$$

That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.

Let's consider how the geometry of these conductors influences the capacity of the pair of plates to store charge. As a capacitor is being charged by a battery, electrons flow into the negative plate and out of the positive plate. If the capacitor plates are large, the accumulated charges are able to distribute themselves over a substantial area and the amount of charge that can be stored on a plate for a given potential difference increases as the plate area is increased. Therefore, it is reasonable that the capacitance is proportional to the plate area A as in the Equation 1.67.
Now consider the region that separates the plates. Imagine moving the plates closer together. Consider the situation before any charges have had a chance to move in response to this change. Because no charges have moved, the electric field between the plates has the same value but extends over a shorter distance. Therefore, the magnitude of the potential difference between the plates is smaller ($\Delta V = Ed$). The difference between this new capacitor voltage and the terminal voltage of the battery appears as a potential difference across the wires connecting the battery to the capacitor, resulting in an electric field in the wires that drives more charge onto the plates and increases the potential difference between the plates. When the potential difference between the plates again matches that of the battery, the flow of charge stops. Therefore, moving the plates closer together causes the charge on the capacitor to increase. If d is increased, the charge decreases.

Cylindrical Capacitors

A cylindrical capacitor is a system made of a solid cylindrical conductor of radius a and charge Q which is in coaxial with a cylindrical shell of negligible thickness of radius $b > a$ and charge -Q, both cylinders have the length l ($l >>> b$)

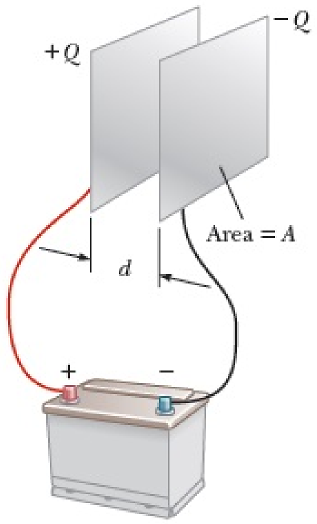


Figure 1.22: *A parallel-plate capacitor consists of two parallel conducting plates, each of area A, separated by a distance d.*

As l is much greater than a and b, we can neglect end effects. In this case, the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them.

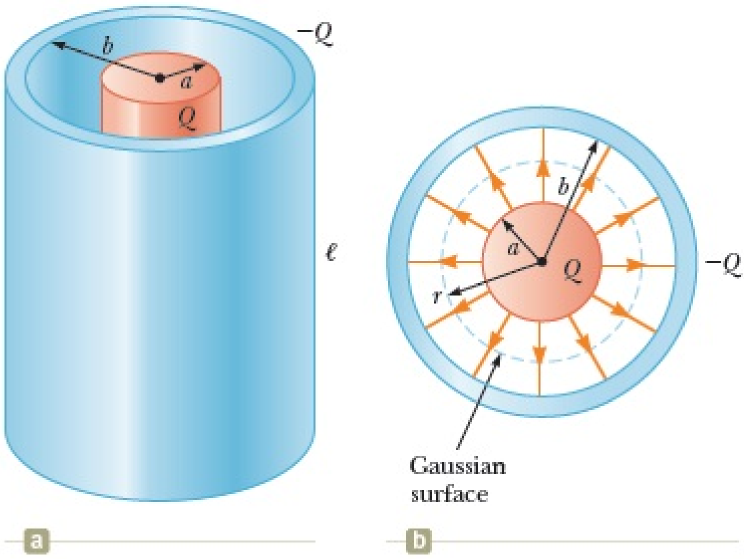


Figure 1.23: *(a) Cylindrical Capacitor, (b) End view, The electric field lines are radial*

The capacitance C is given by

$$C = \frac{Q}{V_a - V_b} \tag{1.68}$$

where V_a and V_b are electric potentials on outer surface on inner cylinder and inner surface of outer cylinder respectively.

We can determine $V_a - V_b$ by finding electric field E at a point between cylinders. Consider a Gaussian surface of the length l and radius r, cross-section of the Gaussian surface is shown on the Figure 1.23b.

Using Gauss's law,

$$\oint \vec{E} \cdot \vec{S} = \frac{Q}{\varepsilon_0} \tag{1.69}$$

That is,

$$E \times 2\pi r l = \frac{Q}{\varepsilon_0} \tag{1.70}$$

We can write the potential difference between the two cylinders as

$$V_a - V_b = -\int \vec{E} \cdot \vec{S} \tag{1.71}$$

Considering the effect that the electric field is perpendicular to the long axis of the cylinders and is confined to the region between them. We obtain,

$$V_a - V_b = -\int \vec{E} \cdot \vec{S} = \int_a^b E dr = \frac{Q}{2\pi\varepsilon_0 l} \int_a^b \frac{dr}{r} \tag{1.72}$$

or

$$V_a - V_b = \frac{Q}{2\pi\varepsilon_0 l} \ln\left(\frac{b}{a}\right) \tag{1.73}$$

By replacing this value in Equation 1.68, we find

$$C = \frac{l}{2k_e \ln(b/a)} \tag{1.74}$$

That is, *the capacitance of a cylindrical capacitor depends only on the geometrical factors.*

Spherical Capacitors

A spherical capacitor consists of a spherical conducting shell of radius b and charge -Q concentric with a smaller conducting sphere of radius a and charge Q.
Its capacitance is given by

$$C = \frac{ab}{k_e(b-a)} \tag{1.75}$$

Note that if the radius b of the outer sphere approaches infinity,the capacitance of the Equation 1.75 becomes,

$$C = \lim_{b\to\infty} \frac{ab}{k_e(b-a)} = \frac{ab}{k_e b} = 4\pi\varepsilon_0 a \tag{1.76}$$

The expression of the isolated spherical conductor (Equation 1.63).

Combinations of Capacitors

Two or more capacitors often are combined in electric circuits. We can calculate the equivalent capacitance of certain combinations using methods described in this section. Throughout this section, we assume the capacitors to be combined are initially uncharged. In studying electric circuits, we use a simplified pictorial representation called *circuit diagram*. Such a diagram uses *circuit symbols* to represent various circuit elements. The circuit symbols are connected by straight lines that represent the wires between the circuit elements. The symbol for the capacitor reflects the geometry of the most common model for a capacitor, a pair of parallel plates. The positive terminal of the battery is at the higher potential and is represented in the circuit symbol by the longer line (see Figure 1.25).

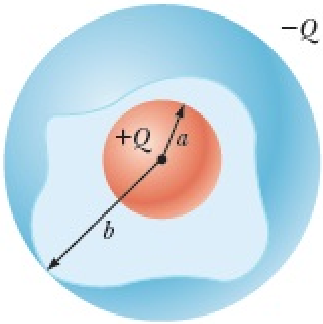


Figure 1.24: *A Spherical Capacitor*

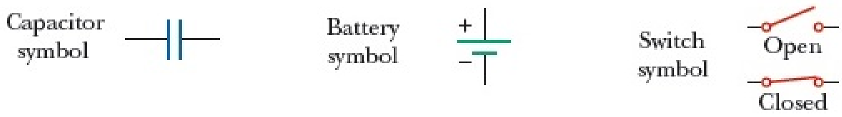


Figure 1.25: *Circuit symbols for capacitors, batteries, and switches*

Parallel Combination

Two capacitors connected as shown in the Figure 1.26 are known as a parallel combination of capacitors.

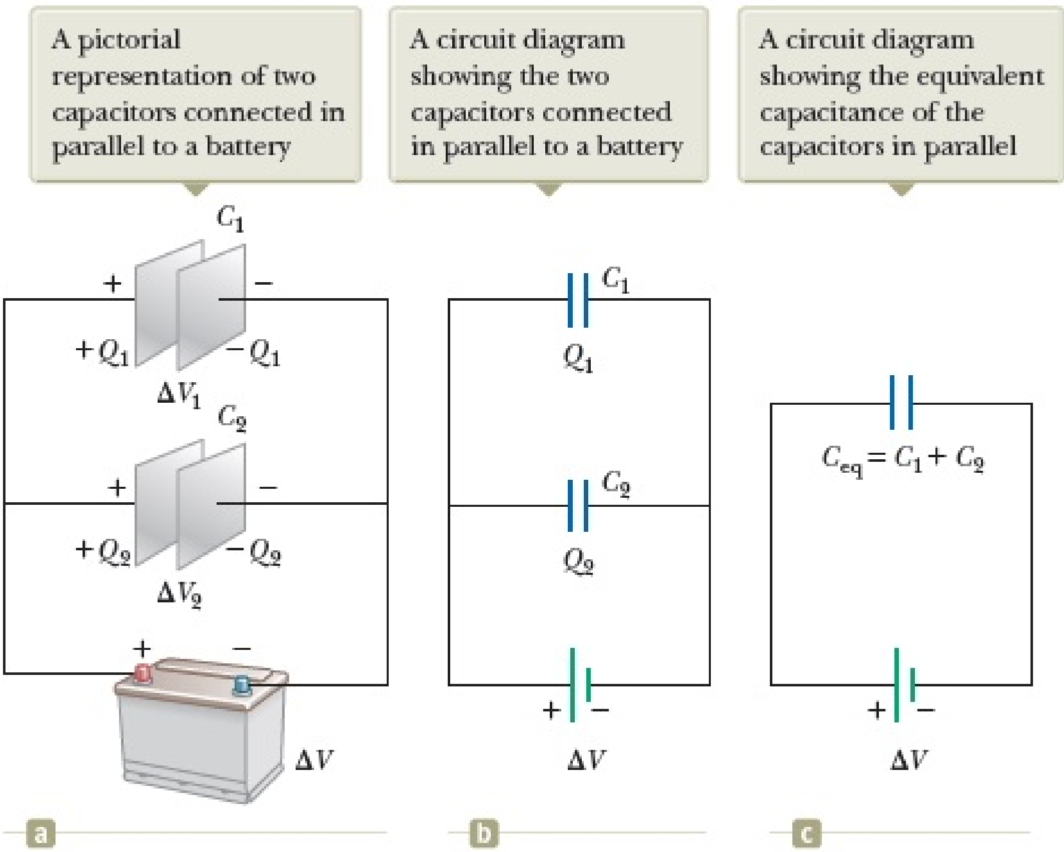


Figure 1.26: *Two capacitors connected in parallel. All three diagrams are equivalent*

The left plates of the capacitors are connected to the positive terminal of the battery by a conducting wire and are therefore both at the same electric potential as the positive terminal. Likewise, the right plates are connected to the negative terminal and so are both at the same potential as the negative terminal. Therefore, the individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination. That is,

$$\Delta V_1 = \Delta V_2 = \Delta V \tag{1.77}$$

where ΔV is the battery terminal voltage.
After the battery is attached to the circuit, the capacitors quickly reach their maximum charge. Let's call the maximum charges on the two capacitors Q_1 and Q_2, where $Q_1 = C_1\Delta V_1$ and $Q_2 = C_2\Delta V_2$. The total charge Q_{tot} stored by the two capacitors is the sum of charges on individual capacitors:

$$Q_{tot} = Q_1 + Q_2 = C_1\Delta V_1 + C_2\Delta V_2 \tag{1.78}$$

Suppose you wish to replace these two capacitors by one equivalent capacitor having a capacitance C_{eq} as in the Figure 1.26c. The effect this equivalent capacitor has on the circuit must be exactly the same as the effect of the combination of the two individual capacitors. That is, the equivalent capacitor must store charge Q_{tot} when connected to the battery. The voltage across the equivalent capacitor is ΔV because the equivalent capacitor is connected directly across the battery terminals. Therefore, for the equivalent capacitor,

$$Q_{tot} = C_{eq}\Delta V \tag{1.79}$$

Substituting this result into Equation 1.78 gives

$$C_{eq}\Delta V = C_1\Delta V_1 + C_2\Delta V_2$$

$$C_{eq} = C_1 + C_2 \tag{1.80}$$

where we have canceled the voltages because they are all the same. If this treatment is extended to three or more capacitors connected in parallel, the equivalent capacitance is found to be

$$C_{eq} = \sum_i C_i \tag{1.81}$$

Therefore, the equivalent capacitance of a parallel combination of capacitors is (1) the algebraic sum of the individual capacitances and (2) greater than any of the individual capacitances. Statement (2) makes sense because we are essentially combining the areas of all the capacitor plates when they are connected with conducting wire, and capacitance of parallel plates is proportional to area.

Series Combination

Two capacitors connected as shown in Figure 1.27a and the equivalent circuit diagram in Figure 1.27b are known as a series combination of capacitors.
The left plate of capacitor 1 and the right plate of capacitor 2 are connected to the terminals of a battery. The other two plates are connected to each other and to nothing else; hence, they form an isolated system that is initially uncharged and must continue to have zero net charge.

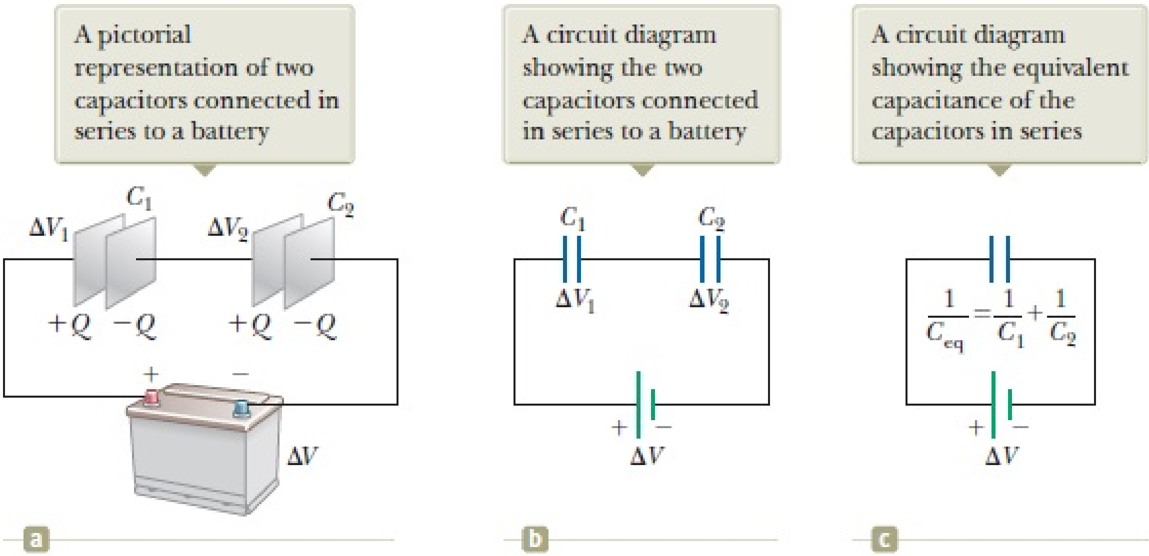


Figure 1.27: *Two capacitors connected in series. All three diagrams are equivalent*

To analyze this combination, let's first consider the uncharged capacitors and then follow what happens immediately after a battery is connected to the circuit. When the battery is connected, electrons are transferred out of the left plate of C_1 and into the right plate of C_2. As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is forced off the left plate of C_2, and this left plate therefore has an excess positive charge. The negative charge leaving the left plate of C_2 causes negative charges to accumulate on the right plate of C_1. As a result, both right plates end up with a charge $-Q$ and both left plates end up with a charge $+Q$. Therefore, the charges on capacitors connected in series are the same:

$$Q_1 = Q_2 = Q \tag{1.82}$$

where Q is the charge that moved between a wire and the connected outside plate of one of the capacitors.

The individual voltages across the capacitors are ΔV_1 and ΔV_2. These voltages add to give the total voltage ΔV_{tot} across the combination:

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \tag{1.83}$$

In general, the total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.
Suppose the equivalent single capacitor in Figure 1.27c has the same effect on the circuit as the series combination when it is connected to the battery. After it is fully charged, the equivalent capacitor must have a charge of $-Q$ on its right plate and a charge of $+Q$ on its left plate. Applying the definition of capacitance to the circuit in Figure 1.27c gives

$$\Delta V_{tot} = \frac{Q}{C_{eq}} \tag{1.84}$$

Substituting this result into Equation 1.83,gives

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \tag{1.85}$$

Canceling the charges because they are all the same gives

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{1.86}$$

When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i} \tag{1.87}$$

This expression shows that (1) the inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances and (2) the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.

Energy Storage in Capacitors and Electric Field Energy

If you transfer electrons from one plate to another, the electric field developed in space between the plates tends to opposite further transfer. Therefore, work must be done by an external agent (e.g. battery) to charge a capacitor. This work done by the external agent is stored as electric potential energy. When the capacitor is discharged, this stored energy is recovered as work done by the electric forces.

Let, during the charging process, q be the charge transferred from one plate to the other, at a given instant. The potential difference ΔV between the plates at that instant is Q/C. If at this stage an extra charge dq is transferred, the work done dW required for this extra charge transfer will be

$$dW = (\Delta V)dq = \frac{q}{C}dq \tag{1.88}$$

The total work W required charging the capacitor from zero to final charge Q is

$$W = \int_0^W dW = \frac{1}{C}\int_0^Q q dq = \frac{Q^2}{2C} \tag{1.89}$$

This work W is stored as electric potential energy U in the capacitor. therefore

$$U = \frac{Q^2}{2C} \tag{1.90}$$

As $Q = C(\Delta V)$, the electric potential energy can also be given by

$$U = \frac{1}{2}C(\Delta V)^2 \tag{1.91}$$

This equation applies to any capacitor, regardless of its geometry. For a given capacitance, the stored energy increases as the charge and the potential difference increase. In practice, there is a limit to the maximum energy (or charge) that can be stored because, at a sufficiently large value of ΔV, discharge ultimately occurs between the plates. For this reason, capacitors are usually labeled with a maximum operating voltage.

The stored energy is related to the electric field between the plates of capacitor. We can consider the electric energy as being stored in the electric field in the space between plates as the capacitor is charged. This description is reasonable because the electric field is proportional to the charge on the capacitor. For a parallel-plate capacitor, the potential

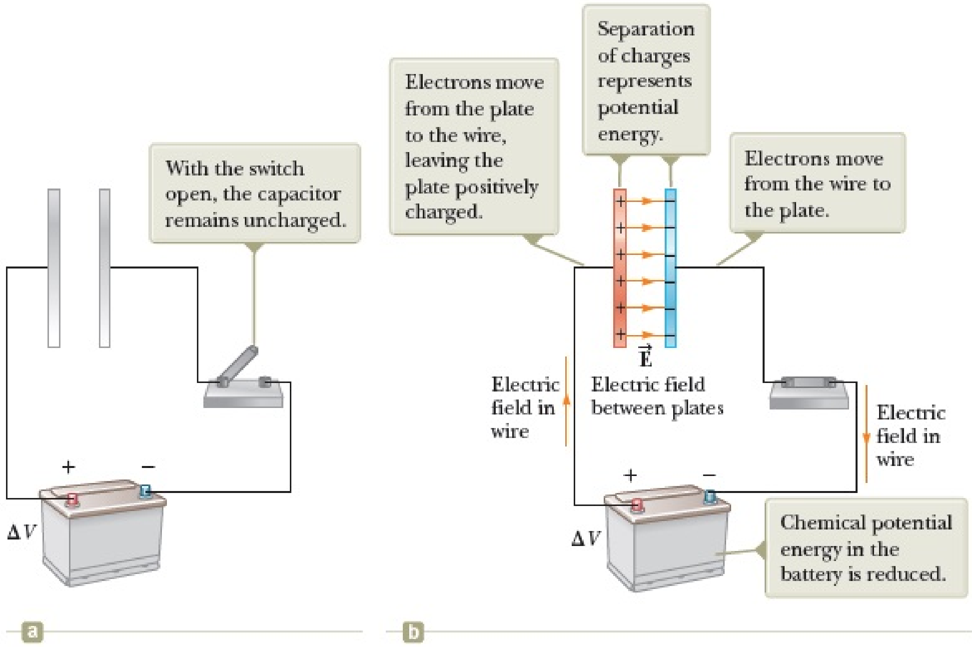


Figure 1.28: *(a) A circuit consisting of a capacitor, a battery, and a switch. (b) When the switch is closed, the battery establishes an electric field in the wire and the capacitor becomes charged*

difference is related to the electric field through the relationship $\Delta V = Ed$. Furthermore, its capacitance is $C = \varepsilon_0 A/d$. Substituting these expressions into Equation 1.92 gives

$$U_E = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\left(\varepsilon_0 A d\right) E^2 \tag{1.92}$$

Because the volume occupied by the electric field is Ad, the *energy per unit volume* $u_E = U_E/Ad$, known as the energy density, is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \tag{1.93}$$

Although the Equation 1.93 was derived for a parallel-plate capacitor, the expression is generally valid regardless of the source of the electric field. That is, the energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

Capacitors with Dielectrics

A *dielectrics* is a nonconducting material such as rubber, glass, or waxed paper. If the space between the plates of a capacitor is filled with a dielectrics, the capacitance of the capacitor will change compared to the situation in which there is vacuum between the plates. The change in the capacitance is caused by a change in the electric field between the plates. The electric field between the capacitor plates will induce dipole moments in the material between the plates. These induced dipole moments will reduce the electric field in the region between the plates. A material in which the induced dipole moment is linearly proportional to the applied electric field is called a linear dielectric. In this type of materials the total electric field between the capacitor plates $\vec{E}$ is related to the electric field $\vec{E}_{free}$ that would exist if no dielectric was present:

$$\vec{E} = \frac{1}{\kappa}\vec{E}_{free} \tag{1.94}$$

where κ is called the dielectric constant. Since the magnitude of the final electric field $\vec{E}$ can never exceed the magnitude of the free electric field $\vec{E}_{free}$, the dielectric constant κ must be larger than 1.

Table 1.1: Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature.

Material	**Dielectric Constant** κ	**Dielectric Strength** $(10^6 V/m)$
Air (dry)	1.000059	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	16	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.00000	–
Water	80	–

The potential difference across a capacitor is proportional to the electric field between the plates. Since the presence of a dielectric reduces the strength of the electric field, it will also reduce the potential difference between the capacitor plates (if the total charge on the plates is kept constant):

$$\Delta V = \frac{1}{\kappa}(\Delta V)_{free} \tag{1.95}$$

The capacitance C of a system with a dielectric is inversely proportional to the potential difference between the plates, and is related to the capacitance C_{free} of a capacitor with no dielectric in the following manner

$$C = \frac{Q}{\Delta V} = \frac{Q}{\Delta V_{free}/\kappa} = \kappa C_{free} \tag{1.96}$$

That is, the capacitance increases by the factor κ when the dielectric completely fills the region between the plates. For a parallel-plate capacitor, we can express the capacitance of a parallel-plate capacitor filled with a dielectric as

$$C = \kappa\frac{\varepsilon_0 A}{d} = \varepsilon\frac{A}{d} \tag{1.97}$$

where $\varepsilon = \kappa\varepsilon_0$ is called **permittivity of dielectric** and ε_0 is the **permittivity of free space**.
From this Equation 1.97, it would appear that the capacitance could be made very large by inserting a dielectric between the plates and decreasing d. In practice, the lowest value of d is limited by the electric discharge that could occur through the dielectric medium separating the plates. For any given separation d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the dielectric strength[1] of the dielectric. If the magnitude of the electric field in the dielectric exceeds the dielectric strength, the insulating properties break down and the dielectric begins to conduct.
Physical capacitors have a specification called by a variety of names, including *working voltage, breakdown voltage, and rated voltage.* This parameter represents the largest voltage that can be applied to the capacitor without exceeding the dielectric strength of the dielectric material in the capacitor. Consequently, when selecting a capacitor for a given application, you must consider its capacitance as well as the expected voltage across the capacitor in the circuit, making sure the expected voltage is smaller than the rated voltage of the capacitor. Insulating materials have values of κ greater than unity and dielectric strengths greater than that of air the table indicates. Therefore, a dielectric provides the following advantages:

- An increase in capacitance
- An increase in maximum operating voltage
- Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing C

1.3 ELECTRIC CURRENT AND ELECTRIC RESISTANCE

1.3.1 Electric Current

The movement of electric charge is known as an electric current, the intensity of which is usually measured in amperes (A). Electric current can consist of any moving charged particles; most commonly these are electrons, but any charge in motion constitutes an electric current.
By historical convention, a positive electric current is defined as having the same direction of flow as any positive charge it contains, or to flow from the most positive part of a circuit to the most negative part. Electric current defined in this manner is called conventional electric current. The motion of negatively charged electrons around an electric circuit, one of the most familiar forms of electric current, is thus deemed positive in the opposite direction to that of the electrons. However, depending on the conditions, an electric current can consist of a flow of charged particles in either direction or even in both directions at once. The positive-to-negative convention is widely used to simplify this situation.
To define current quantitatively, suppose charges are moving perpendicular to a surface of area A as shown in Figure 1.29. (This area could be the cross-sectional area of a wire, for

[1] *The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.*

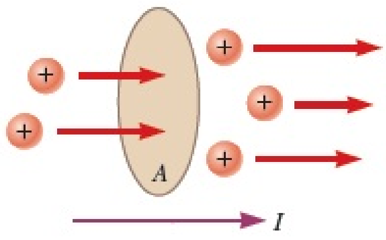


Figure 1.29: *Charges in motion through an area A. The time rate at which charge flows through the area is defined as the current I.*

example.) The current is defined as the rate at which charge flows through this surface. If ΔQ is the amount of charge that passes through this surface in a time interval Δt, the average current I_{avg} is equal to the charge that passes through A per unit time:

$$I_{avg} = \frac{\Delta Q}{\Delta t} \tag{1.98}$$

If the rate at which charge flows varies in time, the current varies in time; we define the instantaneous current I as the limit of the average current as $\Delta t \to 0$:

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \tag{1.99}$$

If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential; hence, the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire; therefore, there is no current. If the ends of the conducting wire are connected to a battery, however, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the electrons in the wire, causing them to move in the wire and therefore creating a current.

Microscopic Model of Current

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal.

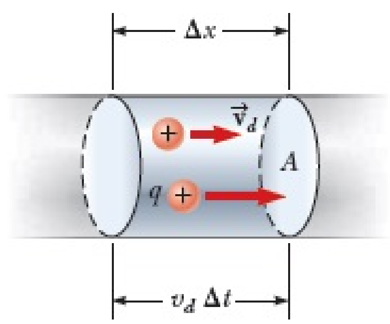


Figure 1.30: *A segment of a uniform conductor of cross-sectional area A.*

Consider the current in a cylindrical conductor of cross-sectional area A (Figure 1.30). The volume of a segment of the conductor of length Δx (between the two circular cross sections shown in Figure 1.30) is $A\Delta x$. If n represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the segment is $nA\Delta x$. Therefore, the total charge ΔQ in this segment is

$$\Delta Q = (nA\Delta x)q \tag{1.100}$$

where q is the charge on each carrier. If the carriers move with a velocity $\vec{v}_d$ parallel to the axis of the cylinder, the magnitude of the displacement they experience in the x direction in a time interval Δt is $\Delta x = v_d \Delta t$. Let Δt be the time interval required for the charge carriers in the segment to move through a displacement whose magnitude is equal to the length of the segment. This time interval is also the same as that required for all the charge carriers in the segment to pass through the circular area at one end. With this choice, we can write ΔQ as

$$\Delta Q = (nAv_d\Delta t)q \tag{1.101}$$

Dividing both sides of the Equation 1.101 by Δt, we find that the average current in the conductor is

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nqv_d A \tag{1.102}$$

In reality, the speed of the charge carriers v_d is an average speed called the *drift speed*. To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated–that is, the potential difference across it is zero, these electrons undergo random motion that is analogous to the motion of gas molecules. The electrons collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzagged. When a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. In addition to the zigzag motion due to the collisions with the metal atoms, the electrons move slowly along the conductor (in a direction opposite that of $\vec{E}$) at the drift velocity $\vec{v}_d$.
You can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by a liquid's molecules flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collisions causes an increase in the atom's vibrational energy and a corresponding increase in the conductor's temperature.

1.3.2 Electrical Resistance

Voltage can be thought of as the pressure pushing charges along a conductor, while the ***electrical resistance*** *of a conductor is a measure of how difficult it is to push the charges along.*

Consider a conductor of cross-sectional area A carrying a current I. The *current density* J in the conductor is defined as the current per unit area. Because the current $I = nqv_d A$, the current density is

$$J = \frac{I}{A} = nqv_d \tag{1.103}$$

This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current.

A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor. In some materials, the current density is proportional to the electric field:

$$J = \sigma E \tag{1.104}$$

where the constant of proportionality σ is called the conductivity of the conductor. Materials that obey the Equation 1.104 are said to follow Ohm's law. More specifically, Ohm's law states the following:

> *"For many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current".*

Materials and devices that obey Ohm's law and hence demonstrate this simple relationship between E and J are said to be ohmic. Experimentally, however, it is found that not all materials and devices have this property. Those that do not obey Ohm's law are said to be non-ohmic.

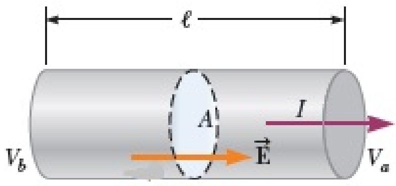


Figure 1.31: *A uniform conductor of length l, and cross-sectional area A.*

We can obtain an equation useful in practical applications by considering a segment of straight wire of uniform cross-sectional area A and length l, as shown in Figure 1.31. A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the magnitude of the potential difference across the wire is related to the field within the wire through the equation $J = \sigma E$,

$$\Delta V = El \tag{1.105}$$

Therefore, we can express the current density in the wire as

$$J = \sigma \frac{\Delta V}{l} \tag{1.106}$$

Because $J = I/A$, the potential difference across the wire is

$$\Delta V = \frac{l}{\sigma} J = \left(\frac{l}{\sigma A} \right) I = RI \tag{1.107}$$

The quantity $R = l/\sigma A$ is called the *resistance* of the conductor. We define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

$$R = \frac{\Delta V}{I} \tag{1.108}$$

The Equation 1.108 shows that resistance has SI units of *volts per ampere*. One volt per ampere is defined to be one *"ohm"* (Ω): $1\Omega = 1V/A$

Most electric circuits use circuit elements called resistors to control the current in the various parts of the circuit. As with capacitors, many resistors are built into integrated circuit chips, but stand-alone resistors are still available and widely used. Two common types are the composition resistor, which contains carbon, and the wire-wound resistor, which consists of a coil of wire.

The inverse of conductivity is resistivity ρ:

$$\rho = \frac{1}{\sigma}$$

That is

$$R = \rho \frac{l}{A} \tag{1.109}$$

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature. In addition, as you can see from the previous equation, the resistance of a sample of the material depends on the geometry of the sample as well as on the resistivity of the material.
The resistance of a given cylindrical conductor such as a wire is proportional to its length and inversely proportional to its cross-sectional area. If the length of a wire is doubled, its resistance doubles. If its cross-sectional area is doubled, its resistance decreases by one half. The situation is analogous to the flow of a liquid through a pipe. As the pipe's length is increased, the resistance to flow increases. As the pipe's cross-sectional area is increased, more liquid crosses a given cross section of the pipe per unit time interval. Therefore, more liquid flows for the same pressure differential applied to the pipe, and the resistance to flow decreases.

Resistance also depends on temperature, usually increasing as the temperature increases the resistance. For reasonably small changes in temperature, the change in resistivity, and therefore the change in resistance, is proportional to the temperature change. This is reflected in the following equations (Equation 1.110 and Equation 1.111):

$$\rho = \rho_0 \left[1 + \alpha(T - T_0)\right] \tag{1.110}$$

and, equivalently,

$$R = R_0 \left[1 + \alpha(T - T_0)\right] \tag{1.111}$$

At low temperatures some materials, known as superconductors, have no resistance at all. Resistance in wires produces a loss of energy (usually in the form of heat), so materials with no resistance produce no energy loss when currents pass through them.

1.3.3 Electrical Power

In typical electric circuits, energy is transferred by electrical transmission from a source such as a battery to some device such as a lightbulb or a radio receiver. Let's determine an expression that will allow us to calculate the rate of this energy transfer.
First, consider the simple circuit in Figure 1.32, where energy is delivered to a resistor. Because the connecting wires also have resistance, some energy is delivered to the wires and some to the resistor. Unless noted otherwise, we shall assume the resistance of the wires is small compared with the resistance of the circuit element so that the energy delivered to the wires is negligible.

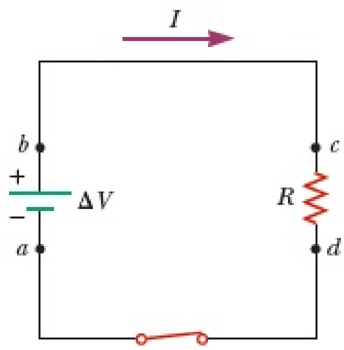


Figure 1.32: *A circuit consisting of a resistor of resistance R and a battery having a potential difference* ΔV *across its terminals.*

Imagine following a positive quantity of charge Q moving clockwise around the circuit in the Figure 1.32 from point a through the battery and resistor back to point a. We identify the entire circuit as our system. As the charge moves from a to b through the battery, the electric potential energy of the system increases by an amount ΔV while the chemical potential energy in the battery *decreases* by the same amount. As the charge moves from c to d through the resistor, however, the electric potential energy of the system decreases due to collisions of electrons with atoms in the resistor. In this process, the electric potential energy is transformed to internal energy corresponding to increased vibrational motion of the atoms in the resistor. Because the resistance of the interconnecting wires is neglected, no energy transformation occurs for paths bc and da. When the charge returns to point a, the net result is that some of the chemical potential energy in the battery has been delivered to the resistor and resides in the resistor as internal energy E_{int} associated with molecular vibration.

The resistor is normally in contact with air, so its increased temperature results in a transfer of energy by heat Q into the air. In addition, the resistor emits thermal radiation, representing another means of escape for the energy. After some time interval has passed, the resistor reaches a constant temperature. At this time, the input of energy from the battery is balanced by the output of energy from the resistor by heat and radiation, and the resistor is a non-isolated system in steady state. Some electrical devices include heat sinks connected to parts of the circuit to prevent these parts from reaching dangerously high temperatures. Heat sinks are pieces of metal with many fins. Because the metal's high thermal conductivity provides a rapid transfer of energy by heat away from the hot component and the large number of fins provides a large surface area in contact with the air, energy can transfer by radiation and into the air by heat at a high rate.
Let's now investigate the rate at which the electric potential energy of the system decreases as the charge Q passes through the resistor:

$$\frac{dU}{dt} = \frac{d}{dt}\left(Q(\Delta V)\right) = \frac{dQ}{dt}(\Delta V) = I(\Delta V) \tag{1.112}$$

where I is the current in the circuit. The system regains this potential energy when the charge passes through the battery, at the expense of chemical energy in the battery. The rate at which the potential energy of the system decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor. Therefore, the power P, representing the rate at which energy is delivered to the resistor, is

$$P = I(\Delta V) \tag{1.113}$$

We derived the Equation 1.113 by considering a battery delivering energy to a resistor. This equation, however, can be used to calculate the power delivered by a voltage source to any device carrying a current I and having a potential difference ΔV between its terminals.

Using the Equation 1.113 and the Equation 1.107 for a resistor, we can express the power delivered to the resistor in the alternative forms

$$P = I^2 R = \frac{(\Delta V)^2}{R} \tag{1.114}$$

When I is expressed in amperes, ΔV in volts, and R in ohms, the SI unit of power is the *watt*. The process by which energy is transformed to internal energy in a conductor of resistance R is often called *"joule heating"*; this transformation is also often referred to as an I^2R loss.

When transporting energy by electricity through power lines, you should not assume the lines have zero resistance. Real power lines do indeed have resistance, and power is delivered to the resistance of these wires. Utility companies seek to minimize the energy transformed to internal energy in the lines and maximize the energy delivered to the consumer. Because $P = I\Delta V$, the same amount of energy can be transported either at high currents and low potential differences or at low currents and high potential differences. Utility companies choose to transport energy at low currents and high potential differences primarily for economic reasons. Copper wire is very expensive, so it is cheaper to use high-resistance wire (that is, wire having a small cross-sectional area). Therefore, in the expression for the power delivered to a resistor, $P = I^2R$, the resistance of the wire is fixed at a relatively high value for economic considerations. The I^2R loss can be reduced by keeping the current I as low as possible, which means transferring the energy at a high voltage.

1.4 DIRECT CURRENT CIRCUITS

1.4.1 Electromotive Force

In the previous section, we discussed a circuit in which a battery produces a current. We will generally use a battery as a source of energy for circuits in our discussion. Because the potential difference at the battery terminals is constant in a particular circuit, the current in the circuit is constant in magnitude and direction and is called direct current. A battery is called either a source of electromotive force or, more commonly, a source of *emf*. (The phrase electromotive force is an unfortunate historical term, describing not a force, but rather a potential difference in volts.) The *emf*, $\mathcal{E}$ of a battery is the maximum possible voltage the battery can provide between its terminals. You can think of a source of *emf* as a "*charge pump*." When an electric potential difference exists between two points, the source moves charges "*uphill*" from the lower potential to the higher.

We shall generally assume the connecting wires in a circuit have no resistance. The positive terminal of a battery is at a higher potential than the negative terminal.

Because a real battery is made of matter, there is resistance to the flow of charge within the battery. This resistance is called internal resistance r. For an idealized battery with zero internal resistance, the potential difference across the battery (called its terminal voltage) equals its *emf*. For a real battery, however, the terminal voltage is not equal to

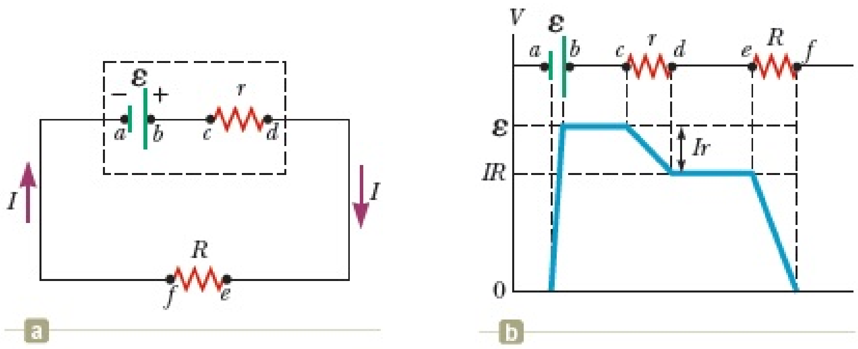


Figure 1.33: *(a) Circuit diagram of a source of emf e (in this case, a battery), of internal resistance r, connected to an external resistor of resistance R. (b) Graphical representation showing how the electric potential changes as the circuit in (a) is traversed clockwise.*

the emf for a battery in a circuit in which there is a current. To understand why, consider the circuit diagram in Figure 1.33a. We model the battery as shown in the diagram; it is represented by the dashed rectangle containing an ideal, resistance-free *emf* $\mathcal{E}$ in series with an internal resistance r. A resistor of resistance R is connected across the terminals of the battery. Now imagine moving through the battery from a to d and measuring the electric potential at various locations. Passing from the negative terminal to the positive terminal, the potential increases by an amount $\mathcal{E}$. As we move through the resistance r, however, the potential decreases by an amount $\boldsymbol{Ir}$, where I is the current in the circuit. Therefore, the terminal voltage of the battery $\Delta V = V_d - V_a$ is

$$\Delta V = \mathcal{E} - Ir \tag{1.115}$$

From this expression, notice that $\mathcal{E}$ is equivalent to the open-circuit voltage, that is, the terminal voltage when the current is zero.The emf is the voltage labeled on a battery; for example, the *emf* of an AA battery is 1.5 V. The actual potential difference between a battery's terminals depends on the current in the battery as described by the previous equation. Figure 1.33b is a graphical representation of the changes in electric potential as the circuit is traversed in the clockwise direction. Figure 1.33a shows that the terminal voltage ΔV must equal the potential difference across the external resistance R, often called the load resistance. The load resistor might be a simple resistive circuit element as in Figure 1.33a, or it could be the resistance of some electrical device (such as a toaster, electric heater, or lightbulb) connected to the battery (or, in the case of household devices, to the wall outlet). The resistor represents a load on the battery because the battery must supply energy to operate the device containing the resistance. The potential difference across the load resistance is $\Delta V = IR$. Combining this expression with the Equation 1.115 , we see that

$$\mathcal{E} = IR - Ir \tag{1.116}$$

Solving for the current gives

$$I = \frac{\mathcal{E}}{R + r} \tag{1.117}$$

The Equation 1.117 shows that the current in this simple circuit depends on both the load resistance R external to the battery and the internal resistance r. If R is much greater than r, as it is in many real-world circuits, we can neglect r. Multiplying the Equation 1.116 by the current I which is in the circuit gives

$$I\mathcal{E} = I^2R - I^2r \tag{1.118}$$

This result (Equation 1.118) indicates that because power $P = I\Delta V$, the total power output $I\mathcal{E}$ is associated with the *emf* of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r.

1.4.2 Resistors in Series and in Parallel

Resistors in Series

When two or more resistors are connected together as are the incandescent lightbulbs in Figure 1.34a, they are said to be in a series combination. Figure 1.34b is the circuit diagram for the lightbulbs, shown as resistors, and the battery. What if you wanted to replace the series combination with a single resistor that would draw the same current from the battery? What would be its value? In a series connection, if an amount of charge Q exits resistor R_1, charge Q must also enter the second resistor R_2. Otherwise, charge would accumulate on the wire between the resistors. Therefore, the same amount of charge passes through both resistors in a given time interval and the currents are the same in both resistors:

$$I = I_1 + I_2 \tag{1.119}$$

where I is the current leaving the battery, I_1 is the current in resistor R_1, and I_2 is the current in the resistor R_2.
The potential difference applied across the series combination of resistors divides between the resistors. In Figure 1.34b, because the voltage drop from a to b equals I_1R_1 and the voltage drop from b to c equals I_2R_2, the *voltage drop*[2] from a to c is

$$\Delta V = \Delta V_1 + \Delta V_2 = I_1R_1 + I_2R_2 \tag{1.120}$$

The potential difference across the battery is also applied to the *equivalent resistance* R_{eq} in Figure 1.22c:

$$\Delta V = IR_{eq} \tag{1.121}$$

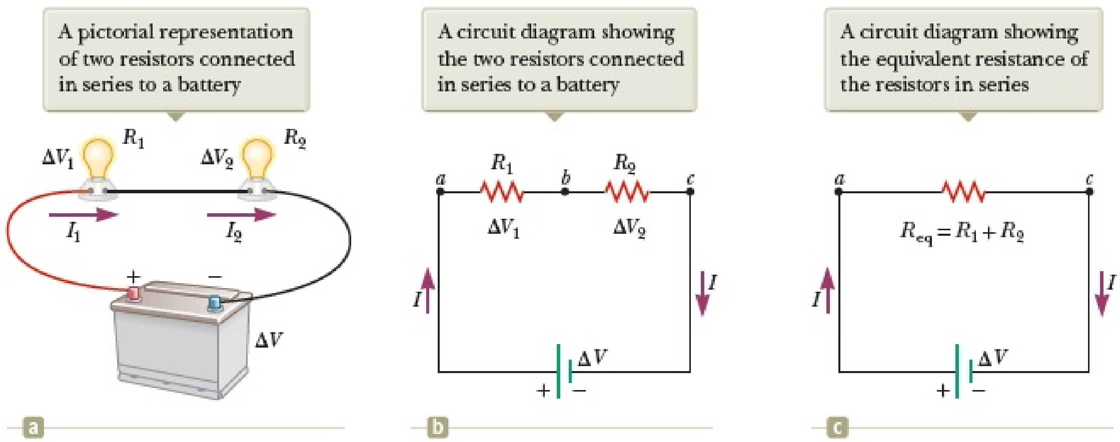


Figure 1.34: *Two lightbulbs with resistances R_1 and R_2 connected in series. All three diagrams are equivalent.*

[2] *The term voltage drop is synonymous with a decrease in electric potential across a resistor.*

where the equivalent resistance has the same effect on the circuit as the series combination because it results in the same current I in the battery. Combining these equations (Equation 1.120 and 1.121)for ΔV gives

$$IR_{eq} = I_1R_1 + I_2R_2 \Rightarrow R_{eq} = R_1 + R_2 \tag{1.122}$$

where we have canceled the currents I, I_1, and I_2 because they are all the same. We see that we can replace the two resistors in series with a single equivalent resistance whose value is the sum of the individual resistances.
The equivalent resistance of two or more resistors connected in series is

$$R_{eq} = \sum_i R_i \tag{1.123}$$

This relationship (Equation 1.123) indicates that the equivalent resistance of a series combination of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.
Looking back at the Equation 1.117, we see that the denominator of the right-hand side is the simple algebraic sum of the external and internal resistances. That is consistent with the internal and external resistances being in series in Figure 1.33a.

If the filament of one lightbulb in Figure 1.34a were to fail, the circuit would no longer be complete (resulting in an open-circuit condition) and the second lightbulb would also go out. This fact is a general feature of a series circuit: if one device in the series creates an open circuit, all devices are inoperative.

Resistors in Parallel

A parallel circuit is a circuit in which the resistors are arranged with their heads connected together, and their tails connected together. The current in a parallel circuit breaks up, with some flowing along each parallel branch and re-combining when the branches meet again. The voltage across each resistor in parallel is the same.
Now consider two resistors in a parallel combination as shown in Figure 1.35. As with the series combination, what is the value of the single resistor that could replace the combination and draw the same current from the battery? Notice that both resistors are connected directly across the terminals of the battery. Therefore, the potential differences across the resistors are the same:

$$\Delta V = \Delta V_1 = \Delta V_2 \tag{1.124}$$

where ΔV is the terminal voltage of the battery.
When charges reach point a in Figure 1.35b, they split into two parts, with some going toward R_1 and the rest going toward R_2. A junction is any such point in a circuit where a current can split. This split results in less current in each individual resistor than the current leaving the battery. Because electric charge is conserved, the current I that enters point a must equal the total current leaving that point:

$$I = I_1 + I_2 = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \tag{1.125}$$

where I_1 is the current in R_1 and I_2 is the current in R_2.

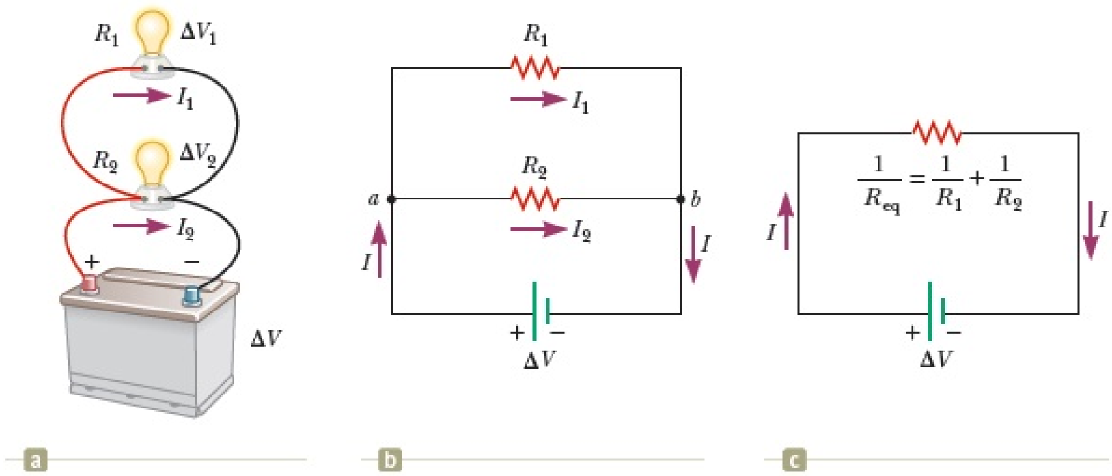


Figure 1.35: *Two lightbulbs with resistances R_1 and R_2 connected in parallel. All three diagrams are equivalent.*

The current in the *equivalent resistance* R_{eq} in Figure 1.23c is

$$I = \frac{\Delta V}{R_{eq}} \tag{1.126}$$

where the equivalent resistance has the same effect on the circuit as the two resistors in parallel; that is, the equivalent resistance draws the same current I from the battery. Combining these equations (Equation 1.126 and 1.125) for I, we see that the equivalent resistance of two resistors in parallel is given by

$$\frac{\Delta V}{R_{eq}} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{1.127}$$

where we have canceled ΔV,ΔV_1, and ΔV_2 because they are all the same. An extension of this analysis to three or more resistors in parallel gives

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i} \tag{1.128}$$

This expression shows that the inverse of the equivalent resistance of two or more resistors in a parallel combination is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.

Household circuits are always wired such that the appliances are connected in parallel. Each device operates independently of the others so that if one is switched off, the others remain on. In addition, in this type of connection, all the devices operate on the same voltage.

Circuits with series and parallel components

Many circuits have a combination of series and parallel resistors. Generally, the total resistance in a circuit like this is found by reducing the different series and parallel combinations step-by-step to end up with a single equivalent resistance for the circuit. This allows the current to be determined easily. The current flowing through each resistor can then be found by undoing the reduction process.
General rules for doing the reduction process include:

- Two (or more) resistors with their heads directly connected together and their tails directly connected together are in parallel, and they can be reduced to one resistor using the equivalent resistance equation for resistors in parallel.
- Two resistors connected together so that the tail of one is connected to the head of the next, with no other path for the current to take along the line connecting them, are in series and can be reduced to one equivalent resistor using the equivalent resistance equation for resistors in series.

Finally, remember that for resistors in series, the current is the same for each resistor, and for resistors in parallel, the voltage is the same for each one.

1.4.3 Kirchhoff's Rules

As we saw in the preceding section, combinations of resistors can be simplified and analyzed using the expression $\Delta V = IR$ and the rules for series and parallel combinations of resistors. Very often, however, it is not possible to reduce a circuit to a single loop using these rules. The procedure for analyzing more complex circuits (multi-loop circuits) is made possible by using two principles, called *Kirchhoff's rules*[3]. These principles also apply to very simple circuits.
Before talking about those rules, it is helpful to define two terms, junction and branch.

- A junction is a point where at least three circuit paths meet.
- A branch is a path connecting two junctions.

Here are the two Kirchhoff's rules:

(i) Junction rule: "At any junction, the sum of the currents must equal zero":

$$\sum_{junction} I = 0 \tag{1.129}$$

(ii) Loop rule: "The sum of the potential differences across all elements around any closed circuit loop must be zero":

$$\sum_{closed\,loop} \Delta V = 0 \tag{1.130}$$

Kirchhoff's first rule is a statement of conservation of electric charge. All charges that enter a given point in a circuit must leave that point because charge cannot build up or disappear at a point. Currents directed into the junction are entered into the sum in the junction rule as $+I$, whereas currents directed out of a junction are entered as $-I$. Applying this rule to the junction in Figure 1.36a gives

$$I_1 - I_2 - I_3 = 0 \tag{1.131}$$

Figure 1.36b represents a mechanical analog of this situation, in which water flows through a branched pipe having no leaks. Because water does not build up anywhere in the pipe,

[3] *Guidelines established by a German Physicist Gustav Kirchhoff (1824-1887)*

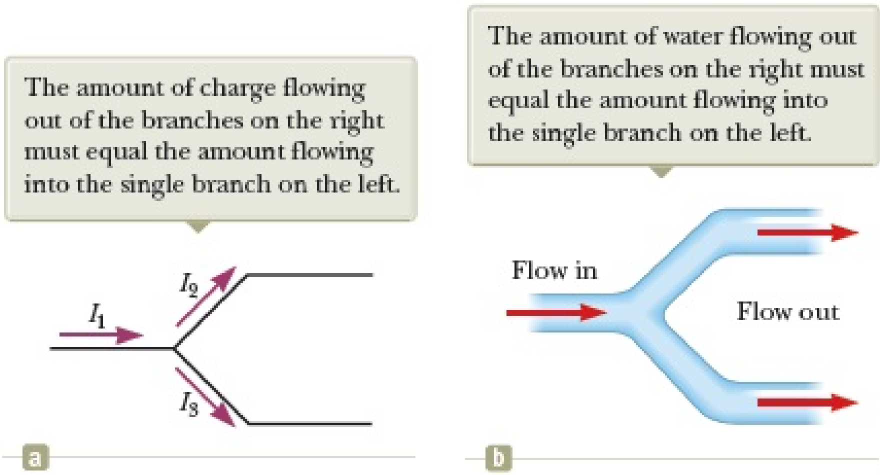


Figure 1.36: *(a) Kirchhoff's junction rule. (b) A mechanical analogy of the junction rule*

the flow rate into the pipe on the left equals the total flow rate out of the two branches on the right.

Kirchhoff's second rule follows from the law of conservation of energy for an isolated system. Let's imagine moving a charge around a closed loop of a circuit. When the charge returns to the starting point, the charge–circuit system must have the same total energy as it had before the charge was moved. The sum of the increases in energy as the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements. The potential energy of the system decreases whenever the charge moves through a potential drop $-IR$ across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal. When applying Kirchhoff's second rule, imagine traveling around the loop and consider changes in electric potential rather than the changes in potential energy described in the preceding paragraph. Imagine traveling through the circuit elements in Figure 1.37 toward the right.
There are two different methods for analyzing circuits. The standard method in physics, which is the one we follow in this study, is the branch current method. There is another method, the loop current method, but we won't worry about that one.
To analyze a circuit using the branch-current method involves three steps:

1. Label the current and the current direction in each branch. Sometimes it is hard to tell which is the correct direction for the current in a particular loop. Simply pick a direction. If you guess wrong, you will get a negative value. The value is correct, and the negative sign means that the current direction is opposite to the way you guessed. You should use the negative sign in your calculations, however.

2. Use Kirchhoff's first rule to write down current equations for each junction that gives you a different equation. For a circuit with two inner loops and two junctions, one current equation is enough because both junctions give you the same equation.

3. Use Kirchhoff's second rule to write down loop equations for as many loops as it takes to include each branch at least once. To write down a loop equation, you choose a starting point, and then walk around the loop in one direction until you get back to the starting point. As you cross batteries and resistors, write down each

potential change. Add these potential gains and losses up and set them equal to zero.

- Charges move from the high-potential end of a resistor toward the low-potential end, so if a resistor is traversed in the direction of the current, the potential difference across the resistor is $-IR$ (Figure 1.37a)
- If a resistor is traversed in the direction opposite the current, the potential difference across the resistor is $+IR$ (Figure 1.37b)
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from negative to positive), the potential difference is $+\mathcal{E}$ (Figure 1.37c)
- If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from positive to negative), the potential difference is $-\mathcal{E}$ (Figure 1.37d)

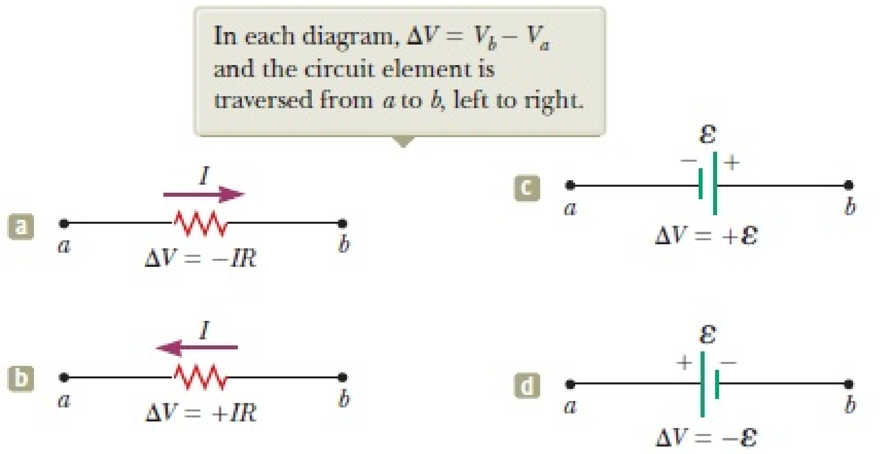


Figure 1.37: *Rules for determining the signs of the potential differences across a resistor and a battery. (The battery is assumed to have no internal resistance.)*

There are limits on the number of times you can usefully apply Kirchhoff's rules in analyzing a circuit. You can use the junction rule as often as you need as long as you include in it a current that has not been used in a preceding junction–rule equation. In general, the number of times you can use the junction rule is one fewer than the number of junction points in the circuit (if you have N *junctions* in the circuit, you need only $N-1$ *equations*). You can apply the loop rule as often as needed as long as a new circuit element (resistor or battery) or a new current appears in each new equation (if the circuit has M *loops* and N *junctions*, for the second rule of Kirchhoff we need $M-N+1$ *equations*. Thus, to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.

1.4.4 RC Circuits

So far, we have analyzed direct-current circuits in which the current is constant. In direct circuits (*DC circuits*) containing capacitors, the current is always in the same direction but may vary in magnitude at different times. A circuit containing a series combination of a resistor and a capacitor is called an *RC circuit*.

Charging a Capacitor

Let us assume that the capacitor in Figure 1.38 is initially uncharged. There is no current while switch S is open (Figure 1.38b). If the switch is closed at $t = 0$, however, charge begins to flow, setting up a current in the circuit, and the capacitor begins to charge. Note that during charging, charges do not jump across the capacitor plates because the gap between the plates represents an open circuit. Instead, charge is transferred between each plate and its connecting wire due to the electric field established in the wires by the battery, until the capacitor is fully charged. As the plates become charged, the potential difference across the capacitor increases. The value of the maximum charge depends on the voltage of the battery. Once the maximum charge is reached, the current in the circuit is zero because the potential difference across the capacitor matches that supplied by the battery.

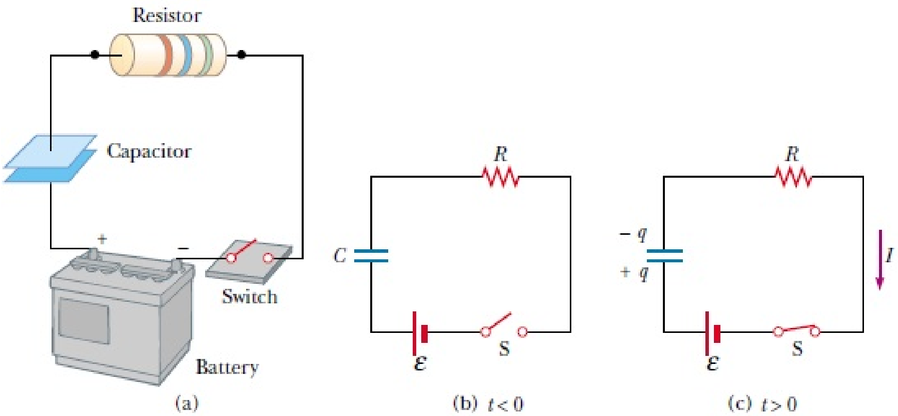


Figure 1.38: *(a) A capacitor in series with a resistor, switch, and battery. (b) Circuit diagram representing this system at time $t < 0$, before the switch is closed. (c) Circuit diagram at time $t > 0$, after the switch has been closed.*

To analyze this circuit quantitatively, let us apply Kirchhoff's loop rule to the circuit after the switch is closed. Traversing the loop clockwise gives

$$\mathcal{E} - \frac{q}{C} - IR = 0 \tag{1.132}$$

where q/C is the potential difference across the capacitor and IR is the potential difference across the resistor.
At the instant the switch is closed ($t = 0$), the charge on the capacitor is zero, and from the previous equation we find that the initial current in the circuit I_0 is a maximum and is equal to

$$I_0 = \frac{\mathcal{E}}{R} \tag{1.133}$$

At this time ($t = 0$), the potential difference from the battery terminals appears entirely across the resistor. Later, when the capacitor is charged to its maximum value Q ,charges cease to flow, the current in the circuit is zero, and the potential difference from the battery terminals appears entirely across the capacitor. Substituting $I = 0$ into the Equation 1.132 gives the charge on the capacitor at this time:

$$Q = C\mathcal{E} \tag{1.134}$$

To determine analytical expression for the time dependance of the charge and current, we must solve the equation Equation 1.132. The current in all parts of series circuit must be the same. Thus, the current in the resistance R must be the same as the current flowing out of and into the capacitor plates. This current is equal to the time rate of change of the charge on the capacitor plates. Thus, we substitute $I = dq/dt$ into Equation 1.132 and rearrange the equation to get

$$\frac{dq}{dt} = -\frac{q - C\mathcal{E}}{RC} \tag{1.135}$$

or

$$\frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}dt \tag{1.136}$$

Integrating this expression, using the fact that $q = 0$ at $t = 0$, we obtain

$$\int_0^q \frac{dq}{q - C\mathcal{E}} = -\frac{1}{RC}\int_0^t dt \tag{1.137}$$

That is

$$\ln\left(\frac{q - C\mathcal{E}}{-C\mathcal{E}}\right) = -\frac{t}{RC} \tag{1.138}$$

From the definition of the natural logarithm, we can write this expression as

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q\left(1 - e^{-t/RC}\right) \tag{1.139}$$

We can find an expression for the charging current by differentiating the previous equation with respect to time. Using $I = dq/dt$ we find that

$$I(t) = \frac{\mathcal{E}}{R}e^{-t/RC} \tag{1.140}$$

The quantity RC is called the *time constant* τ of the circuit. The time constant represents the time interval during which the current decreases to $1/e$ of its initial value; that is, after a time intervalτ, the current decreases to $i = e^{-1}I_i = 0.368I_i$. After a time interval 2τ, the current decreases to $i = e^{-2}I_i = 0.135I_i$ and so forth. Likewise, in the time interval τ, the charge increases from zero to $C\mathcal{E}[1 - e^{-1}] = 0.632C\mathcal{E}$
The dimensional analysis shows that τ has units of time.

The energy output of the battery as the capacitor is fully charged is $Q\mathcal{E} = C\mathcal{E}^2$. After the capacitor is fully charged, the energy stored in the capacitor is $\frac{1}{2}Q\mathcal{E} = \frac{1}{2}C\mathcal{E}^2$, which is just half the energy output of the battery. The remaining half of the energy supplied by the battery appears as internal energy in the resistor.

Discharging a Capacitor

Now let us consider the circuit shown in Figure 1.39, which consists of a capacitor carrying an initial charge Q, a resistor, and a switch. The initial charge Q is not the same as the maximum charge Q in the previous discussion, unless the discharge occurs after the capacitor is fully charged (as described earlier). When the switch is open, a potential difference Q/C exists across the capacitor and there is zero potential difference across the resistor because $I = 0$. If the switch is closed at $t = 0$ the capacitor begins to discharge through the resistor. At some time t during the discharge, the current in the circuit is I

and the charge on the capacitor is q (Figure 1.39b). The circuit in Figure 1.39 is the same as the circuit in Figure 1.38 except for the absence of the battery. Thus, we eliminate the *emf* from the Equation 1.132 to obtain the appropriate loop equation for the circuit in Figure 1.39:

$$-\frac{q}{C} - IR = 0 \tag{1.141}$$

When we substitute $I = dq/dt$ into Equation 1.141, it becomes

$$-R\frac{dq}{dt} = \frac{q}{C} \quad or \quad \frac{dq}{q} = -\frac{1}{RC}dt \tag{1.142}$$

Integrating Equation 1.142, using the fact that $q = Q$ at $t = 0$, gives

$$\int_Q^q \frac{dq}{q} = -\frac{1}{RC}\int_0^t dt \tag{1.143}$$

Thus,

$$\ln\left(\frac{q}{Q}\right) = -\frac{t}{RC}$$

or

$$q(t) = Qe^{-t/RC} \tag{1.144}$$

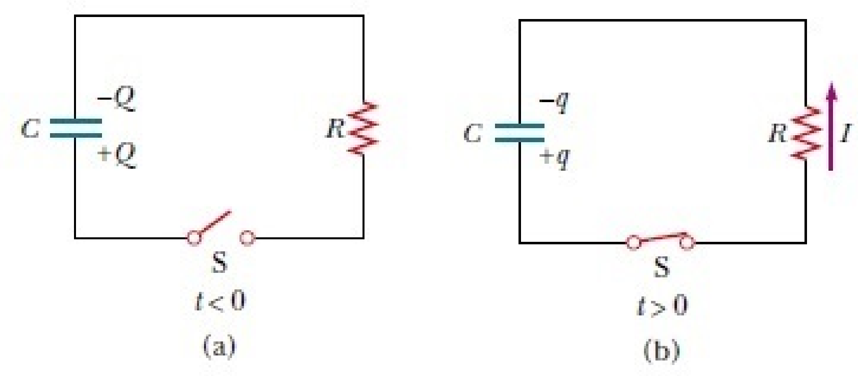


Figure 1.39: *(a) A charged capacitor connected to a resistor and a switch, which is open at $t < 0$ (b) After the switch is closed, a current that decreases in magnitude with time is set up in the direction shown, and the charge on the capacitor decreases exponentially with time.*

Differentiating the expression of the Equation 1.144 with respect to time gives the instantaneous current as a function of time:

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-t/RC}\right) = -\frac{Q}{RC}e^{-t/RC} \tag{1.145}$$

where $Q/RC = I_0$ is the initial current. The negative sign indicates that the current direction now that the capacitor is discharging is opposite the current direction when the capacitor was being charged. We see that both the charge on the capacitor and the current decay exponentially at a rate characterized by the time constant $\tau = RC$.

1.4.5 Electrical Instruments

The Ammeter

A device that measures current is called an ammeter. The current to be measured must pass directly through the ammeter, so the ammeter must be connected in series with other elements in the circuit, as shown in Figure 1.40. When using an ammeter to measure direct currents, you must be sure to connect it so that current enters the instrument at the positive terminal and exits at the negative terminal.

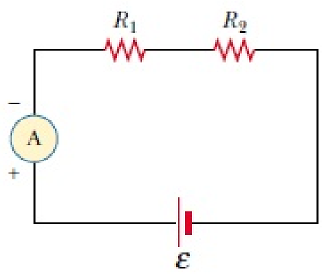


Figure 1.40: *Current can be measured with an ammeter connected in series with the resistor and battery of a circuit. An ideal ammeter has zero resistance.*

Ideally, an ammeter should have zero resistance so that the current being measured is not altered. In the circuit shown in Figure 1.40, this condition requires that the resistance of the ammeter be much less than $R_1 + R_2$. Because any ammeter always has some internal resistance, the presence of the ammeter in the circuit slightly reduces the current from the value it would have in the meter's absence.

The Voltmeter

A device that measures potential difference is called a voltmeter. The potential difference between any two points in a circuit can be measured by attaching the terminals of the voltmeter between these points without breaking the circuit, as shown in Figure 1.41. The potential difference across resistor R_2 is measured by connecting the voltmeter in parallel with R_2 . Again, it is necessary to observe the polarity of the instrument. The positive terminal of the voltmeter must be connected to the end of the resistor that is at the higher potential, and the negative terminal to the end of the resistor at the lower potential.
An ideal voltmeter has infinite resistance so that no current passes through it. In Figure Figure 1.41, this condition requires that the voltmeter have a resistance much greater than R_2. In practice, if this condition is not met, corrections should be made for the known resistance of the voltmeter.

The Galvanometer

The galvanometer is the main component in analog ammeters and voltmeters. Figure 1.42 illustrates the essential features of a common type called the D'Arsonval galvanometer. It consists of a coil of wire mounted so that it is free to rotate on a pivot in a magnetic field provided by a permanent magnet. The basic operation of the galvanometer makes use of the fact that a torque acts on a current loop in the presence of a magnetic field (Chapter 2). The torque experienced by the coil is proportional to the current through

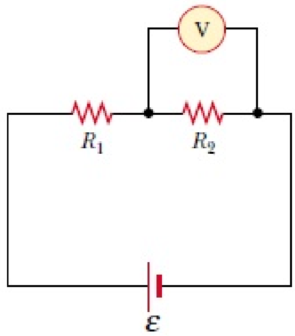


Figure 1.41: *The potential difference across a resistor can be measured with a voltmeter connected in parallel with the resistor. An ideal voltmeter has infinite resistance.*

it: the larger the current, the greater the torque and the more the coil rotates before the spring tightens enough to stop the rotation. Hence, the deflection of a needle attached to the coil is proportional to the current. Once the instrument is properly calibrated, it can be used in conjunction with other circuit elements to measure either currents or potential differences.

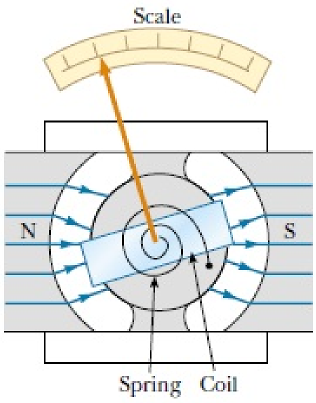


Figure 1.42: *The principal components of a D'Arsonval galvanometer. When the coil situated in a magnetic field carries a current, the magnetic torque causes the coil to twist. The angle through which the coil rotates is proportional to the current in the coil because of the counteracting torque of the spring.*

A typical off-the-shelf galvanometer is often not suitable for use as an ammeter, primarily because it has a resistance of about $60\,\Omega$. An ammeter resistance this great considerably alters the current in a circuit. You can understand this by considering the following example: The current in a simple series circuit containing a *3-V* battery and a $3\,\Omega$ resistor is 1 A. If you insert a $60\,\Omega$ galvanometer in this circuit to measure the current, the total resistance becomes $63\,\Omega$ and the current is reduced to *0.048 A*!
A second factor that limits the use of a galvanometer as an ammeter is the fact that a typical galvanometer gives a full-scale deflection for currents of the order of *1 mA* or less. Consequently, such a galvanometer cannot be used directly to measure currents greater than this value. However, it can be converted to a useful ammeter by placing a shunt resistor R_p in parallel with the galvanometer, as shown in Figure 1.43a. The value of R_p must be much less than the galvanometer resistance so that most of the current to be measured passes through the shunt resistor.

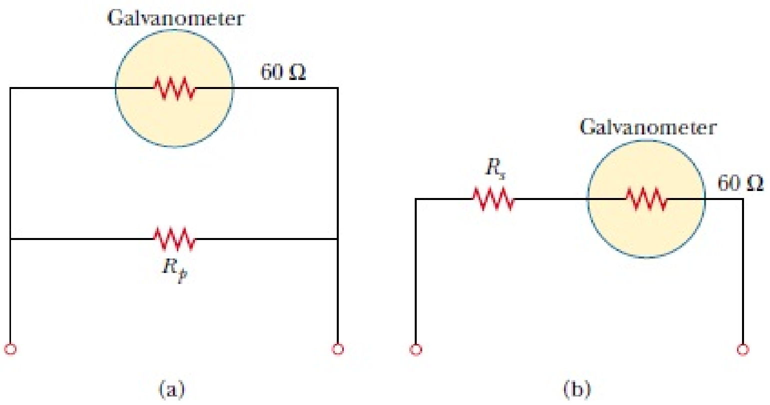


Figure 1.43: *(a) When a galvanometer is to be used as an ammeter, a shunt resistor R_p is connected in parallel with the galvanometer. (b) When the galvanometer is used as a voltmeter, a resistor R_s is connected in series with the galvanometer.*

A galvanometer can also be used as a voltmeter by adding an external resistor R_s in series with it, as shown in Figure 1.43b. In this case, the external resistor must have a value much greater than the resistance of the galvanometer to ensure that the galvanometer does not significantly alter the voltage being measured.

The Wheatstone Bridge

An unknown resistance value can be accurately measured using a circuit known as a Wheatstone bridge (Figure 1.44). This circuit consists of the unknown resistance R_x , three known resistances R_1 , R_2 , and R_3 (where R_1 is a calibrated variable resistor), a galvanometer, and a battery. The known resistor R_1 is varied until the galvanometer reading is zero—that is, until there is no current from a to b. Under this condition the bridge is said to be balanced. Because the electric potential at point a must equal the potential at point b. when the bridge is balanced, the potential difference across R_1 must equal the potential difference across R_2 . Likewise, the potential difference across R_3 must equal the potential difference across R_x.

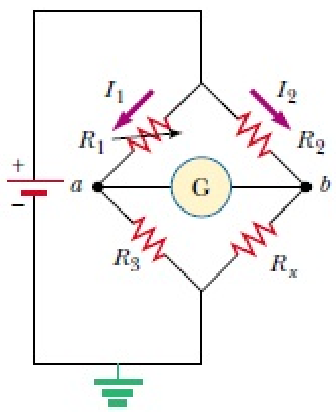


Figure 1.44: *Circuit diagram for a Wheatstone bridge, an instrument used to measure an unknown resistance R_x in terms of known resistances R_1 , R_2 , and R_3. The arrow superimposed on the circuit symbol for resistor R_1 indicates that the value of this resistor can be varied by the person operating the bridge.*

From these considerations we see that

$$(1)\ \ I_1R_1 = I_2R_2 \qquad and \qquad (2)\ \ I_1R_3 = I_2R_x \tag{1.146}$$

Dividing Equation (1) by Equation (2) eliminates the currents, and solving for R_x, we find that

$$R_x = \frac{R_2R_3}{R_1} \tag{1.147}$$

A number of similar devices also operate on the principle of null measurement (that is, adjustment of one circuit element to make the galvanometer read zero). One example is the capacitance bridge used to measure unknown capacitances. These devices do not require calibrated meters and can be used with any voltage source.
Wheatstone bridges are not useful for resistances above $10^5\ \Omega$, but modern electronic instruments can measure resistances as high as $10^{12}\,\Omega$. Such instruments have an extremely high resistance between their input terminals. For example, input resistances of $10^{10}\,\Omega$ are common in most digital multimeters, which are devices that are used to measure voltage, current, and resistance.

The Potentiometer

A potentiometer is a circuit that is used to measure an unknown *emf* $\mathcal{E}_x$ by comparison with a known *emf*. In Figure 1.45, point d represents a sliding contact that is used to vary the resistance (and hence the potential difference) between points a and d. The other required components are a galvanometer, a battery of known *emf* $\mathcal{E}_0$, and a battery of unknown *emf* $\mathcal{E}_x$. With the currents in the directions shown in Figure 1.45, we see from Kirchhoff's junction rule that the current in the resistor R_x is $I - I_x$ where I is the current in the left branch (through the battery of emf $\mathcal{E}_0$) and I_x is the current in the right branch. Kirchhoff's loop rule applied to loop *abcda* traversed clockwise gives

$$-\mathcal{E}_x + (I - I_x)R_x = 0 \tag{1.148}$$

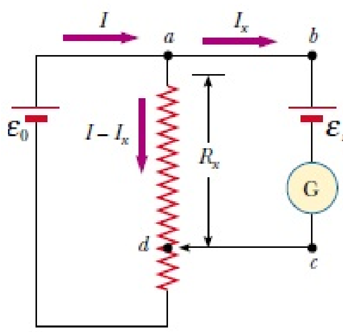


Figure 1.45: *Circuit diagram for a potentiometer. The circuit is used to measure an unknown emf* $\mathcal{E}_x$.

Because current I_x passes through it, the galvanometer displays a nonzero reading. The sliding contact at d is now adjusted until the galvanometer reads zero (indicating a balanced circuit and that the potentiometer is another null-measurement device). Under this condition, the current in the galvanometer is zero, and the potential difference between a and d must equal the unknown emf $\mathcal{E}_x$.

$$\mathcal{E}_x = IR_x \tag{1.149}$$

Next, the battery of unknown emf is replaced by a standard battery of known *emf* $\mathcal{E}_s$, and the procedure is repeated. If R_s is the resistance between a and d when balance is achieved this time, then

$$\mathcal{E}_s = IR_s \tag{1.150}$$

where it is assumed that I remains the same.
Combining Equation 1.150 with Equation 1.149, we see that

$$\mathcal{E}_x = \frac{R_x}{R_s}\mathcal{E}_s \tag{1.151}$$

If the resistor is a wire of resistivity ρ, its resistance can be varied by using the sliding contact to vary the length L, indicating how much of the wire is part of the circuit. With the substitutions $R_s = \rho L_s/A$ and $R_x = \rho L_x/A$ the Equation 1.151 becomes

$$\mathcal{E}_x = \frac{L_x}{L_s}\mathcal{E}_s \tag{1.152}$$

where L_x is the resistor length when the battery of unknown *emf* $\mathcal{E}_x$ is in the circuit and L_s is the resistor length when the standard battery is in the circuit.
The sliding–wire circuit of Figure 1.45 without the unknown *emf* and the galvanometer is sometimes called a *Potential (Voltage) Divider*.

In general, a potentiometer divider is a line of resistors in series that are used to give different voltages in parts of an electronic circuit. As the name says, in this case it divides the electric potential (voltage) into different amounts. These types of potential dividers are widely used in electronic circuits for setting and adjusting voltages.
For example, you may find that you need a supply of 6 V or 3 V and you have a 9-V–battery, your only option may be to make a potential divider.

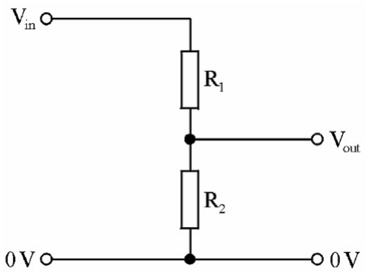


Figure 1.46: *A Potential divider*

Knowing the values of the resistors of the Figure 1.46, we ca establish the following formula

$$(1) \quad V_{in} = I(R_1 + R_2) \qquad and \qquad (2) \quad V_{out} = IR_2 \tag{1.153}$$

Dividing Equation (1) by Equation (2) eliminates the currents, and solving for V_{out}, we find that

$$V_{out} = \frac{R_2}{R_1 + R_2}V_{in} \tag{1.154}$$

1.4.6 Household Wiring and Electrical Safety

Many considerations are important in the design of an electrical system of a home that will provide adequate electrical service for the occupants while maximizing their safety. We discuss some aspects of a home electrical system in this section.

Household Wiring

Household circuits represent a practical application of some of the ideas presented in this chapter. In our world of electrical appliances, it is useful to understand the power requirements and limitations of conventional electrical systems and the safety measures that prevent accidents.
In a conventional installation, the utility company distributes electric power to individual homes by means of a pair of wires, with each home connected in parallel to these wires. One wire is called the live wire (*Live wire is a common expression for a conductor whose electric potential is above or below ground potential*) as illustrated in Figure 1.47, and the other is called the neutral wire. The neutral wire is grounded; that is, its electric potential is taken to be zero. The potential difference between the live and neutral wires is approximately 230 V (or 120 V according to the countries' or territories' convention). This voltage alternates in time, and the potential of the live wire oscillates relative to ground. Much of what we have learned so far for the constant-*emf* situation (direct current) can also be applied to the alternating current that power companies supply to businesses and households.

To record a household's energy consumption, a meter is connected in series with the live wire entering the house. After the meter, the wire splits so that there are several separate circuits in parallel distributed throughout the house. Each circuit contains a circuit breaker (or, in older installations, a fuse). A circuit breaker is a special switch that opens if the current exceeds the rated value for the circuit breaker. The wire and circuit breaker for each circuit are carefully selected to meet the current requirements for that circuit. If a circuit is to carry currents as large as 30 A, a heavy wire and an appropriate circuit breaker must be selected to handle this current. A circuit used to power only lamps and small appliances often requires only 20 A. Each circuit has its own circuit breaker to provide protection for that part of the entire electrical system of the house.

As an example, consider a circuit in which a toaster oven, a microwave oven, and a coffee maker are connected (corresponding to R_1, R_2, and R_3 in Figure 1.47). We can calculate the current in each appliance by using the expression $P = I\Delta V$. The toaster oven, rated at 1 000 W, draws a current of 1000 $W/120\ V = 8.33\ A$. The microwave oven, rated at 1 300 W, draws 10.8 A, and the coffee maker, rated at 800 W, draws 6.67 A. When the three appliances are operated simultaneously, they draw a total current of 25.8 A. Therefore, the circuit must be wired to handle at least this much current. If the rating of the circuit breaker protecting the circuit is too small–say, 20 A–the breaker will be tripped when the third appliance is turned on, preventing all three appliances from operating. To avoid this situation, the toaster oven and coffee maker can be operated on one 20-A circuit and the microwave oven on a separate 20-A circuit.
Many heavy-duty appliances such as electric ranges and clothes dryers require 240 V (0r 380 V according to countries' or territories' convention)for their operation. The power company supplies this voltage by providing a third wire that is 120 V below ground potential. The potential difference between this live wire and the other live wire (which is

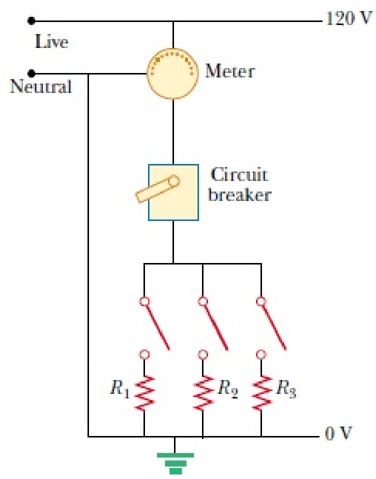


Figure 1.47: *Wiring diagram for a household circuit. The resistances represent appliances or other electrical devices that operate with an applied voltage of 120 V.*

120 V above ground potential) is 240 V. An appliance that operates from a 240-V line requires half as much current compared with operating it at 120 V; therefore, smaller wires can be used in the higher-voltage circuit without overheating.

Electrical Safety

When the live wire of an electrical outlet is connected directly to ground, the circuit is completed and a short-circuit condition exists. A short circuit occurs when almost zero resistance exists between two points at different potentials, and the result is a very large current. When that happens accidentally, a properly operating circuit breaker opens the circuit and no damage is done. A person in contact with ground, however, can be electrocuted by touching the live wire of a frayed cord or other exposed conductor. An exceptionally effective (and dangerous!) ground contact is made when the person either touches a water pipe (normally at ground potential) or stands on the ground with wet feet. The latter situation represents effective ground contact because normal, non-distilled water is a conductor due to the large number of ions associated with impurities. This situation should be avoided at all cost.

Electric shock can result in fatal burns or can cause the muscles of vital organs such as the heart to malfunction. The degree of damage to the body depends on the magnitude of the current, the length of time it acts, the part of the body touched by the live wire, and the part of the body in which the current exists. Currents of 5 mA or less cause a sensation of shock, but ordinarily do little or no damage. If the current is larger than about 10 mA, the muscles contract and the person may be unable to release the live wire. If the body carries a current of about 100 mA for only a few seconds, the result can be fatal. Such a large current paralyzes the respiratory muscles and prevents breathing. In some cases, currents of approximately 1 A can produce serious (and sometimes fatal) burns. In practice, no contact with live wires is regarded as safe whenever the voltage is greater than 24 V.
Many outlets are designed to accept a three-pronged power cord. One of these prongs is the live wire at a nominal potential of 120 V (or 230 accordingly). The second is the neutral wire, nominally at 0 V, which carries current to ground. Figure 1.48a shows a connection to an electric drill with only these two wires. If the live wire accidentally makes

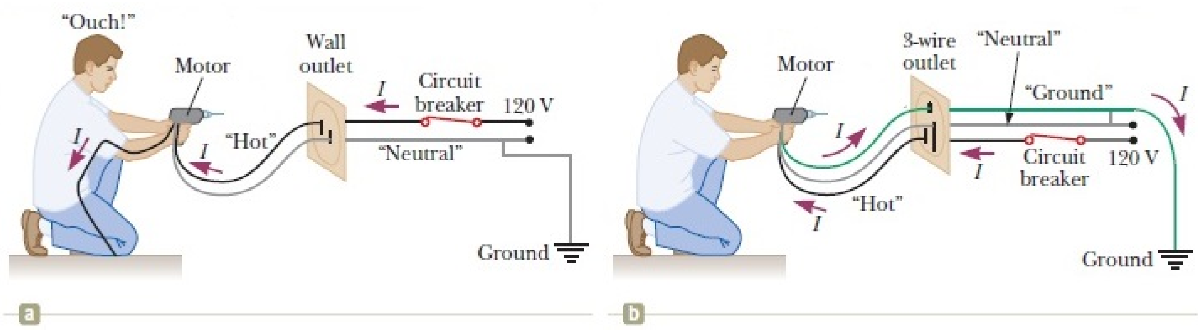


Figure 1.48: *(a) A diagram of the circuit for an electric drill with only two connecting wires. The normal current path is from the live wire through the motor connections and back to ground through the neutral wire. (b) This shock can be avoided by connecting the drill case to ground through a third ground wire.*

contact with the casing of the electric drill (which can occur if the wire insulation wears off), current can be carried to ground by way of the person, resulting in an electric shock. The third wire in a three-pronged power cord, the round prong, is a safety ground wire that normally carries no current. It is both grounded and connected directly to the casing of the appliance. If the live wire is accidentally shorted to the casing in this situation, most of the current takes the low–resistance path through the appliance to ground as shown in Figure 1.48.

Special power outlets called *ground-fault circuit interrupters* (GFCIs) are used in kitchens, bathrooms, basements, exterior outlets, and other hazardous areas of homes. These devices are designed to protect persons from electric shock by sensing small currents (less than 5 mA) leaking to ground. When an excessive leakage current is detected, the current is shut off in less than 1 ms.

1.5 Exercises on Chapter I

1. Three objects are brought close to one another, two at a time. When objects A and B are brought together, they attract. When objects B and C are brought together, they repel. Which of the following are necessarily true? (a) Objects A and C possess charges of the same sign. (b) Objects A and C possess charges of opposite sign. (c) All three objects possess charges of the same sign. (d) One object is neutral. (e) Additional experiments must be performed to determine information about the charges on the objects. [Ans. (e)]

2. Two identical charged spheres, each having a mass of $3.00 \times 10^{-2}\ kg$, hang in equilibrium as shown in the figure 1.49a The length L of each string is 0.150 m, and the angle θ is 5.00^o. Find the magnitude of the charge on each sphere. [Ans. 4.42×10^{-8} C]

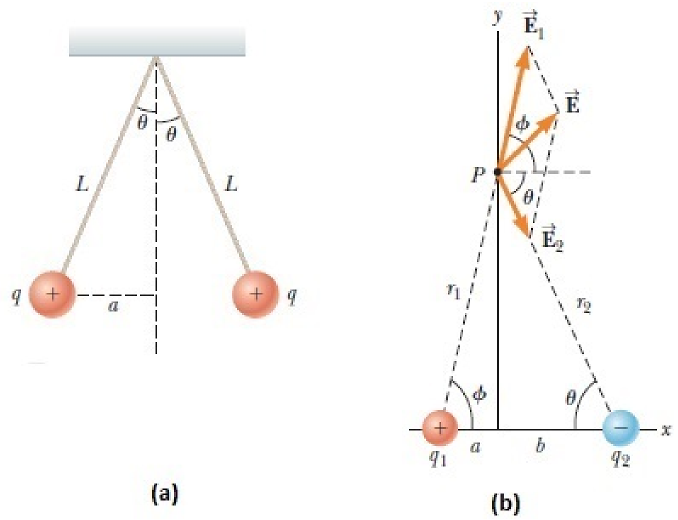


Figure 1.49: Exercise 2 and exercise 4.

3. A water droplet of mass $3.00 \times 10^{-12}\ kg$ is located in the air near the ground during a stormy day. An atmospheric electric field of magnitude $6.00 \times 10^3\ N/C$ points vertically downward in the vicinity of the water droplet. The droplet remains suspended at rest in the air. What is the electric charge on the droplet? [Ans. $-\times 10^{-15}$ C]

4. Charges q_1 and q_2 are located on x axis, at distance a and b, respectively, from the origin as shown in the figure1.49b. (a) Find the component of the net electric field at point P, which is at position *(0,y)*. (b) Evaluate the electric field at point P in the special case that $|q_1| = |q_2|$ and $a = b$. (c) Find the electric field due to the electric dipole when point P is a distance $y \gg a$ from the origin.

5. Find the ratio of the Coulomb electric force F_e to the gravitation force F_g between two electrons in vacuum. [Ans. 4.2×10^{42}]

6. Two coins lie 1.5 m apart on a table. They carry identical charges. Approximately how large is the charge on each if a coin experiences a force of 2 N? [Ans. 2×10^{-5} C]

7. Repeat the previous problem if the coins are separated by a distance of 1.5 m in a large vat of water. The dielectric constant of water is about 80. [Ans. 2×10^{-4} C]

8. A helium nucleus has charge $+2e$, and a neon nucleus $+10e$, where e is the quantum of charge. Find the repulsive force exerted on one by the other when they are 3.0 nm apart. Assume the system to be in vacuum. [Ans. 0.51 N]

9. In the Bohr model of the hydrogen atom, an electron circles a proton in an orbit of radius 5.3×10^{-11} m. The attraction of the proton for the electron furnishes the centripetal force to hold the electron in orbit. Find (a) the force of electrical attraction between the particles and (b) the electron's speed. [Ans. 82 nN; $2.2 \times 10^{6} m/s$]

10. The charges shown in the figure 1.50a are stationary. Find the force on the 4.0 μC charge due to the other two. [Ans. 3.9 N; $\theta = 97^o$]

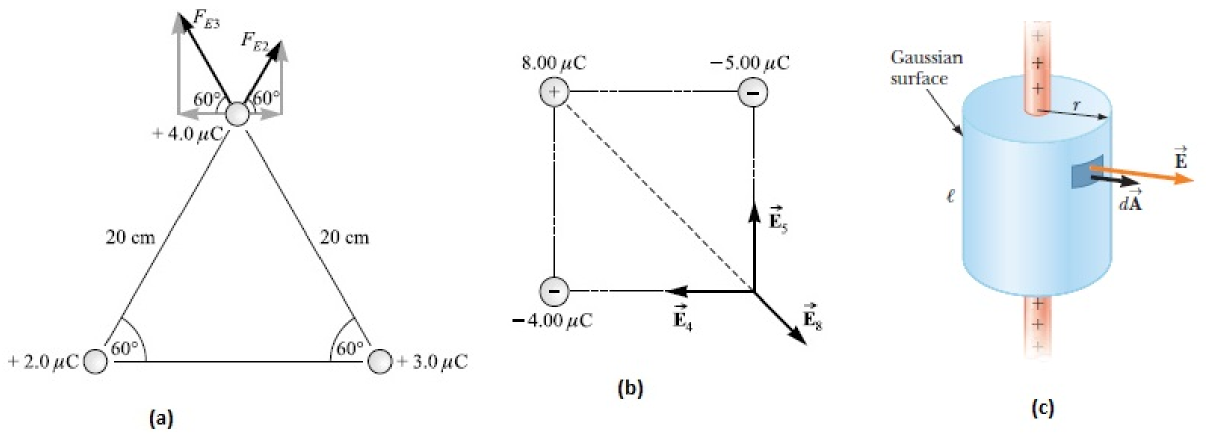


Figure 1.50: Exercise 10, exercise 11 and exercise 12.

11. Three charges are placed on three corners of a square, as shown in the figure 1.50b. Each side of the square is 30.0 cm. Compute $\vec{E}$ at the fourth corner. What would be the force on 6.00 μC charge placed at the vacant corner? [Ans. $E = 2.47 \times 10^{-7}\ N/C;\ \theta = 118^o;\ F = 1.48\ N$]

12. Find the electric field at a distance r from a line of positive charge of infinite length and constant charge per unit length λ (1.50c). [Ans. $E = 2k_e\frac{\lambda}{r}$]

13. Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ. [Ans. $E = \frac{\sigma}{2\varepsilon_0}$]

14. A flat surface of area 3.20 m^2 is rotated in a uniform electric field of magnitude $E = 6.20 \times 10^5\ N/C$ Determine the electric flux through this area (a) when the electric field is perpendicular to the surface and (b) when the electric field is parallel to the surface. [Ans. $(a)\ 19.84 \times 10^5\ Nm/C;\ (b)\ 0$]

15. Two charged metal plates are 15 cm apart as in the figure 1.51a. The electric field between the plates s uniform and has a strength of $E = 3000\ N/C$. An electron is released from rest at point P just outside the negative plate. (a) How long will it take to reach other plate? (b) How fast will be going just before it hits? [Ans. $(a);\ 2.38 \times 10^{-8}\ s;\ (b)\ 1.28 \times 10^7\ m/s$]

16. A battery has a specified potential difference ΔV between its terminals and establishes that potential difference between conductors attached to the terminals. A 12 V battery is connected on the two parallel plates. The separation between the plates is $d = 0.30\ cm$, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates. [Ans. $4.0 \times 10^3\ V/m$]

17. How much electrical work does a proton lose if it falls through a potential drop of 5 kV?. [Ans. $-8.0 \times 10^{-16}\ J$]

18. In the figure 1.51b, the potential difference between the metal plates is 40 V. (a) Which plate is at the higher potential? (b) How much work must be done to carry a $+3.0\ C$ charge form B to A? From A to B? (c) How do we know that the electric field is in the direction indicated? (d) If the plate separation is 5.0 mm, what is the magnitude of $\vec{E}$. [Ans. (a) A; (b) 0.12 kJ; (d) 8.0 kV]

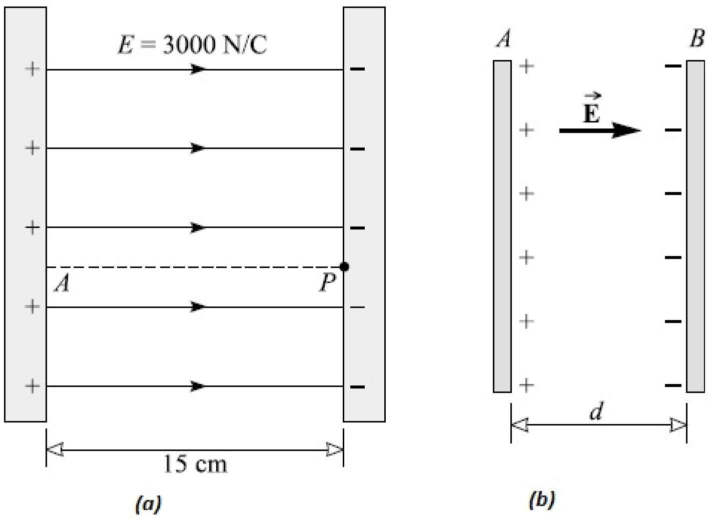


Figure 1.51: Exercise 15 and exercise 18.

19. How much work is required to carry an electron from the positive terminal of 12 V battery to the negative terminal? [Ans. 1.9×10^{-18} J]

20. (a) What is the absolute potential at each of the following distance from a charge of $2.0\ \mu C$: $r = 10\ cm$ and $r = 50\ cm$? (b) How much work is required to carry a $0.05\ \mu C$ charge from point at $r = 50\ cm$ to that at $r = 10\ cm$? [Ans. (a) 1.8×10^5 V; 36 kV]

21. A tin nucleus has a charge $+50e$. (a) Find the absolute potential V at the radius of $1.0 \times 10^{-12}\ m$ from the nucleus. (b) If the proton is released from this point, how fast will it be moving when it is 1.0 m from the nucleus? [Ans. 72 kV; $3.7 \times 10^6\ m/s$]

22. Find the electric potential energy of three point charges placed as follows on the x-axis: $+2.0\ \mu C$ at $x = 0$,$+3.0\ \mu C$ at $x = 20\ cm$, and $+6.0\ \mu C$ at $x = 50\ cm$. Take the PE_E to be zero when the charges are far separated. [Ans. 1.0 J]

23. Two protons are held at rest, 5.0×10^{-12} m apart. When released, they fly apart. How fast will each be moving when they are far each other. [Ans. 1.7×10^5 m/s]

24. A capacitor has a capacitance of 8.0 μF with air between its plates. Determine its capacitance when a dielectric with dielectric constant 6.0 is placed between its plates. [Ans. 48 μF]

25. What is the charge on a 300 pF capacitor when it is charged to a voltage of 1.0 kV? [Ans. 0.3 μC]

26. A metal sphere mounted on insulating rod carries a charge of 6.0 nC when its potential is 200 V higher than its surroundings. What is the capacitance of the capacitor formed by the sphere and its surroundings? [Ans. 30 pF]

27. The series combination of two capacitors shown in the figure 1.52a is connected across 100 V. Compute (a) the equivalent capacitance of the combination, (b) the magnitude of the charges on the capacitors, (c) the potential difference across the capacitors, and (d) the energy stored in the capacitors. [Ans. (a) 2.0 pF; (b) 2.0 nC; (c) 0.67 kV and 0.33 kV; (d) 1.0 μJ]

28. The parallel capacitor combination shown in the figure 1.52b is connected across a 120 V source. Determine the equivalent capacitor, the charge on each capacitor, and the charge on the combination. [Ans. 8.0 pF; 0.96 nC]

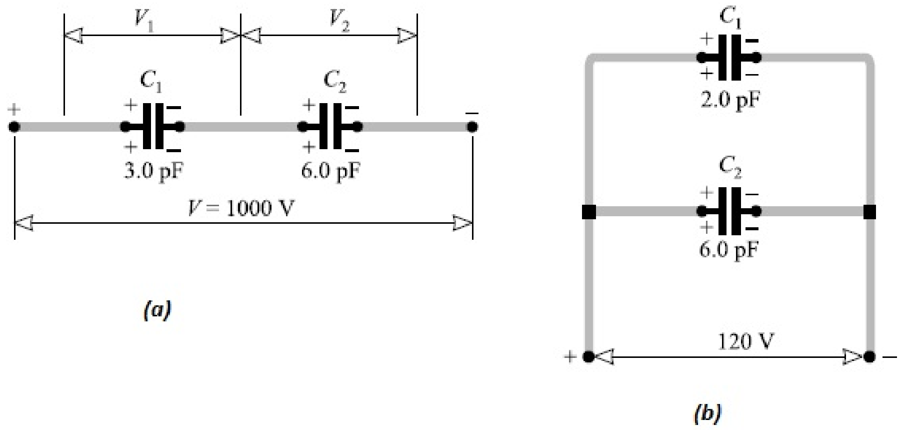


Figure 1.52: Exercise 27 and exercise 28.

29. A certain parallel-plate capacitor consists of two, each with area 200 cm^2, separated by a $0.40 - cm$ air gap. (a) Computer its capacitance. (b) If the capacitor is connected across a 500 V source, find the charge on it, the energy stored in it, and the value of E between the plates. (c) If a liquid with $\kappa = 2.60$ is poured between the plates so as fill the air gap, how much additional charge will flow into the capacitor from the 500 V source? [Ans. 44 pF; 1.3×10^5 V/m; 57 nC]

30. Two capacitors, 3.0 μC and 4.0 μC, are individually charge across a 6.0-V battery. After being disconnected from the battery, they are connected together with a negative plate of one attached to the positive plate of the other. What is the final charge on each capacitor? [Ans. 3.4 μC; 2.6 μC]

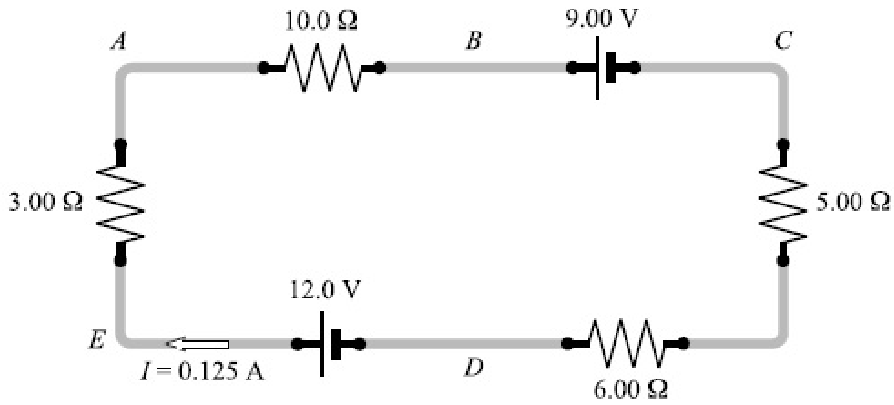


Figure 1.53: Exercise 31.

31. The current in the figure 1.53 is 0.125 A in the direction shown. For each of the following pairs of points, what is their potential difference, and which point is at higher potential? (a) A,B; (b) B, C; (c) C, D; (d) D, E; (e) C, E; (f) E; C. [Ans. (a) -1.25 V, A;(b) 9 V, B; (c) -1.38 V, C; (d) 12.0 V, E; (e) 10.6 V, E; (f) -10.6 V, E]

32. A steady current of 0.5 A flows through a wire. How much charge passes through the wire in one minute? [Ans. 30 C]

33. A light bulb has a resistance of 240 Ω when lit. How much current will flow through it when connected across 120 V, its normal operating voltage? [Ans. 0.500 A]

34. A dry cell has an emf of 1.52 V. Its terminal potential drops to zero when a current of 25A passes through it. What is its internal resistance? [Ans. 0.061 Ω]

35. A direct-current generator has an emf of 120 V, that is, *its terminal voltage is 120 V when no current is flowing from it.* At an output of 20 A the potential is 115 V. (a) What is the internal resistance of the generator? (b) What will be the terminal voltage at an output of 40 A? [Ans. 0.25 Ω; 110 V]

36. Number 10 wire has a diameter of 2.59 mm. How many meters of number 10 aluminium wire are needed to give a resistance of 1.0 Ω? ρ for aluminium is $2.8 \times 10^{-8}\ \Omega \cdot m$. [Ans. 0.19 km]

37. The resistance of a coil of copper is 3.35 Ω at 0 ^oC. What is its resistance at 50 ^oC? For copper, $\alpha = 4.3 \times 10^{-3}\ {^oC}^{-1}$. [Ans. 4.1 Ω]

38. In the Bohr model, the electron of hydrogen atom moves in a circular orbit of radius $5.3 \times 10^{-11}\ m$ with a speed of $2.2 \times 10^6\ m/s$. Determine its frequency f and the current I in the orbit. [Ans. $6.6 \times 10^{15}\ rev/s$; 1.1 mA]

39. A wire that has a resistance of 5.0 Ω is passed through an extruder so as to make it into a new wire three times as long as the original. What is the new resistance? [Ans. 45 Ω]

40. How much current does a 60 W light bulb draw when connected to its proper voltage, 120 V? [Ans. 0.50 A]

41. An electric motor takes 5.0 A from a 110 V line. Determine the power input and the energy supplied to the motor in 2.0 h. [Ans. 0.55 kW; $1.1 kW \cdot h$]

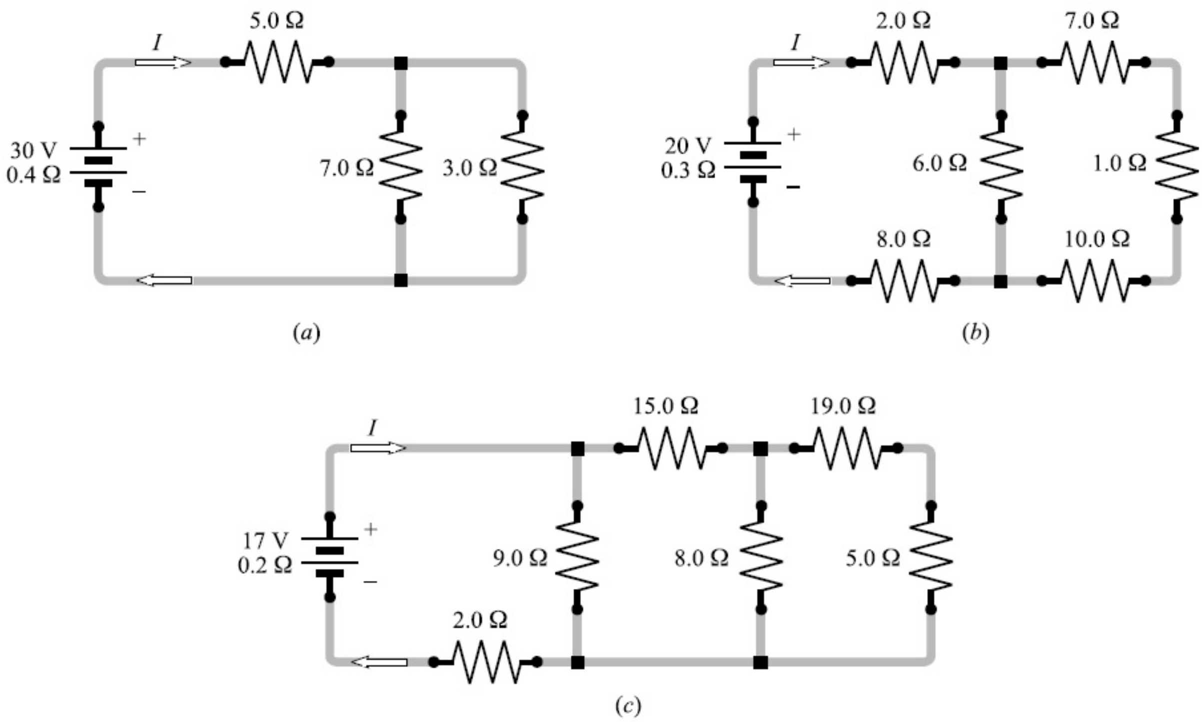


Figure 1.54: Exercise 46.

42. A line having a total resistance of 0.20 Ω delivers 10.0 kW at 250 V to a small factory. What is the efficiency of the transmission. [Ans. 97.0%]

43. A hoist motor supplied by 240 V requires 12.0 A to lift a 800-kg load at a rate of 9.00m/min. Determine the power input to the motor and the power output, and the overall efficiency of the system. [Ans. 2.8 kW; 1.2 kW; 40.8%]

44. A 120-V house circuit has the following light bulbs turned on: 40.0 W, 60.0 W, and 75.0 W. Find the equivalent resistance of the lights. [Ans. 82.3 Ω]

45. What resistance must be placed in parallel with 12 Ω to obtain a combined resistance of 4 Ω [Ans. 6 Ω]

46. For each circuit shown in figure 1.54, determine the current I through the battery. [Ans. 4.0 A; 1.4 A; 2.0 A]

47. The current in the figure 1.55a are steady. Find I_1, I_2, I_3, and the charge on the capacitor. [Ans. $I_1 = I_3 = 1.25\ A$; $I_2 = 0\ Q = 0.5\ \mu C$]

48. Find the ammeter reading and the voltmeter reading in the circuit in figure 1.55b. Assume both meters to be ideal. [Ans. 0.50 A; $V_b - V_a = 2.5V$]

49. Find the currents in the circuit shown in figure 1.56a [Ans. $I_1 = 1.43\ A$; $I_2 = 2.4\ A$; $I_3 = -3.8\ A$]

50. In figure 1.56b find I_1, I_2 and I_3 if switch S is (a) open and (b) closed.

    [Ans. (a) $I_1 = I_2 = 0.20\ A,\ I_3 = 0$ (b)$I_1 = 0.93\ A,\ I_2 = -0.44\ A,\ I_3 = -1.37\ A$]

51. An uncharged capacitor and a resistor are connected in series to a battery as shown in figure 1.57, where $\mathcal{E} = 12.0\ V$, $C = 5.00\ \mu F$, and $R = 8.00 \times 10^5\ \Omega$. The switch is

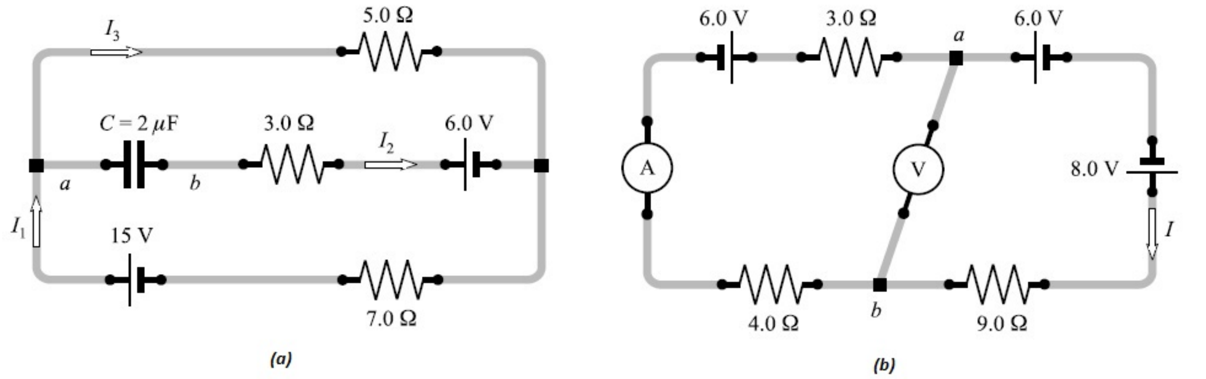


Figure 1.55: Exercise 47 and exercise 48.

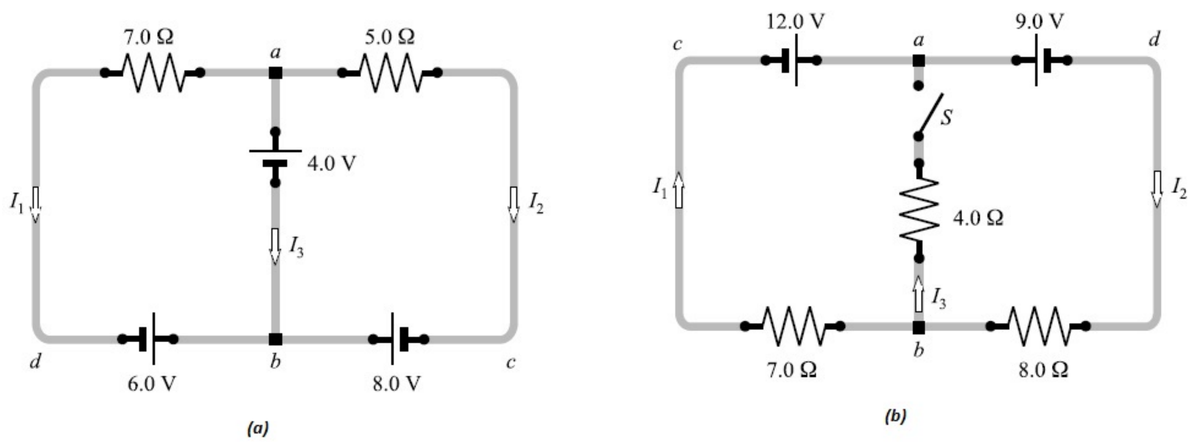


Figure 1.56: Exercise 49 and exercise 50.

thrown to position a. Find the time constant of the circuit, the maximum charge on the capacitor, the maximum current in the circuit, and the charge and the current as functions of time. [Ans. $q(t) = 60.0\left(1 - e^{-t/4.00}\right)$, $I(t) = 15.0e^{-t/4.00}$]

52. Consider a capacitor of capacitance C that is being discharged through a resistance R as shown in Figure 1.57. (a) After how many time constants is the charge on the capacitor one-fourth its initial value? (b) The energy stored in the capacitor decreases with time as the capacitor discharges. After how many time constants is this stored energy one-fourth its initial value? [Ans. (a) $t = 1.39\tau$, $(b)\ t = 0.693\tau$]

53. A $5.00 - \mu F$ capacitor is charged to a potential difference of 800 V and then discharged through a resistor. How much energy is delivered to the resistor in the time interval required to fully discharge the capacitor. [1.60 J]

54. An electric heater is rated at 1500 W, a toaster at 750 W, and an electric grill at 1000 W. The three appliances are connected to a common 120-V circuit. (a) How much current does each draw? (b) Is a 25.0-A circuit breaker sufficient in this situation? Explain your answer. [Ans. (a) 12.5 A; 6.25 A; 8.33 A; (b) No, the total current is greater than 25 A]

55. An 8.00-feet extension cord has two 18-gauge copper wires, each with a diameter of 1.024 mm. What is the I^2R loss in this cord when it carries a current of (a) 1.00 A? (b) 10.0 A? $\rho_{copper} = 1.7 \times 10^{-8}\ \Omega \cdot m$. [Ans. 0.050 W; 5.0 W]

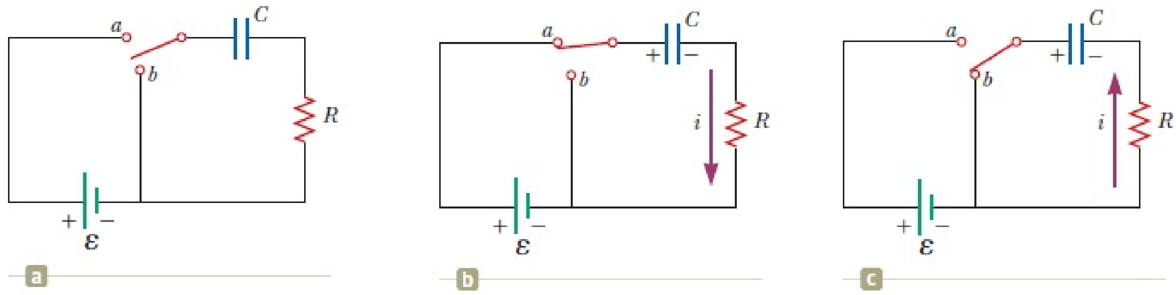


Figure 1.57: Exercise 52.

56. Sometimes aluminum wiring has been used instead of copper for economic reasons. According to the International Electrical Code, the maximum allowable current for 12-gauge copper wire with rubber insulation is 20 A. What should be the maximum allowable current in a 12-gauge aluminum wire if it is to have the same I^2R loss per unit length as the copper wire? $\rho_{copper} = 2.82 \times 10^{-8}\ \Omega \cdot m$ and $\rho_{copper} = 1.7 \times 10^{-8}\ \Omega \cdot m$. [Ans. 15.53 A]

Chapter 2

MAGNETISM

Introduction

Many historians of science believe that the compass, which uses a magnetic needle, was used in China as early as the 13th century B.C., its invention being of Arabic or Indian origin. The early Greeks knew about magnetism as early as 800 B.C. They discovered that the stone magnetite (Fe_3O_4) attracts pieces of iron. Legend ascribes the name magnetite to the shepherd Magnes, the nails of whose shoes and the tip of whose staff stuck fast to chunks of magnetite while he pastured his flocks.

In 1269 a Frenchman named Pierre de Maricourt mapped out the directions taken by a needle placed at various points on the surface of a spherical natural magnet. He found that the directions formed lines that encircled the sphere and passed through two points diametrically opposite each other, which he called the poles of the magnet. Subsequent experiments showed that every magnet, regardless of its shape, has two poles, called north and south poles that exert forces on other magnetic poles just as electric charges exert forces on one another. That is, like poles repel each other, and unlike poles attract each other.

The poles received their names because of the way a magnet behaves in the presence of the Earth's magnetic field. If a bar magnet is suspended from its midpoint and can swing freely in a horizontal plane, it will rotate until its North Pole points to the Earth's geographic North Pole and its South Pole points to the Earth's geographic South Pole (The same idea is used in the construction of a simple compass.)

In 1600 William Gilbert (1540-1603) extended de Maricourt's experiments to a variety of materials. Using the fact that a compass needle orients in preferred directions, he suggested that the Earth itself is a large permanent magnet. In 1750 experimenters used a torsion balance to show that magnetic poles exert attractive or repulsive forces on each other and that these forces vary as the inverse square of the distance between interacting poles. Although the force between two magnetic poles is similar to the force between two electric charges, there is an important difference. Electric charges can be isolated (witness the electron and proton), whereas a single magnetic pole has never been isolated. That is, magnetic poles are always found in pairs. All attempts thus far to detect an isolated magnetic pole have been unsuccessful. No matter how many times a permanent magnet is cut in two, each piece always has a north and a south pole.

The relationship between magnetism and electricity was discovered in 1819 when, during a lecture demonstration, the Danish scientist Hans Christian Oersted found that an electric current in a wire deflected a nearby compass needle. Shortly thereafter, André Ampère (1775-1836) formulated quantitative laws for calculating the magnetic force exerted by one current-carrying electrical conductor on another. He also suggested that on the atomic level, electric current loops are responsible for all magnetic phenomena.

In the 1820s, further connections between electricity and magnetism were demonstrated by Faraday and independently by Joseph Henry (1797-878). They showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. These observations demonstrate that a changing magnetic field creates an electric field. Years later, theoretical work by Maxwell showed that the reverse is also true: A changing electric field creates a magnetic field. A similarity between electric and magnetic effects has provided methods of making permanent magnets. In section 1 of previous chapter, we learned that when glass and silk are rubbed together, both become charged-one positively and the other negatively. In an analogous fashion, one can magnetize an unmagnetized piece of iron by stroking it with a magnet. Magnetism can also be induced in iron (and other materials) by other means. For example, if a piece of unmagnetized iron is placed near (but not touching) a strong magnet, the unmagnetized piece eventually becomes magnetized.

2.1 MAGNETIC FORCE AND MAGNETIC FIELD

2.1.1 Magnetic field lines

In our study of electricity, we described the interactions between charged objects in terms of electric fields. Recall that an electric field surrounds any stationary or moving electric charge. In addition to an electric field, the region of space surrounding any moving electric charge also contains a magnetic field, as we shall see in the followings. A magnetic field also surrounds any magnetic substance. Historically, the symbol $\vec{B}$ has been used to represent a magnetic field, and this is the notation we use in this lecture. The direction of the magnetic field $\vec{B}$ at any location is the direction in which a compass needle points at that location.

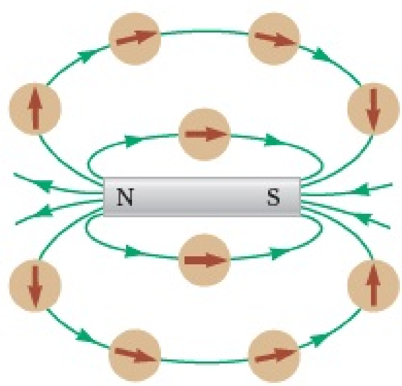


Figure 2.1: *Compass needles can be used to trace the magnetic field lines in the region outside a bar magnet.*

Figure (2.1) shows how the magnetic field of a bar magnet can be traced with the aid of a compass. Note that the magnetic field lines outside the magnet point away from north poles and toward south poles. One can display magnetic field patterns of a bar magnet using small iron filings, as shown in figure (2.1):
We can define a magnetic field $\vec{B}$ at some point in space in terms of the magnetic force $\vec{F}_B$ that the field exerts on a test object, for which we use a charged particle moving with a velocity $\vec{v}$. For the time being, let us assume that no electric or gravitational fields are present at the location of the test object. Many experiments on various charged particles moving in a magnetic field gave the following results:

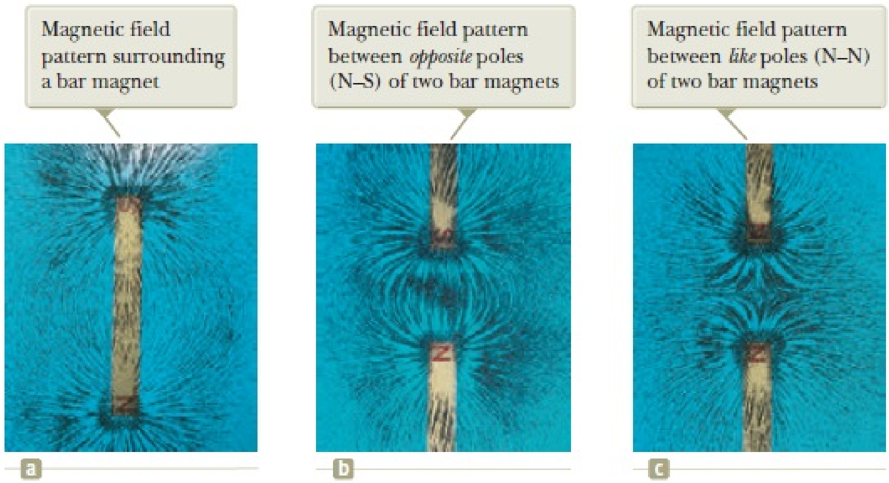


Figure 2.2: *Magnetic field patterns can be displayed with iron filings sprinkled on paper near magnets.*

- The magnitude $\vec{F}_B$ of the magnetic force exerted on the particle is proportional to the charge q and to the speed v of the particle.
- The magnitude and direction of $\vec{F}_B$ depend on the velocity of the particle and on the magnitude and direction of the magnetic field $\vec{B}$.
- When a charged particle moves parallel to the magnetic field vector, the magnetic force acting on the particle is zero.
- When the particle's velocity vector makes any angle $\theta \neq 0$ with the magnetic field, the magnetic force acts in a direction perpendicular to both $\vec{v}$ and $\vec{B}$; that is, $\vec{F}_B$ is perpendicular to the plane formed by $\vec{v}$ and $\vec{B}$ (Fig. 2.3a).
- The magnetic force exerted on a positive charge is in the direction opposite the direction of the magnetic force exerted on a negative charge moving in the same direction (Fig. 2.3b).
- The magnitude of the magnetic force exerted on the moving particle is proportional to $\sin\theta$, where θ is the angle that the particle's velocity vector makes with the direction of $\vec{B}$.

We can summarize these observations by writing the magnetic force in the form

$$\vec{F}_B = q(\vec{v} \times \vec{B}) \tag{2.1}$$

Where the direction of $\vec{F}_B$ is in the direction of $\vec{v} \times \vec{B}$ if q is positive, which by definition of the cross product is perpendicular to both $\vec{v}$ and $\vec{B}$. We can regard this equation as an operational definition of the magnetic field at some point in space. That is, the magnetic field is defined in terms of the force acting on a moving charged particle. Figure 2.4a reviews the right-hand rule for determining the direction of the cross product $\vec{v} \times \vec{B}$. You point the four fingers of your right hand along the direction of $\vec{v}$ with the palm facing $\vec{B}$ and curl them toward $\vec{B}$. The extended thumb, which is at a right angle to the fingers, points in the direction of $\vec{v} \times \vec{B}$. An alternative rule is shown in figure 2.4b.

The magnitude of the magnetic force is given by

$$F_B = |q|vB\sin\theta \tag{2.2}$$

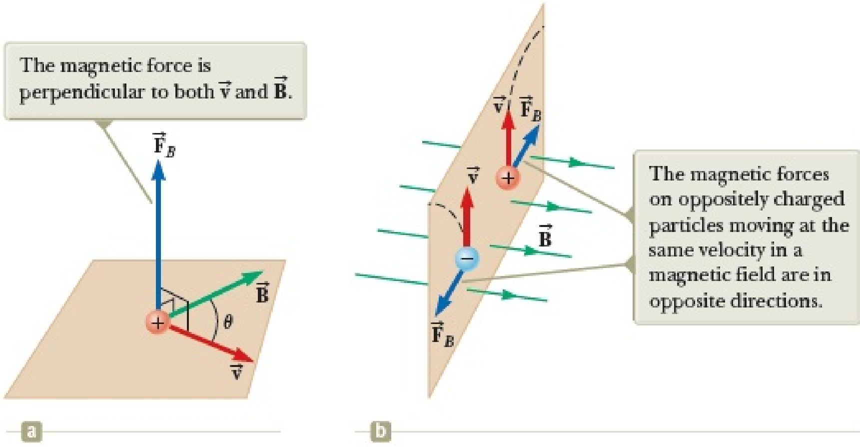


Figure 2.3: *The direction of the magnetic force $\vec{F}_B$ acting on a charged particle moving with a velocity v in the presence of a magnetic field $\vec{B}$.*

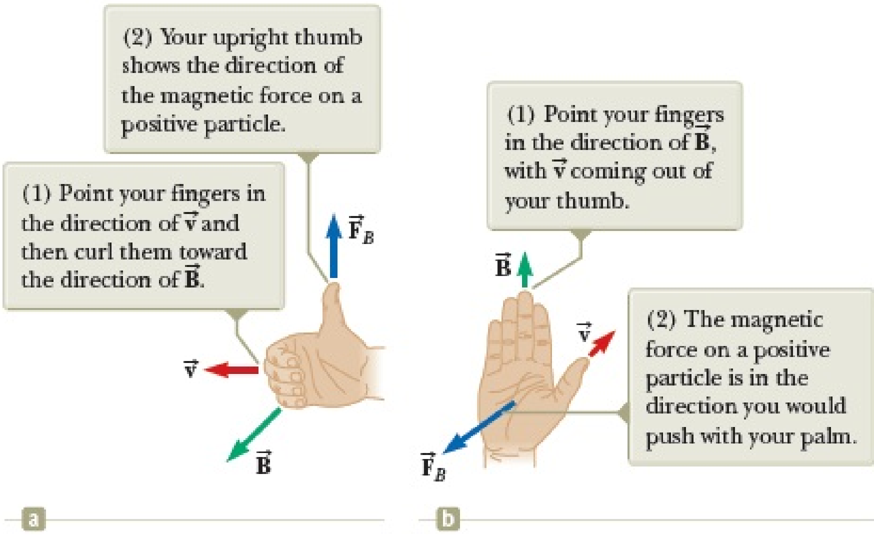


Figure 2.4: *Two right angle rules for determining the direction of magnetic force.(a) In this rule, the magnetic force is in the direction in which your thumb points. (b) In this rule, the magnetic force is in the direction of your palm, as if you are pushing the particle with your hand.*

where θ is the angle between $\vec{v}$ and $\vec{B}$
There are several important differences between electric and magnetic forces:

- The electric force acts in the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.
- The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.
- The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced.

Notice that from the last statement and on the basis of the work–kinetic energy theorem, we conclude that the kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. In other words,

> *"When a charged particle moves with a velocity v through a magnetic field, the field can alter the direction of the velocity vector but cannot change the speed or kinetic energy of the particle".*

It is clear form the Equation 2.1 that the magnitude of $\vec{B}$ is given by

$$B = \frac{F_B}{|q| v \sin\theta} \tag{2.3}$$

From Equation (2.3), we see that the SI unit of magnetic field is the newton per coulomb-meter per second, which is called the *tesla* (T):

2.1.2 Motion of a Charged Particle in a Uniform Magnetic Field

We found that the magnetic force acting on a charged particle moving in a magnetic field is perpendicular to the velocity of the particle and that consequently the work done on the particle by the magnetic force is zero. Let us now consider the special case of a positively charged particle moving in a uniform magnetic field with the initial velocity vector of the particle perpendicular to the field. Let us assume that the direction of the magnetic field is into the page. Figure 2.5 shows that the particle moves in a circle in a plane perpendicular to the magnetic field. The particle moves in this way because the magnetic force $\vec{F}_B$ is at right angles to $\vec{v}$ and $\vec{B}$ and has a constant magnitude qvB. As the force deflects the particle, the directions of $\vec{v}$ and $\vec{F}_B$ change continuously, as Figure 2.5 shows. Because $\vec{F}_B$ always points toward the center of the circle, it changes only the direction of $\vec{v}$ and not its magnitude.

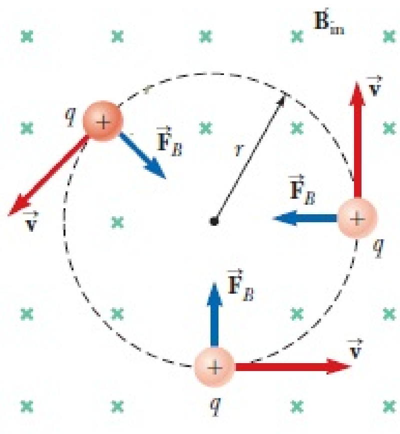


Figure 2.5: *Circulation of a charged particle in a uniform magnetic field*

As Figure 2.5 illustrates, the rotation is counter-clockwise for a positive charge. If q were negative, the rotation would be clockwise. We can use the second Newton's law to equate this magnetic force to the radial force required to keep the charge moving in a circle:

$$\sum F = ma$$

Because the particle moves in a circle, we also model it as a particle in uniform circular motion and we replace the acceleration with centripetal acceleration:

$$F_B = qvB = m\frac{v^2}{r} \tag{2.4}$$

This expression leads to the following equation for the radius of the circular path:

$$r = \frac{mv}{|q|B} \tag{2.5}$$

That is, the radius of the path is proportional to the linear momentum mv of the particle and inversely proportional to the magnitude of the charge on the particle and to the magnitude of the magnetic field. The angular speed of the particle is

$$\omega = \frac{v}{r} = \frac{|q|B}{m} \tag{2.6}$$

The period of the motion (the time interval the particle requires to complete one revolution) is equal to the circumference of the circle divided by the speed of the particle:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} \tag{2.7}$$

These results shows that the angular speed of the particle and the period of the circular motion do not depend on the linear speed of the particle or on the radius of the orbit. The angular speed ω is often referred to as cyclotron frequency because charged particles circulate at this angular speed in the type of accelerator called a cyclotron.

If a charged particle moves in a uniform magnetic field with its velocity at some arbitrary angle with respect to $\vec{B}$, its path is a helix.

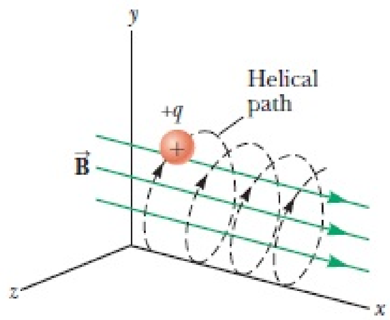


Figure 2.6: *A charged particle moving in a uniform magnetic field with its velocity at some arbitrary angle with respect to* $\vec{B}$

If a charged particle q with velocity $\vec{v}$ moves in a combined electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force on the charged particles is given by:

$$\vec{F} = \vec{F}_e + \vec{F}_B = q(\vec{E} + \vec{v} \times \vec{B}) \tag{2.8}$$

If the electrons then pass through a region where there exists a downward uniform electric field, the negatively charged electrons will be deflected upward. However, if in addition to the electric field, a magnetic field directed into the page is also applied, then the electrons will experience an additional downward magnetic force $-e\vec{v} \times \vec{B}$. When the two forces exactly cancel, the electrons will move in a straight path. From Equation (2.8), we see that when the condition for the cancellation of the two forces is given by $eE = evB$, which implies

$$v = \frac{E}{B}.$$

In other words, only those particles with speed $v = E/B$ will be able to move in a straight line.

2.1.3 Hall effect

The Hall Effect experiment is used to study some of the physics of charge transport in metal and semiconductor samples. When a magnetic field is applied to a current carrying conductor, a magnetic force is transferred to the wire. In addition a voltage develops transverse to the current direction; this separation of charge in the wire is called the Hall Effect. The Hall Effect is a well-known method to determine the carrier concentration, carrier type, and when coupled with a resistivity measurement, the mobility of materials.

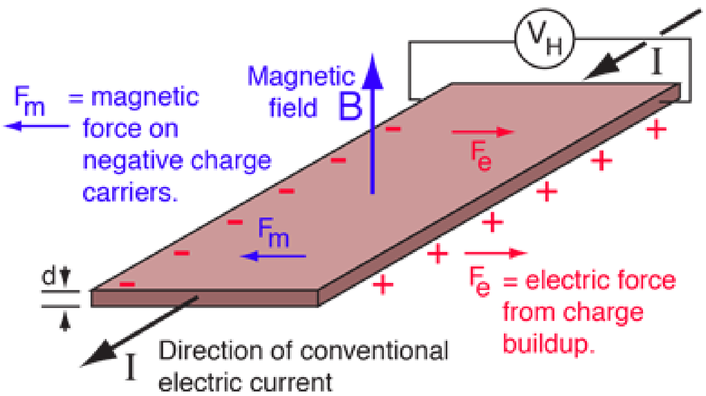


Figure 2.7: *Set-up illustrating the generation of a Hall voltage*

If a current passes through a conducting material along the X direction and there is a magnetic field in the Y direction then a voltage difference V_H appears along the Z axis; that is between the faces C and D of the material, as shown in Figure 2.7. If a charge e moves with velocity $\vec{v_d}$ through a magnetic field $\boldsymbol{B}$ then a force $\vec{F}$ acts on the charge, where $\vec{F} = e\,\vec{v_d} \times \vec{B}$
If the charge moves along the X-axis and the magnetic field is along the Y axis then the force is along the Z axis. If the charges are positive, then the force acts in the direction of the positive Z axis. Positive charges will therefore move upwards towards face C and because an excess charge accumulates on face C with a corresponding shortage on face D, a voltage, the Hall voltage, develops in the Z direction. The Hall voltage increases until it exactly balances the force due to the magnetic field and an equilibrium is reached.

$$V_H = \frac{I}{enw}B \tag{2.9}$$

The Hall constant is defined as:

$$R_H = \frac{1}{ne}$$

In the metallic conductors the charge carriers are generally electrons of high mobility and density. In a semiconductor the carriers may be electrons or positive holes.

It can be seen from the Equation 2.9 that the Hall voltage changes linearly with magnetic field and such a Hall probe device is often used to measure magnetic fields. It is not practical to use metals in such devices as n is very large for these materials, making V_H very small. Of course for insulating materials, such as glass, n is very small but very large voltages are then required to increase I to some measurable value.

2.1.4 Magnetic Force acting on a Current-carrying conductor

If a magnetic force is exerted on a single charged particle when the particle moves through a magnetic field, it should not surprise you that a current-carrying wire also experiences a force when placed in a magnetic field. This follows from the fact that the current is a collection of many charged particles in motion; hence, the resultant force exerted by the field on the wire is the vector sum of the individual forces exerted on all the charged particles making up the current. The force exerted on the particles is transmitted to the wire when the particles collide with the atoms making up the wire.

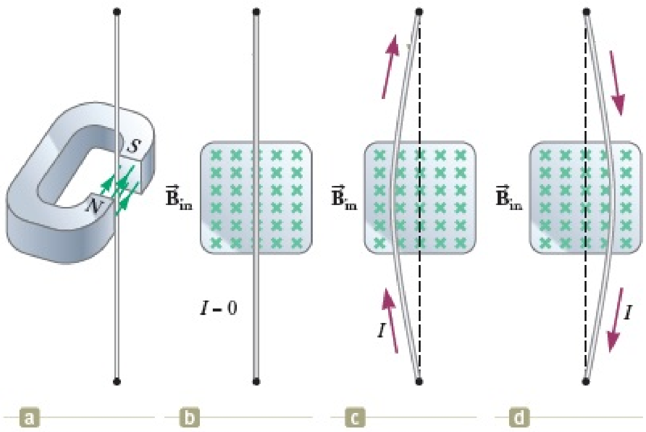


Figure 2.8: *(a) A wire suspended vertically between the poles of a magnet. (b)–(d) The setup shown in (a) as seen looking at the south pole of the magnet so that the magnetic field (green crosses) is directed into the page.*

One can demonstrate the magnetic force acting on a current-carrying conductor by hanging a wire between the poles of a magnet, as shown in 2.8a. For ease in visualization, part of the horseshoe magnet in part (a) is removed to show the end face of the South Pole in parts (b), (c), and (d) of Figure 2.8. The magnetic field is directed into the page and covers the region within the shaded circles. When the current in the wire is zero, the wire remains vertical, as shown in Figure 2.8b. However, when a current directed upward flows in the wire, as shown in Figure 2.8c, the wire deflects to the left. If we reverse the current, as shown in Figure 2.8d, the wire deflects to the right.

Let us quantify this discussion by considering a straight segment of wire of length L and cross-sectional area A, carrying a current I in a uniform magnetic field $\vec{B}$, as shown in figure 2.9. The magnetic force exerted on a charge q moving with a drift velocity $\vec{v}_d$ is $q(\vec{v}_d \times \vec{B})$. To find the total force acting on the wire, we multiply the force exerted on one charge by the number of charges in the segment. Because the volume of the segment is AL, the number of charges in the segment is nAL, where n is the number of charges per unit volume.

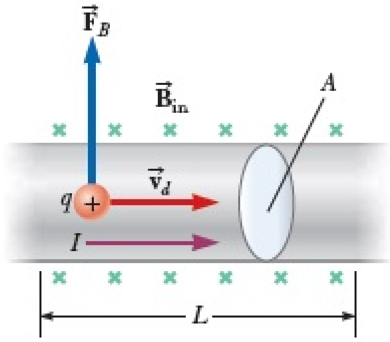


Figure 2.9: *A segment of a current-carrying wire located in a magnetic field* $\vec{B}$

Hence, the total magnetic force on the wire of length L is

$$\vec{F} = q(\vec{v}_d \times \vec{B})nAL \tag{2.10}$$

We can write this expression in a more convenient form by noting that, from the fact that the current in the wire is $I = nqv_dA$. Therefore,

$$\vec{F_B} = I\ \vec{L} \times \vec{B} \tag{2.11}$$

where $\vec{L}$ is a vector that points in the direction of the current I and has a magnitude equal to the length L of the segment. This expression applies only to a straight segment of wire in a uniform magnetic field.

Now consider an arbitrarily shaped wire segment of uniform cross section in a magnetic field as shown in Figure 2.10. It follows from Equation 2.11 that the magnetic force exerted on a small segment of vector length $d\vec{s}$ in the presence of a field $\vec{B}$ is

$$d\vec{F_B} = I\ d\vec{s} \times \vec{B} \tag{2.12}$$

where $d\vec{F_B}$ is directed out of the page for the directions of $\vec{B}$ and $d\vec{s}$ in Figure 2.10. Equation 2.12 can be considered as an alternative definition of $\vec{B}$. That is,we can define the magnetic field $\vec{B}$ in terms of a measurable force exerted on a current element, where the force is a maximum when $\vec{B}$ is perpendicular to the element and zero when $\vec{B}$ is parallel to the element.

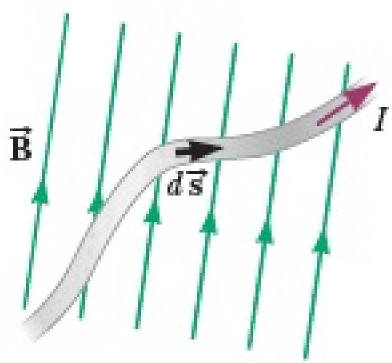


Figure 2.10: *A wire segment of arbitrary shape carrying a current I in a magnetic field $\vec{B}$ experiences a magnetic force.*

To calculate the total force $\vec{F}_B$ acting on the wire shown in Figure 2.10, we integrate Equation 2.12 over the length of the wire:

$$\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B} \tag{2.13}$$

where a and b represent the end points of the wire. When this integration is carried out, the magnitude of the magnetic field and the direction the field makes with the vector $d\vec{s}$ may differ at different points.

Example: Current loop in B-field

What happens when we place a rectangular loop carrying a current I in the xy plane and switch on a uniform magnetic field $\vec{B} = B\vec{i}$ which runs parallel to the plane of the loop,as shown in Figure 2.11.

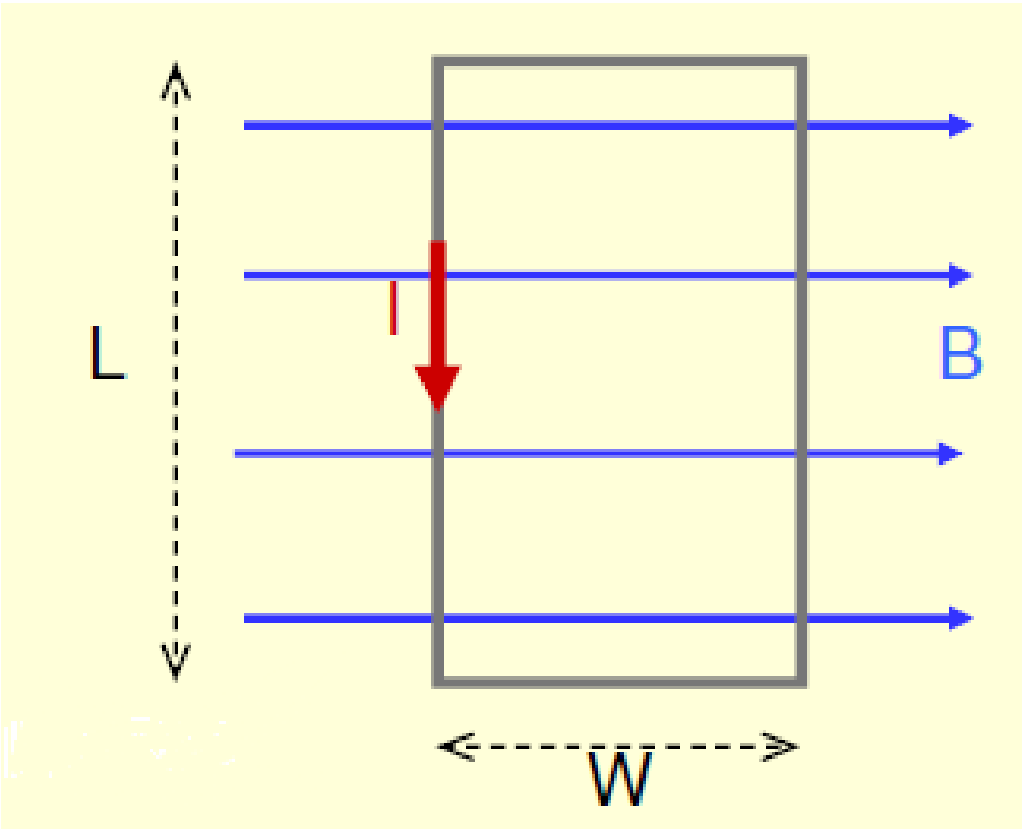


Figure 2.11: *A rectangular current loop placed in a uniform magnetic field. The magnetic forces acting on sides 2 and 4*

From Equation (2.13), we see the magnetic forces acting on sides 1 and 3 vanish because the length vectors $\vec{s}_1 = -b\vec{i}$ and $\vec{s}_3 = b\vec{i}$ are parallel and anti–parallel to B and their cross products vanish. The magnetic forces acting on segments 2 and 4 are non–vanishing,

$$\vec{F}_2 = I(-a\vec{j}) \times B\vec{i} = IaB\vec{k} \;, and \quad \vec{F}_4 = I(a\vec{j}) \times B\vec{i} = -IaB\vec{k}$$

Thus, the force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$$

Even though the force on the loop vanishes, the forces $\vec{F}_2$ and $\vec{F}_4$ will produce a torque that causes the loop to rotate about the y-axis as shown in Figure 2.11. The torque about the center of the loop is

$$\vec{\tau} = -(b/2)\vec{i} \times \vec{F}_2 + (b/2)\vec{i} \times \vec{F}_4 = -(b/2)\vec{i} \times (IaB\vec{k}) + (b/2)\vec{i} \times (-IaB\vec{k}) = IaB\vec{j}$$

where $A = ab$ represents the area of the loop and the positive sign indicates that the rotation is clockwise about the y-axis as shown in Figure 2.12.

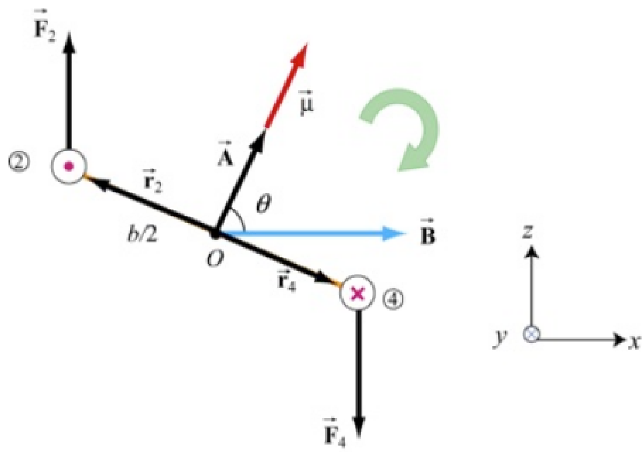


Figure 2.12: *Rotation of a rectangular current loop in a uniform magnetic field*

It is convenient to introduce the area vector $\vec{A} = A\vec{n}$ where $\vec{n}$ is a unit vector in the direction normal to the plane of the loop. The direction of the positive sense of $\vec{n}$ is set by the conventional right-hand rule. In our case, we have $\vec{n} = +\vec{k}$. The above expression for torque can then be rewritten as

$$\vec{\tau} = IA\vec{n} \times \vec{B}$$

For a loop consisting of N turns, the magnitude of the toque is

$$\vec{\tau} = NIA\vec{n} \times \vec{B}$$

The quantity $NI\vec{A}$ is called the **magnetic dipole moment** $\vec{\mu}$.

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{2.14}$$

The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpendicular to the plane of the loop) and is determined by the right-hand rule. The SI unit for the magnetic dipole moment $\vec{\mu}$ is $ampere - meter^2 \quad (Am^2)$.

2.2 SOURCES OF THE MAGNETIC FIELDS

2.2.1 The Biot–Savart Law

Shortly after Oersted's discovery in 1819 that a compass needle is deflected by a current–carrying conductor, Jean Baptiste Biot (1774–1862) and Félix Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet. From their experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{s}$ of a wire carrying a steady current I (Figure 2.13):

- The vector $d\vec{B}$ is perpendicular both to $d\vec{s}$ (which points in the direction of the current) and to the unit vector $\hat{r}$ directed from $d\vec{s}$ to P.
- The magnitude of $d\vec{B}$ is inversely proportional to r^2, where r is the distance from $d\vec{s}$ to P.
- The magnitude of $d\vec{B}$ is proportional to the current and the magnitude ds of the length element $d\vec{s}$.
- The magnitude of $d\vec{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{s}$ and $\hat{r}$.

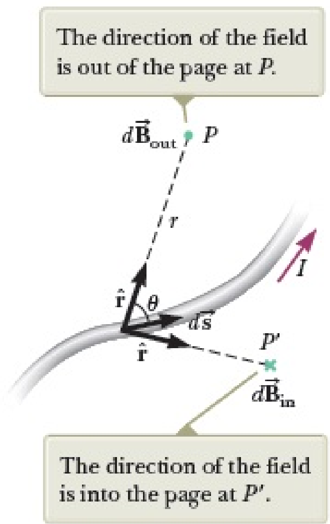


Figure 2.13: *A current–carrying wire*

These observations are summarized in the mathematical formula known today as the Biot–Savart law:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{r}}{r^2} \tag{2.15}$$

where μ_0 is a constant called *permeability of free space*:

$$\mu_0 = 4\pi \times 10^{-7}\, Tm/A \tag{2.16}$$

It is important to note that the field $d\vec{B}$ is created by the current in only a small length element $d\vec{s}$ of the conductor. To find the total magnetic field $\vec{B}$ created at some point by a current of finite size, we must sum up contribute current elements $Id\vec{B}$ that make up the current. That is, we must evaluate $\vec{B}$ by integrating Equation 2.15:

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{s} \times \hat{r}}{r^2} \tag{2.17}$$

With the integral taken over the entire current distribution.
This expression must be handled with special care because the integrand is a cross product and therefore a vector quantity. Although we developed the Biot–Savart law for a current–carrying wire, it is also valid for a current consisting of charges flowing through space, such as the electron beam in a television set. In that case, $d\vec{s}$ represents the length of a small segment of space in which the charges flow.

Interesting similarities exist between the Biot–Savart law for magnetism and Coulomb's law for electrostatics. The current element produces a magnetic field, whereas a point charge produces an electric field. Furthermore, the magnitude of the magnetic field varies as the inverse square of the distance from the current element, as does the electric field due to a point charge. However, the directions of the two fields are quite different. The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\vec{s}$ and the unit vector, as described by the cross product in Equation 2.15. Hence, if the conductor lies in the plane of the page as shown in Figure 2.13, $d\vec{B}$ points out of the page at P and into the page at P.
Another difference between electric and magnetic fields is related to the source of the field. An electric field is established by an isolated electric charge. The Biot–Savart law gives the magnetic field of an isolated current element at some point, but such an isolated current element cannot exist the way an isolated electric charge can. A current element must be part of an extended current distribution because we must have a complete circuit in order for charges to flow. Thus, the Biot–Savart law is only the first step in a calculation of a magnetic field; it must be followed by integration over the current distribution as in the Equation 2.17.

2.2.2 Magnetic Field Surrounding a Thin, Straight Conductor

Consider a thin, straight wire of finite length carrying a constant current I and placed along the x axis as shown in Figure 2.14. Determine the magnitude and direction of the magnetic field at point P due to this current. One of the applications of Biot-Savart law

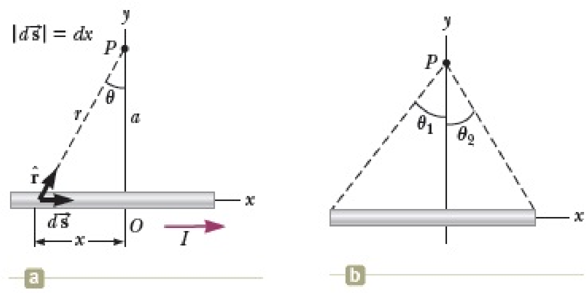


Figure 2.14: *A thin, straight wire carrying a current I*

is to get a magnetic field surrounding a thin straight conductor.
From the Biot–Savart law, we expect that the magnitude of the field is proportional to the current in the wire and decreases as the distance a from the wire to point P increases. We also expect the field to depend on the angles θ_1 and θ_2 in Figure 2.14b. We place the origin at O and let point P be along the positive y axis, with $\hat{k}$ being a unit vector pointing out of the page.

Let's start by considering a length element $d\vec{s}$ located a distance r from P. The direction of the magnetic field at point P due to the current in this element is out of the page

because $d\vec{s} \times \hat{r}$ is out of the page. In fact, because all the current elements $Id\vec{s}$ lie in the plane of the page, they all produce a magnetic field directed out of the page at point P. Therefore, the direction of the magnetic field at point P is out of the page and we need only find the magnitude of the field.
Evaluating the cross product in Biot–Savart law we get:

$$d\vec{s} \times \hat{r} = |\vec{s} \times \hat{r}|\hat{k} = \left[dx \sin\left(\frac{\pi}{2} - \theta\right)\right]\hat{k} = (dx\cos\theta)\hat{k}$$

Substituting into Equation 2.15, gives

$$(1) \qquad d\vec{B} = (dB)\hat{k} = \frac{\mu_0 I}{4\pi}\frac{dx\cos\theta}{r^2}\hat{k}$$

From the geometry in Figure 2.14a, we express r in terms of θ:

$$(2) \qquad r = \frac{a}{\cos\theta}$$

Notice that $\tan\theta = -x/a$ from the right triangle in Figure 2.14a (the negative sign is necessary because $d\vec{s}$ is located at a negative value of x) and solve for x:

$$x = -a\tan\theta$$

That is

$$(3) \qquad dx = -a\sec^2\theta d\theta = -\frac{a\,d\theta}{\cos^2\theta}$$

Substitute Equations (2) and (3) into the expression for the z component of the field from Equation (1):

$$(4) \qquad dB = -\frac{\mu_0 I}{4\pi}\left(\frac{a\,d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right)\cos\theta = -\frac{\mu_0 I}{4\pi a}\cos\theta\,d\theta$$

Integrate Equation (4) over all length elements on the wire, where the subtending angles range from θ_1 to θ_2 as defined in Figure 2.14b:

$$B = -\frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2) \qquad (2.18)$$

We can use this result to find the magnitude of the magnetic field of *any straight current-carrying wire* if we know the geometry and hence the angles θ_1 and θ_2. Consider the special case of an infinitely long, straight wire. If the wire in Figure 2.14b becomes infinitely long, we see that $\theta_1 = \pi/2$ and $\theta_2 = -\pi/2$ for length elements ranging between positions $x = -\infty$ and $x = +\infty$. Because $(\sin\theta_1 - \sin\theta_2) = [\sin\pi/2 = \sin(-\pi/2)] = 2$, Equation 2.18 becomes

$$B = \frac{\mu_0 I}{2\pi a} \qquad (2.19)$$

Equations 2.18 and 2.19 both show that the magnitude of the magnetic field is proportional to the current and decreases with increasing distance from the wire, as expected.

The result of this example 2.19 is important because a current in the form of a long, straight wire occurs often. Figure 2.15 is a three-dimensional view of the magnetic field surrounding a long, straight current-carrying wire. Because of the symmetry of the wire, the magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire. The magnitude of $\vec{B}$ is constant on any circle of radius a and is given by Equation 2.19. A convenient rule for determining the direction of $\vec{B}$ is to grasp the wire with the right hand, positioning the thumb along the direction of the current. The four fingers wrap in the direction of the magnetic field.

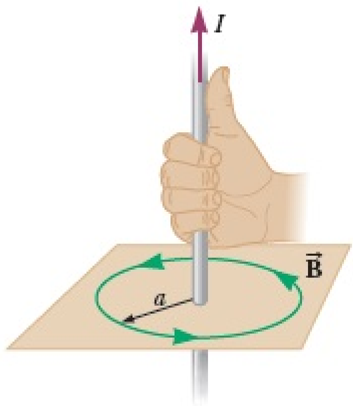


Figure 2.15: *The right-hand rule for determining the direction of the magnetic field surrounding a long, straight wire carrying a current. Notice that the magnetic field lines form circles around the wire.*

2.2.3 The Magnetic Force Between Two Parallel Conductors

We already described the magnetic force that acts on a current-carrying conductor placed in an external magnetic field. Because a current in a conductor sets up its own magnetic fields, it is easy to understand that two current-carrying conductors exert magnetic forces on each other.

Consider two long, straight, parallel wires separated by a distance a, and carrying currents I_1 and I_2 in the same direction, as illustrated in Figure 2.16. We can determine the force exerted on one wire due to the magnetic field set up by the other wire.

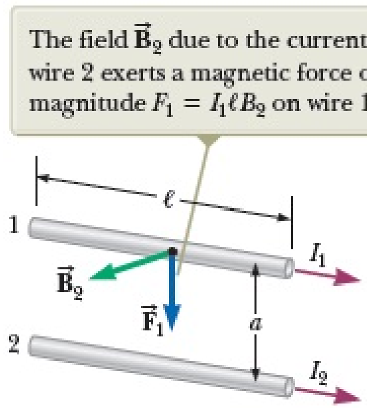


Figure 2.16: *Two parallel wires that each carry a steady current exert a magnetic force on each other. The force is attractive if the currents are parallel and repulsive if the currents are antiparallel.*

Wire 2, which carries a current I_2, creates a magnetic field $\vec{B}_2$ at the location of wire 1. The direction of $\vec{B}_2$ is perpendicular to wire 1, as shown in Figure 2.16.

According to the equation $\vec{F}_B = I\vec{l} \times \vec{B}$, the magnetic force on a length l of wire 1 is $\vec{F}_1 = I_1\vec{l} \times \vec{B}_2$. Because l is perpendicular to $\vec{B}_2$ in this situation, the magnitude of $\vec{F}_1$ is $F_1 = I_1 l B_2$. Because the magnitude of $\vec{B}_2$ is given by $B_2 = \frac{\mu_0 I_2}{2\pi a}$ (Equation 2.19) , we see that:

$$F_1 = I_1 l B_2 = I_1 l \left(\frac{\mu_0 I_2}{2\pi a} \right) = \frac{\mu_0 I_1 I_2}{2\pi a} l \tag{2.20}$$

The direction of F_1 is toward wire 2 because $\vec{l} \times \vec{B}_2$ is in that direction. If the field set up at wire 2 by wire 1 is calculated, the force F_2 acting on wire 2 in found to be equal in magnitude and opposite in direction to F_1. This is what we expect because Newton's third law must be obeyed. When the currents are opposite directions (that is, when one of the currents is reversed in Figure 2.16), the forces are reversed and the wires repel each other. Hence, we find that parallel conductors carrying currents in the same direction attract each other, and parallel conductors carrying currents in opposite directions repel each other. Because the magnitudes of the forces are the same on both wires, we denote the magnitude of the magnetic force between the wires as simply $\vec{F}_B$. We can rewrite this magnitude in terms of terms of the force per unit length:

$$\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a} \tag{2.21}$$

The force between two parallel wires is used to define the ampere as follows:

> "When the magnitude of the force per unit length between two long, parallel wires that carry identical currents and are separated by *1 m* is $2.3 \times 10^{27} N/m$, the current in each wire is defined to be *1 A*".

2.2.4 Magnetic field created by a circular current loop

A circular loop of radius R in the xy-plane carries a steady current I, as shown in Figure 2.17. What is the magnetic field at a point P on the axis of the loop, at a distance z from the center?

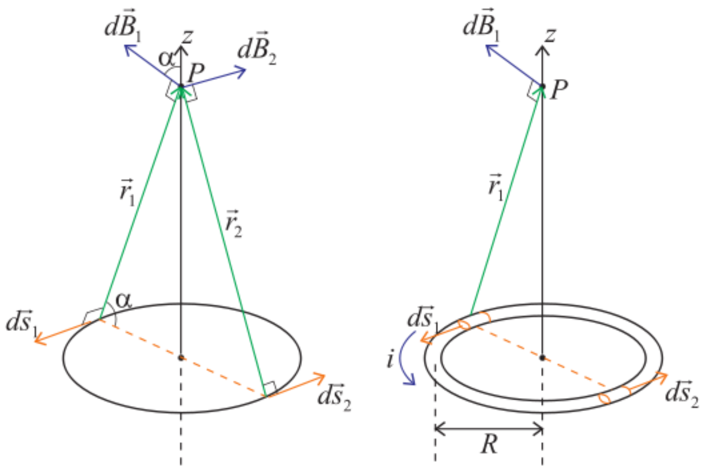


Figure 2.17: *Magnetic field due to a circular loop carrying a steady current.*

Notice that for every current element $id\vec{s}_1$, generating a magnetic field $d\vec{B}_1$ at point P, there is an opposite current element $id\vec{s}_2$, generating B-field $d\vec{B}_2$ so that

$$d\vec{B}_1 \sin\alpha = -d\vec{B}_2 \sin\alpha$$

Only vertical component of B-field needs to be considered at point P.

$$dB = \frac{\mu_0}{4\pi}\frac{i\ ds\ sin90}{r^2} \quad where \quad ds = R\ d\theta$$

Considering the vertical component, $B = \int dB\ cos\alpha$.

$$B = \int_0^{2\pi} \frac{\mu_0}{4\pi}\frac{i\ R\ cos\alpha}{r^2} d\theta = \frac{\mu_0 i R^2}{2r^3}$$

$$B = \frac{\mu_0 i R^2}{2(R^2+z^2)^{3/2}}\ ; \tag{2.22}$$

The direction of B-field determined from right-hand screw rule.
Limiting Cases :

(1) B-field at center of loop:

$$z = 0; \quad B = \frac{\mu_0 i}{2R}$$

(2) For $z \gg R$;

$$B \approx \frac{\mu_0 i R^2}{2z^3} \propto \frac{1}{z^3}$$

Recall E-field for an electric dipole: $E = p/(4\pi\epsilon_0 x^3)$ A circular current loop is also called a **magnetic dipole**. Recall from Equation (2.14), we define the magnetic dipole moment of a rectangular current loop:

$$\vec{\tau} = NIA\vec{n} \times \vec{B} = \vec{\mu} \times \vec{B}$$

where where $\vec{n}$ = area unit vector with direction determined by the right-hand rule; N = Number of turns in current loop and A = Area of current loop.
This is actually a general definition of a magnetic dipole, i.e. we use it for current loops of all shapes.
For example, magnetic field at point P (height z above the ring of are $A = \pi R^2$), can be written

$$\vec{B} = \frac{\mu_0 i R^2 \vec{n}}{2(R^2+z^2)^{3/2}} = \frac{\mu_0 \vec{\mu}}{2\pi(R^2+z^2)^{3/2}} \tag{2.23}$$

Similar to electric dipole in a E-field, we can consider the work done in rotating the magnetic dipole.

$$dW = -dU \Rightarrow U = \vec{\mu} \cdot \vec{B}$$

where U is potential energy of dipole
Note :
(1) We cannot define the potential energy of a magnetic field in general. However, we can define the potential energy of a magnetic dipole in a constant magnetic field.
(2) In a non-uniform external B-field, the magnetic dipole will experience a net force (not only net torque)

2.2.5 Ampère's Law

Oersted's 1819 discovery about deflected compass needles demonstrates that a current-carrying conductor produces a magnetic field. Figure 2.18a shows how this effect can be demonstrated.
Several compass needles are placed in a horizontal plane near a long vertical wire. When no current is present in the wire, all the needles point in the same direction (that of the Earth's magnetic field). When the wire carries a strong, steady current, the needles all deflect in a direction tangent to the circle, as shown in Figure 2.18b. These observations demonstrate that the direction of the magnetic field produced by the current in the wire is consistent with the right-hand rule described in Figure 2.15. When the current is reversed, the needles in Figure 2.18b also reverse. Because the compass needles point in the direction of $\vec{B}$, we conclude that the lines of $\vec{B}$ form circles around the wire.

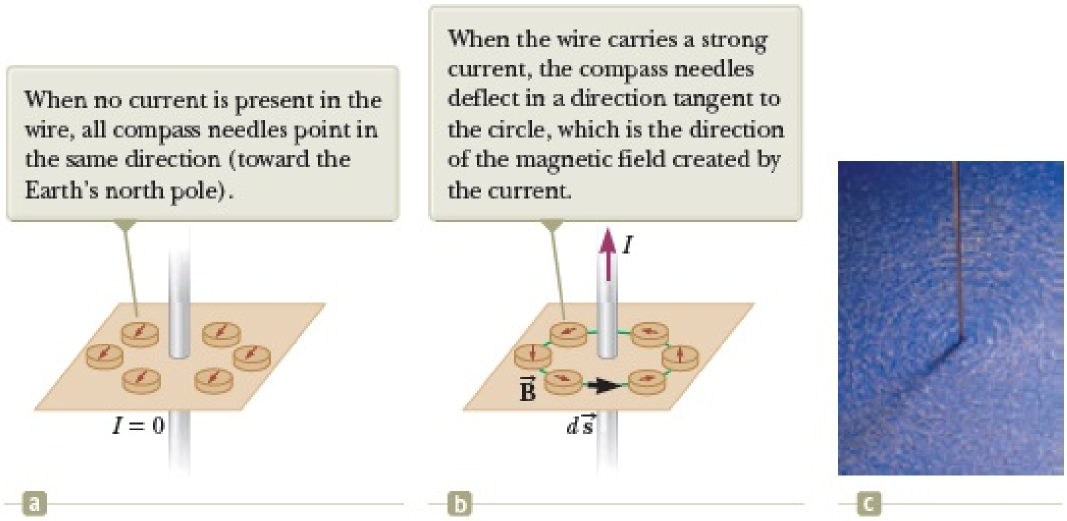


Figure 2.18: *(a) and (b) Compasses show the effects of the current in a nearby wire. (c) Circular magnetic field lines surrounding a current-carrying conductor, displayed with iron filings.*

Now let's evaluate the product $\vec{B} \cdot d\vec{s}$ for a small length element $d\vec{s}$ on the circular path defined by the compass needles, and sum the products for all elements over the closed circular path. Along this path, the vectors $d\vec{s}$ and $\vec{B}$ are parallel at each point (see Figure. 2.18.b), so $\vec{B} \cdot d\vec{s} = Bds$. Furthermore, the magnitude of $\vec{B}$ is constant on this circle and is given by $B = \frac{\mu_0 I}{2\pi r}$. Therefore, the sum of the products $\vec{B} \cdot d\vec{s} = Bds$ over the closed path, which is equivalent to the line integral of$\vec{B} \cdot d\vec{s} = Bds$, is

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I = \mu_0 I_{net\ through\ path} \tag{2.24}$$

Where $\oint ds$ is the circumference of the circular path. Although this result was calculated for the special case of a circular path surrounding a wire, it holds for a closed of any shape surrounding that exists in an unbroken circuit. The general case, known as Ampère's law, can be stated as follows:

> *"The line integral of $\vec{B} \cdot d\vec{s}$ around any closed path equals $\mu_0 I$, where I is the total continuous current passing through any surface bounded by the closed path".*

Ampère's law describes the creation of magnetic fields by all continuous current configurations. Its use is similar to that of Gauss's law in calculating electric fields for highly symmetric charge distributions.

2.2.6 The Magnetic Field of a Solenoid

A solenoid is a long wire wound in the form of a helix. With this configuration, a reasonably uniform magnetic field can be produced in the space surrounded by the turns of wire–which we shall call the interior of the solenoid—when the solenoid carries a current. When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.
Figure 2.19*c* shows the magnetic field lines surrounding a loosely wound solenoid. Note that the field lines in the interior are nearly parallel to one another, are uniformly distributed, and are close together, indicating that the field in this space is uniform and strong. The field lines between current elements on two adjacent turns tend to cancel each other because the field vectors from the two elements are in opposite directions.

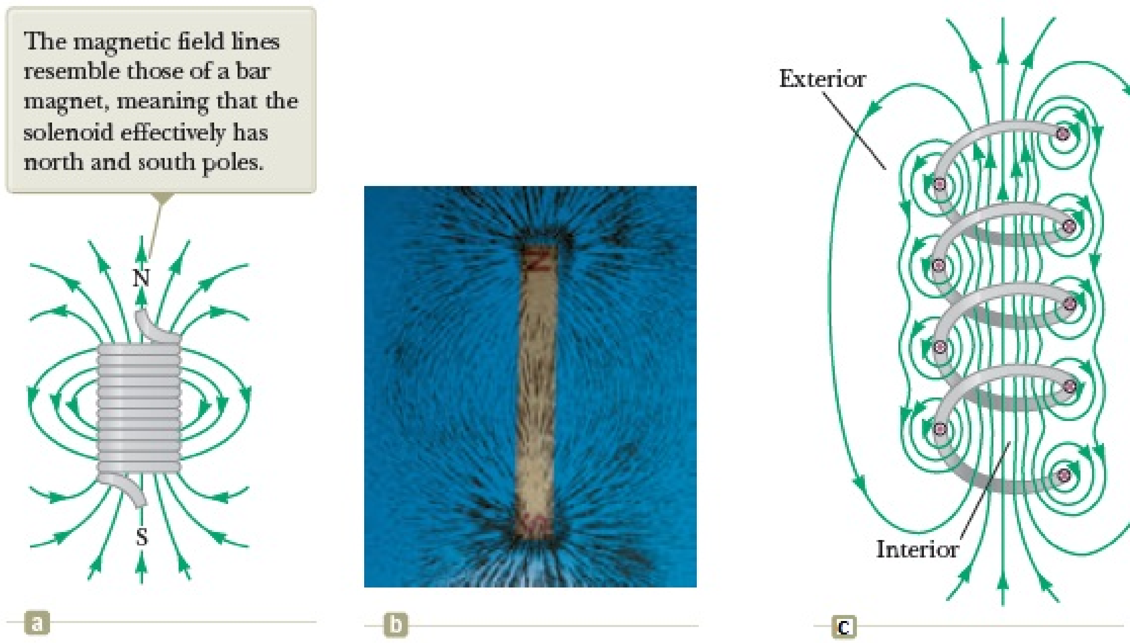


Figure 2.19: *(a) Magnetic field lines for a tightly wound solenoid of finite length, carrying a steady current. The field in the interior space is strong and nearly uniform. (b) The magnetic field pattern of a bar magnet, displayed with small iron filings on a sheet of paper. (c) The magnetic field lines for a loosely wound solenoid.*

The field at exterior points such as P is weak because the field due to current elements on the right-hand portion of a turn tends to cancel the field due to current elements on the left-hand portion.

If the turns are closely spaced and the solenoid is of finite length, the magnetic field lines are as shown in Figure 2.19a. This field line distribution is similar to that surrounding a bar magnet (see Fig. 2.19b). Hence, one end of the solenoid behaves like the north pole of a magnet, and the opposite end behaves like the south pole. As the length of the solenoid increases, the interior field becomes more uniform and the exterior field becomes

weaker. An ideal solenoid is approached when the turns are closely spaced and the length is much greater than the radius of the turns. In this case, the external field is zero, and the interior field is uniform over a great volume.
Using the Ampere's law, the interior magnetic field of an ideal solenoid is found to be:

$$B = \mu_0 n I \tag{2.25}$$

where $n = N/l$ is the number of turns per unit length.

2.2.7 The Magnetic Field of a Toroid (a circular solenoid)

Consider a toroid that consists of N turns, with inner radius a and outer radius b, as shown in Figure 2.20. Find the magnetic field everywhere.

Figure 2.20: *A toroid with N turns*

One can think of a toroid as a solenoid wrapped around with its ends connected. Thus, the magnetic field is completely confined inside the toroid and the field points in the azimuthal direction (clockwise due to the way the current flows, as shown in Figure 2.20). Applying Ampere's law using a circular loop of radius r, for the region $a < r < b$, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \oint B ds = B \oint ds = B 2\pi r = \mu_0 N I \tag{2.26}$$

The magnitude of the magnetic field is

$$B = \frac{\mu_0 N I}{2\pi r} \;;\; a < r < b \tag{2.27}$$

where r is the distance measured from the center of the toroid. Unlike the magnetic field of a solenoid, the magnetic field inside the toroid is non-uniform and decreases as $1/r$.
For the region $r < b$, there is no current enclosed in a circular Amperian loop of radius r, so the magnetic field is *zero*. In the region $r >$, the enclosed current in a circular Amperian loop of radius r is $I_{enc} = NI - NI = 0$ because windings cut through the loop in opposite directions. Therefore the magnetic field is *zero* for $r > b$.

2.2.8 Magnetic Flux

The flux associated with a magnetic field is defined in a manner similar to that used to define electric flux.

Consider an element of area $d\vec{A}$ on an arbitrarily shaped surface, as shown in Figure . If the magnetic field at this element is $\vec{B}$, the magnetic flux through the element is $\vec{B} \cdot d\vec{A}$ where $d\vec{A}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dA. Hence, the total magnetic flux Φ_B through the surface is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \tag{2.28}$$

Consider the special case of a plane of area A in a uniform field $\vec{B}$ that makes an angle θ with $d\vec{A}$. The magnetic flux through the plane in this case is

$$\Phi_B = BA\cos\theta \tag{2.29}$$

2.2.9 Gauss's Law in Magnetism

In previous chapter we found that the electric flux through a closed surface surrounding a net charge is proportional to that charge (Gauss's law). In other words, the number of electric field lines leaving the surface depends only on the net charge within it. This property is based on the fact that electric field lines originate and terminate on electric charges.

The situation is quite different for magnetic fields, which are continuous and form closed loops. In other words, magnetic field lines do not begin or end at any point—as illustrated by the magnetic field lines of the bar magnet in Figure 29.a. Note that for any closed surface, such as the one outlined by the dashed red line in Figure 29.a, the number of lines entering the surface equals the number leaving the surface; thus, the net magnetic flux is zero. In contrast, for a closed surface surrounding one charge of an electric dipole (Fig. 29.b), the net electric flux is not zero.

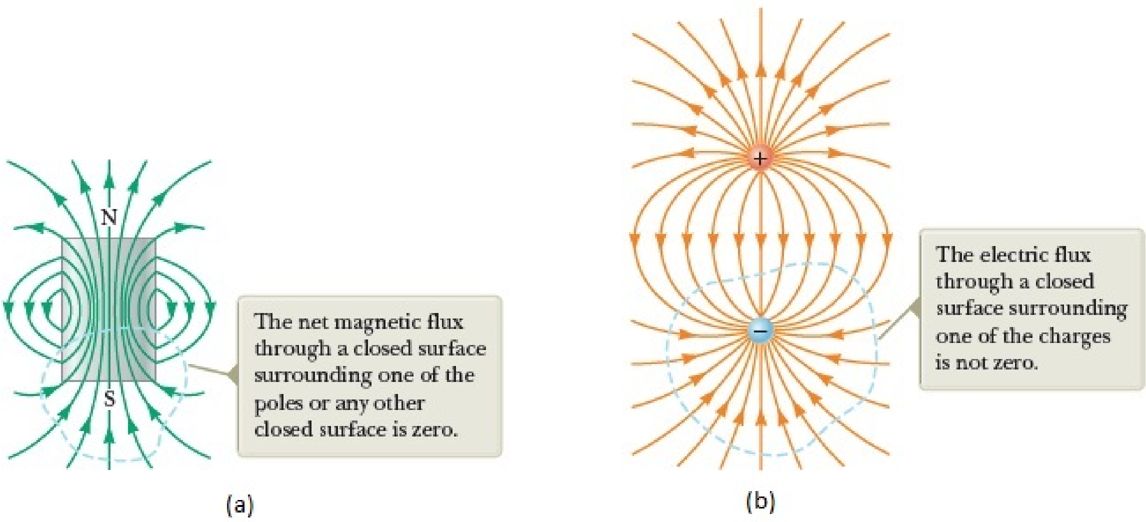


Figure 2.21: *(a) The magnetic field lines of a bar magnet and, (b) The electric field lines surrounding an electric dipole.*

Gauss's law in magnetism states that:

The net magnetic flux through any closed surface is always zero. Or, mathematically

$$\oint \vec{B} \cdot d\vec{A} = 0 \tag{2.30}$$

This statement is based on the experimental fact that isolated magnetic poles (monopoles) have never been detected and perhaps do not exist. Nonetheless, scientists continue the search because certain theories that are otherwise successful in explaining fundamental physical behavior suggest the possible existence of magnetic monopoles.

2.2.10 Magnetic Materials

The introduction of material media into the study of magnetism has very different consequences as compared to the introduction of dielectric material media into the study of electrostatics. When we dealt with dielectric materials in electrostatics, their effect was always to reduce $\vec{E}$ below what it would otherwise be, for a given amount of *free* electric charge. In contrast, when we deal with magnetic materials, their effect can be one of the following:

(i) reduce $\vec{B}$ below what it would otherwise be, for the same amount of *free* electric current (**diamagnetic** materials);

(ii) increase $\vec{B}$ a little above what it would otherwise be (**paramagnetic** materials);

(iii) increase $\vec{B}$ a lot above what it would otherwise be (**ferromagnetic** materials).

Magnetic materials consist of many permanent or induced magnetic dipoles. One of the concepts crucial to the understanding of magnetic materials is the average magnetic field produced by many magnetic dipoles that are all aligned.

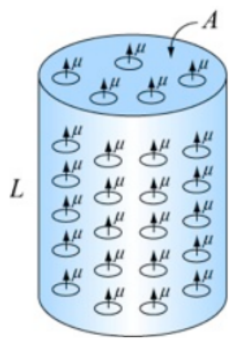


Figure 2.22: *A cylinder with N magnetic dipole moments*

Suppose we have a piece of material in the form of a long cylinder with area A and height L, and that it consists of N magnetic dipoles, each with magnetic dipole moment $\vec{\mu}$ of magnitude μ, spread uniformly throughout the volume of the cylinder, as shown in Figure 2.22. We also assume that all of the magnetic dipole moments $\vec{\mu}$ are aligned with the axis of the cylinder. In the absence of any external magnetic field, what is the average magnetic field due to these dipoles alone?

To answer this question, we note that each magnetic dipole has its own magnetic field associated with it. Let's define the magnetization vector $\vec{M}$ to be the net magnetic dipole moment vector per unit volume,

$$\vec{M} = \frac{1}{V}\sum_i \vec{\mu} \tag{2.31}$$

where V is the volume. In the case of the cylinder, where all the dipoles are aligned, the magnitude of $\vec{M}$ is simply $M = N\mu/AL$.
The average magnetic field inside the material produced by the equivalent current system (in most materials, except ferromagnets) is given by:

$$\vec{B}_M = \mu_0\vec{M} \tag{2.32}$$

For magnetic field in a material:

$$\vec{B}_{net} = \vec{B}_0 + \vec{B}_M \tag{2.33}$$

Where $\vec{B}_0$ is applied B-field and $\vec{B}_M$ B-field produced by induced dipoles. In many materials (except ferromagnets), $\vec{B}_0 \propto \vec{B}_M$. We define:

$$\vec{B}_M = \chi_m\ \vec{B}_0$$

χ_m is a number called **magnetic susceptibility**.

$$\vec{B}_{net} = \vec{B}_0 + \chi_m\ \vec{B}_0 = (1+\chi_m)\vec{B}_0 = \kappa_m\ \vec{B}_0$$

$\kappa_m = 1 + \chi_m$ is a number called **relative permeability**.

Three types of magnetic materials:

(1) ***Paramagnetic***: $\kappa_m \geq 1$ or $\chi_m \geq 0$: Induced magnetic dipoles aligned with the applied B-field.

e.g. Al ($\chi_m \approx= 2.2\times10^{-5}$), Mg ($\chi_m \approx 1.2\times10^{-5}$), O_2 ($\chi_m \approx 2.0\times10^{-6}$)

(2) ***Diamagnetic***: $\kappa_m \leq 1$ or $\chi_m \leq 0$: Induced magnetic dipoles aligned opposite with the applied B-field.

e.g. Cu ($\chi_m \approx -1\times10^{-5}$), Ag ($\chi_m \approx -2.6\times10^{-5}$), N_2 ($\chi_m \approx -5\times10^{-9}$)

(3) ***Ferromagnetic***: Magnetization not linearly proportional to applied field. B_{net}/B_{app} not a constant (can be as big as $\sim 5000 - 100,000$).

e.g. Fe, Co, Ni

2.3 ELECTROMAGNETIC INDUCTION

2.3.1 Faraday's Law of Electromagnetic Induction

To see how an *emf* can be induced by a changing magnetic field, let us consider a loop of wire connected to a galvanometer, as illustrated in Figure 2.23.

When a magnet is moved toward the loop, the galvanometer needle deflects in one direction, arbitrarily shown to the right in Figure 2.23a. When the magnet is moved away from the loop, the needle deflects in the opposite direction, as shown in Figure 2.23c. When the magnet is held stationary relative to the loop (Fig. 2.23b), no deflection is observed. Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects. From these observations, we conclude that the loop "knows" that the magnet is moving relative to it because it experiences a change in magnetic field. Thus, it seems that a relationship exists between current and changing magnetic field.

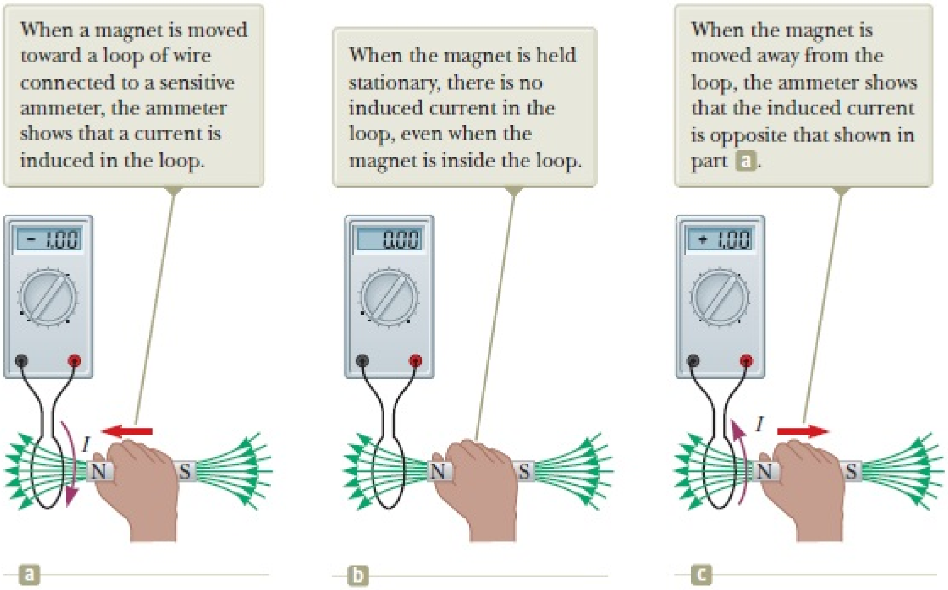


Figure 2.23: *A simple experiment showing that a current is induced in a loop when a magnet is moved toward or away from the loop.*

These results are quite remarkable in view of the fact that a current is set up even though no batteries are present in the circuit! We call such a current an induced current and say that it is produced by an induced emf.

Now let us describe an experiment conducted by Faraday and illustrated in Figure 2.24. A primary coil is connected to a switch and a battery. The coil is wrapped around a ring, and a current in the coil produces a magnetic field when the switch is closed. A secondary coil also is wrapped around the ring and is connected to a galvanometer. No battery is present in the secondary circuit, and the secondary coil is not connected to the primary coil.

Any current detected in the secondary circuit must be induced by some external agent. Initially, you might guess that no current is ever detected in the secondary circuit. However, something quite amazing happens when the switch in the primary circuit is either suddenly closed or suddenly opened. At the instant the switch is closed, the galvanometer needle deflects in one direction and then returns to zero. At the instant the switch is opened, the needle deflects in the opposite direction and again returns to zero. Finally, the galvanometer reads zero when there is either a steady current or no current in the primary circuit.

The key to under- standing what happens in this experiment is to first note that when the switch is closed, the current in the primary circuit produces a magnetic field in the region of the circuit, and it is this magnetic field that penetrates the secondary circuit.

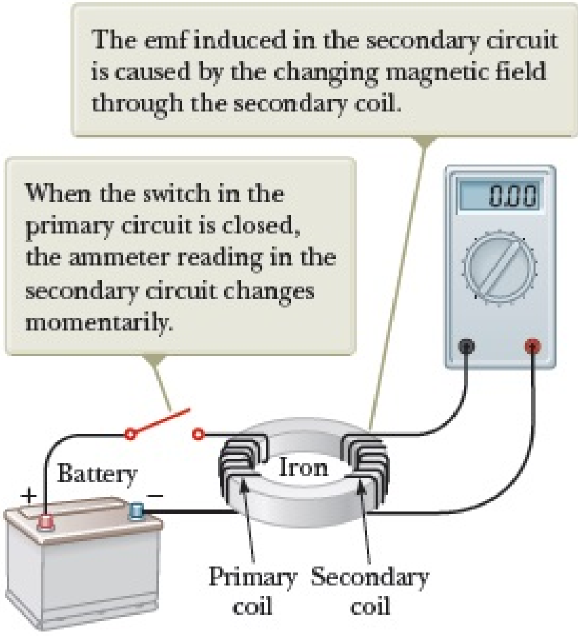


Figure 2.24: *Faraday's experiment.*

Furthermore, when the switch is closed, the magnetic field produced by the current in the primary circuit changes from zero to some value over some finite time, and it is this changing field that induces a current in the secondary circuit. As a result of these observations, Faraday concluded that an electric current can be induced in a circuit (the secondary circuit in our setup) by a changing magnetic field. The induced current exists for only a short time while the magnetic field through the secondary coil is changing. Once the magnetic field reaches a steady value, the current in the secondary coil disappears. In effect, the secondary circuit behaves as though a source of *emf* were connected to it for a short time. It is customary to say that an induced emf is produced in the secondary circuit by the changing magnetic field.

The experiments shown in Figures 2.23 and 2.24 have one thing in common: In each case, an *emf* is induced in the circuit when the magnetic flux through the circuit changes with time. In general, this emf is directly proportional to the time rate of change of the magnetic flux through the loop. This statement can be written mathematically as Faraday's law of induction:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} \tag{2.34}$$

where $\Phi_B = \int \vec{B} \cdot d\vec{A}$ is the magnetic flux through the loop.
If the circuit is a coil consisting of N loops all of the same area, an *emf* is induced in every loop; thus, the total induced emf in the coil is given by

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} \tag{2.35}$$

The negative sign is of important physical significance known as Lenz's law (to be discussed in the followings):

2.3.2 Lenz's Law

Faraday's law (Equation 2.35) indicates that the induced emf and the change in flux have opposite algebraic signs. This feature has a very real physical interpretation that has come to be known as Lenz's law:

> "The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop."

That is, the induced current tends to keep the original magnetic flux through the loop from changing. We shall show that this law is a consequence of the law of conservation of energy.
To understand Lenz's law, let's return to the example of a bar moving to the right on two parallel rails in the presence of a uniform magnetic field (the external magnetic field, shown by the green crosses in Figure 2.25a).

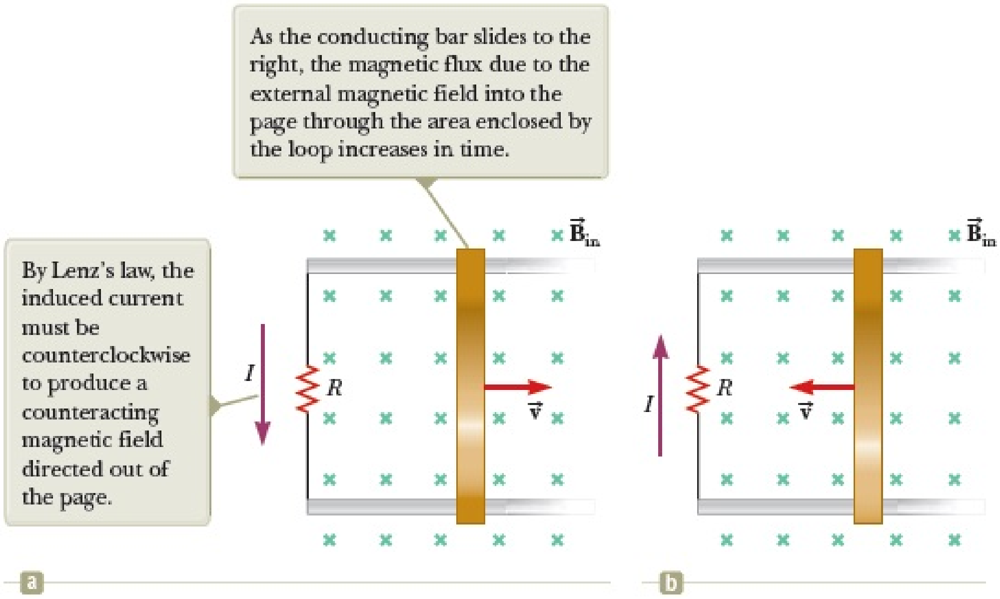


Figure 2.25: *Faraday's experiment.*

As the bar moves to the right, the magnetic flux through the area enclosed by the circuit increases with time because the area increases. Lenz's law states that the induced current must be directed so that the magnetic field it produces opposes the change in the external magnetic flux. Because the magnetic flux due to an external field directed into the page is increasing, the induced current–if it is to oppose this change–must produce a field directed out of the page. Hence, the induced current must be directed counterclockwise when the bar moves to the right. (Use the right-hand rule to verify this direction.) If the bar is moving to the left as in Figure 2.25b, the external magnetic flux through the area enclosed by the loop decreases with time. Because the field is directed into the page, the direction of the induced current must be clockwise if it is to produce a field that also is directed into the page. In either case, the induced current attempts to maintain the original flux through the area enclosed by the current loop.
Let's examine this situation using energy considerations. Suppose the bar is given a slight push to the right. In the preceding analysis, we found that this motion sets up a

counterclockwise current in the loop. What happens if we assume the current is clockwise such that the direction of the magnetic force exerted on the bar is to the right? This force would accelerate the rod and increase its velocity, which in turn would cause the area enclosed by the loop to increase more rapidly. The result would be an increase in the induced current, which would cause an increase in the force, which would produce an increase in the current, and so on. In effect, the system would acquire energy with no input of energy. This behavior is clearly inconsistent with all experience and violates the law of conservation of energy. Therefore, the current must be counterclockwise.

One of the application of Lenz's law is a magnet placed near a metal loop as shown in Figure 2.26a As the magnet moves to the right toward the loop, the external magnetic flux through the loop increases with time. To counteract this increase in flux due to a field toward the right, the induced current produces its own magnetic field to the left as illustrated in Figure 2.26b; hence, the induced current is in the direction shown. Knowing that like magnetic poles repel each other, we conclude that the left face of the current loop acts as a noth pole and the right face acts like a south pole.

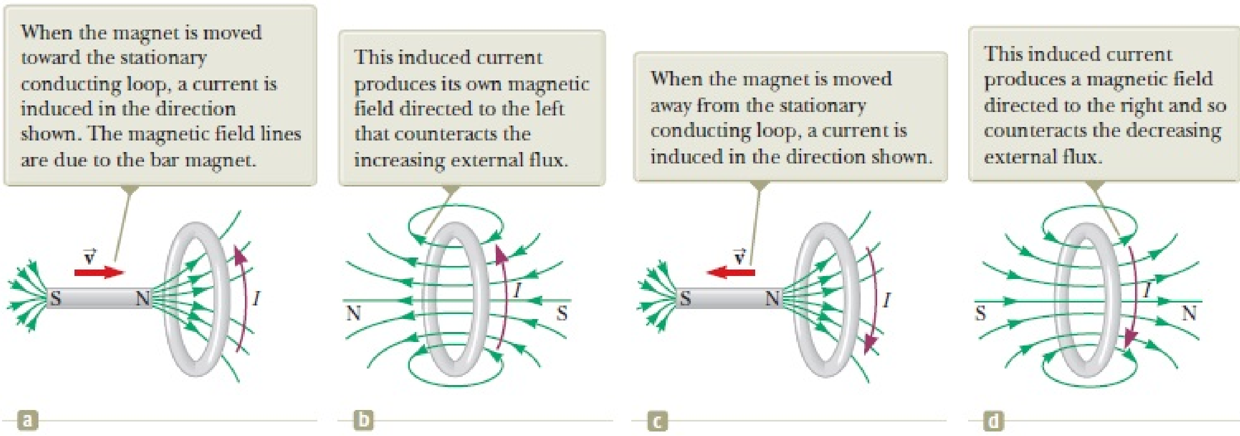


Figure 2.26: *Faraday's experiment.*

If the magnet moves to the left as in Figure 2.26c, its flux through the area enclosed by the loop decreases in time. Now the induced current in the loop is in the direction shown in Figure 2.26d because this current direction produces a magnetic field in the same direction as the external field. In this case, the left face of the loop is a south pole and the right face is a north pole.

2.3.3 Motional Electromotive Force

The straight conductor of length l , shown in the Figure 2.27a is moving through a uniform magnetic field directed into the page. For simplicity, let's assume the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent. From the magnetic version of the particle in a field model, the electrons in the conductor experience a force $\vec{F}_B = q\vec{v} \times \vec{B}$ that is directed along the length l perpendicular to both $\vec{v}$ and $\vec{B}$. Under the influence of this force, the electrons move to the lower end of the conductor and accumulate there, leaving a net positive charge at the upper end. As a result of this charge separation, an electric field $\vec{v}$ is produced inside the conductor.

Therefore, the electrons are also described by the electric version of the particle in a field model. The charges accumulate at both ends until the downward magnetic force qvB on charges remaining in the conductor is balanced by the upward electric force qE. The electrons are then described by the particle in equilibrium model. The condition for equilibrium requires that the forces on the electrons balance:

$$qE = qvB \qquad or \qquad E = vB$$

The magnitude of the electric field produced in the conductor is related to the potential difference across the ends of the conductor according to the relationship $\Delta V = El$. Therefore, for the equilibrium condition,

$$\Delta V = El = Blv \tag{2.36}$$

where the upper end of the conductor in Figure 2.27a is at a higher electric potential than the lower end. Therefore, a potential difference is maintained between the ends of the conductor as long as the conductor continues to move through the uniform magnetic field. If the direction of the motion is reversed, the polarity of the potential difference is also reversed.

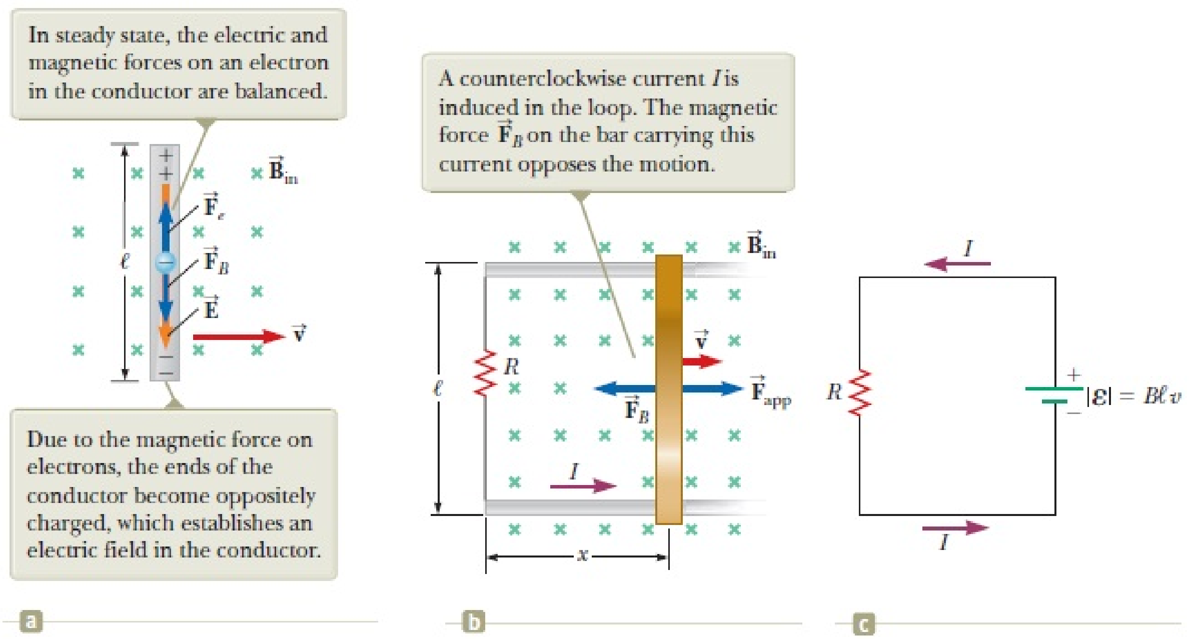


Figure 2.27: *(a) straight electrical conductor of length l, moving with a velocity $\vec{v}$ through a uniform magnetic field $\vec{B}$ directed perpendicular to $\vec{v}$. (b) A conducting bar sliding with a velocity $\vec{v}$ along two conducting rails under the action of an applied force $\vec{F}_{app}$. (c) The equivalent circuit diagram for the setup shown in (b).*

A more interesting situation occurs when the moving conductor is part of a closed conducting path. This situation is particularly useful for illustrating how a changing magnetic flux causes an induced current in a closed circuit.
Consider a circuit consisting of a conducting bar of length l, sliding along two fixed, parallel conducting rails as shown in Figure 2.27b. For simplicity, let's assume the bar has zero resistance and the stationary part of the circuit has a resistance R. A uniform and constant magnetic field $\vec{B}$ is applied perpendicular to the plane of the circuit. As

the bar is pulled to the right with a velocity $\vec{v}$ under the influence of an applied force $\vec{F}_{app}$, free charges in the bar are moving particles in a magnetic field that experience a magnetic force directed along the length of the bar. This force sets up an induced current because the charges are free to move in the closed conducting path. In this case, the rate of change of magnetic flux through the circuit and the corresponding induced motional emf across the moving bar are proportional to the change in area of the circuit.
Because the area enclosed by the circuit at any instant is lx, where x is the position of the bar, the magnetic flux through that area is

$$\Phi_B = Blx$$

Using Faraday's law and noting that x changes with time at a rate $dx/dt = v$, we find that the induced motional emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt}$$

Thus,

$$\mathcal{E} = -Blx \tag{2.37}$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\mathcal{E}|}{R} = \frac{Blv}{R} \tag{2.38}$$

The equivalent circuit diagram for this analysis is shown in Figure 2.27c.

2.3.4 Induced emf and Electric Fields

We have seen that a changing magnetic flux induces an *emf* and a current in a conducting loop. In our study of electricity, we related a current to an electric field that applies electric forces on charged particles. In the same way, we can relate an induced current in a conducting loop to an electric field by claiming that an electric field is created in the conductor as a result of the changing magnetic flux. We also noted in our study of electricity that the existence of an electric field is independent of the presence of any test charges. This independence suggests that even in the absence of a conducting loop, a changing magnetic field generates an electric field in empty space.
This induced electric field is nonconservative, unlike the electrostatic field produced by stationary charges. To illustrate this point, consider a conducting loop of radius r situated in a uniform magnetic field that is perpendicular to the plane of the loop as in Figure 2.28. If the magnetic field changes with time, an *emf* $\mathcal{E} = -d\Phi_B/dt$ is, according to Faraday's law, induced in the loop. The induction of a current in the loop implies the presence of an induced electric field $\vec{E}$, which must be tangent to the loop because that is the direction in which the charges in the wire move in response to the electric force. The work done by the electric field in moving a charge q once around the loop is equal to $q\mathcal{E}$. Because the electric force acting on the charge is $q\vec{E}$, the work done by the electric field in moving the charge once around the loop is $qE(2\pi r)$, where $2\pi r$ is the circumference of the loop.
These two expressions for the work done must be equal; therefore,

$$q\mathcal{E} = qE(2\pi r)$$

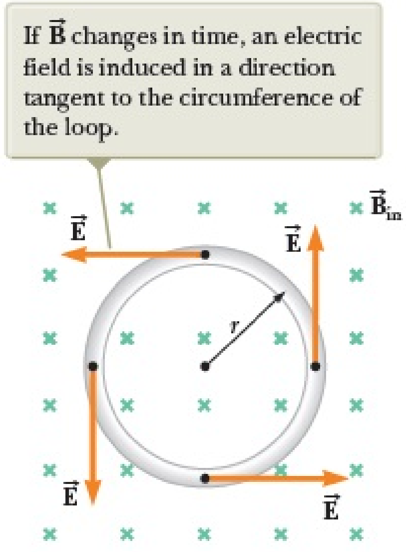


Figure 2.28: *A conducting loop of radius r in a uniform magnetic field perpendicular to the plane of the loop.*

Using this result along with Equation 2.34 and that $\Phi_B = BA = B\pi r^2$ for a circular loop, the induced electric field can be expressed as

$$E = -\frac{1}{2\pi}\frac{d\Phi_B}{dt} = -\frac{r}{2}\frac{dB}{dt} \tag{2.39}$$

If the time variation of the magnetic field is specified, the induced electric field can be calculated from Equation 2.39.
The emf for any closed path can be expressed as the line integral of $\vec{E}\cdot d\vec{s}$ over that path: $\mathcal{E} = \oint \vec{E}\cdot d\vec{s}$. In more general cases, E may not be constant and the path may not be a circle. Hence, Faraday's law of induction, $\mathcal{E} = -d\Phi_B/dt$, can be written in the general form

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \tag{2.40}$$

$$\oint \vec{E}_{induced}\cdot d\vec{s} = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{A} \tag{2.41}$$

The induced electric field $\vec{E}$ in Equation 2.41 is a nonconservative field that is generated by a changing magnetic field. The field $\vec{E}$ that satisfies Equation 2.41 cannot possibly be an electrostatic field because were the field electrostatic and hence conservative, the line integral of $\vec{E}\cdot d\vec{s}$ over a closed loop would be zero, which would be in contradiction to Equation 2.40.

2.3.5 Generators and Motors

Electricity generation is the process of generating electricity from other forms of energy. The fundamental principles of electricity generation were discovered during the 1820s and early 1830s by the British scientist Michael Faraday. His basic method is still used today: electricity is generated by the movement of a loop of wire, or disc of copper between the poles of a magnet.
There are seven fundamental methods of directly transforming other forms of energy into electrical energy:

(a) Static electricity: from the physical separation and transport of charge (examples: turboelectric effect and lightning);

(b) Electromagnetic induction: where an electrical generator, dynamo or alternator transforms kinetic energy (energy of motion) into electricity;

(c) Electrochemistry: the direct transformation of chemical energy into electricity, as in a battery, fuel cell or nerve impulse;

(d) Photoelectric effect: the transformation of light into electrical energy, as in solar cells

(e) Thermoelectric effect: Direct conversion of temperature differences to electricity, as in thermocouple, thermopile, and thermionic converters;

(f) Piezoelectric effect: from the mechanical strain of electrically anisotropic molecules or crystals

(g) Nuclear transformation: the creation and acceleration of charged particles (examples: betavoltaics or alpha particle emission).

Static electricity was the first form discovered and investigated, and the electrostatic generator is still used even in modern devices such as the Van de Graaff generator and Magnetohydrodynamic generators (MHD). Electrons are mechanically separated and transported to increase their electric potential.
Almost all commercial electrical generation is done using electromagnetic induction, in which mechanical energy forces an electrical generator to rotate. There are many different methods of developing the mechanical energy, including heat engines, hydro, wind and tidal power.
The direct conversion of nuclear energy to electricity by beta decay is used only on a small scale. In a full-size nuclear power plant, the heat of a nuclear reaction is used to run a heat engine. This drives a generator, which converts mechanical energy into electricity by magnetic induction.
Most electric generation is driven by heat engines. The combustion of fossil fuels supplies most of the heat to these engines, with a significant fraction from nuclear fission and some from renewable sources. The modern steam turbine invented by Sir Charles Parsons in 1884 - today generates about 80 percent of the electric power in the world using a variety of heat sources.

Induction Generators and Motors

To understand how generation by electromagnetic induction operate, let us consider the alternating current (AC) generator. In its simplest form, it consists of a loop of wire rotated by some external means in a magnetic field (Figure 2.29).
In commercial power plants, the energy required to rotate the loop can be derived from a variety of sources. For example, in a hydroelectric plant, falling water directed against the blades of a turbine produces the rotary motion; in a coalfired plant, the energy released by burning coal is used to convert water to steam, and this steam is directed against the turbine blades.

As a loop rotates in a magnetic field, the magnetic flux through the area enclosed by the loop changes with time, and this change induces an *emf* and a current in the loop

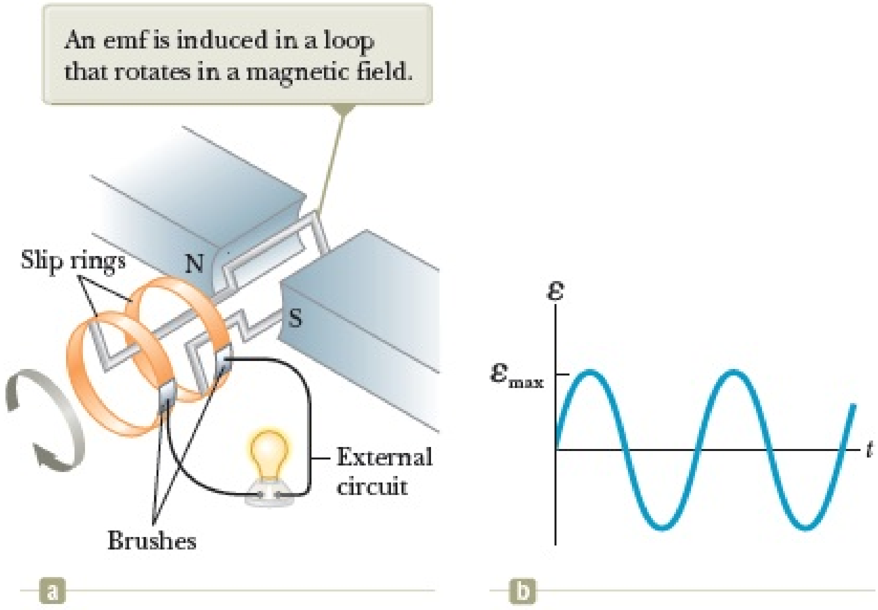


Figure 2.29: *(a) Schematic diagram of an AC generator. (b) The alternating emf induced in the loop plotted as a function of time.*

according to Faraday's law. The ends of the loop are connected to slip rings that rotate with the loop. Connections from these slip rings, which act as output terminals of the generator, to the external circuit are made by stationary metallic brushes in contact with the slip rings.

Instead of a single turn, suppose a coil with N turns (a more practical situation), with the same area A, rotates in a magnetic field with a constant angular speed ω. If θ is the angle between the magnetic field and the normal to the plane of the coil as in Figure 2.29c, the magnetic flux through the coil at any time t is

$$\Phi_B = BA\cos\omega t$$

where we have used the relationship $\theta = \omega t$ between angular position and angular speed. (We have set the clock so that $t = 0$ when $\theta = 0$.) Hence, the induced *emf* in the coil is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t) = NBA\omega\sin\omega t \tag{2.42}$$

This result shows that the emf varies sinusoidally with time as plotted in Figure 2.29b. Equation 2.42 shows that the maximum emf has the value

$$\mathcal{E}_{max} = NBA\omega \tag{2.43}$$

which occurs when $\omega t = \pi/2, \quad or \quad 3\pi/2$. In other words, $\mathcal{E} = \mathcal{E}_{max}$ when the magnetic field is in the plane of the coil and the time rate of change of flux is a maximum. Furthermore, the *emf* is zero when $\omega t = 0, \quad or \quad \pi$, that is, when $\vec{B}$ is perpendicular to the plane of the coil and the time rate of change of flux is zero.

The frequency for commercial generators is, according to the country or region, 60 Hz or 50 Hz.

2.4 INDUCTANCE

In this section, we need to distinguish carefully between *emfs* and currents that are caused by physical sources such as batteries and those that are induced by changing magnetic fields. When we use a term (such as emf or current) without an adjective, we are describing the parameters associated with a physical source. We use the adjective induced to describe those *emfs* and currents caused by a changing magnetic field.

2.4.1 Self-Induction and Inductance

Consider a circuit consisting of a switch, a resistor, and a source of *emf* as shown in Figure 2.30. The circuit diagram is represented in perspective to show the orientations of some of the magnetic field lines due to the current in the circuit. When the switch is thrown to its closed position, the current does not immediately jump from zero to its maximum value $\mathcal{E}/R$. Faraday's law of electromagnetic induction can be used to describe this effect as follows.

As the current increases with time, the magnetic field lines surrounding the wires pass through the loop represented by the circuit itself. This magnetic field passing through the loop causes a magnetic flux through the loop. This increasing flux creates an induced *emf* in the circuit. The direction of the induced *emf* is such that it would cause an induced current in the loop (if the loop did not already carry a current), which would establish a magnetic field opposing the change in the original magnetic field. Therefore, the direction of the induced *emf* is opposite the direction of the *emf* of the battery, which results in a gradual rather than instantaneous increase in the current to its final equilibrium value. Because of the direction of the induced *emf*, it is called a back emf (because it tends to reduce the supplied current). This effect is called *"self-induction"* because the changing flux through the circuit and the resultant induced *emf* arise from the circuit itself. The *emf* $\mathcal{E}_L$ set up in this case is called a *"self-induced emf"*. To obtain a quantitative description of self-induction, recall from Faraday's law that the induced *emf* is equal to the negative of the time rate of change of the magnetic flux. The magnetic flux is proportional to the magnetic field, which in turn is proportional to the current in the circuit. Therefore, a self-induced emf is always proportional to the time rate of change of the current.

For any loop of wire, we can write this proportionality as

$$\mathcal{E}_L = -L\frac{di}{dt} \tag{2.44}$$

where L is a proportionality constant-called the *"inductance"* of the loop—that depends on the geometry of the loop and other physical characteristics. If we consider a closely spaced coil of N turns carrying a current i and containing N turns, Faraday's law tells us that $\mathcal{E}_L = -N d\Phi_B/dt$. Combining this expression with Equation 2.44 gives

$$L = \frac{N\Phi_B}{i} \tag{2.45}$$

where it is assumed the same magnetic flux passes through each turn and L is the inductance of the entire coil.

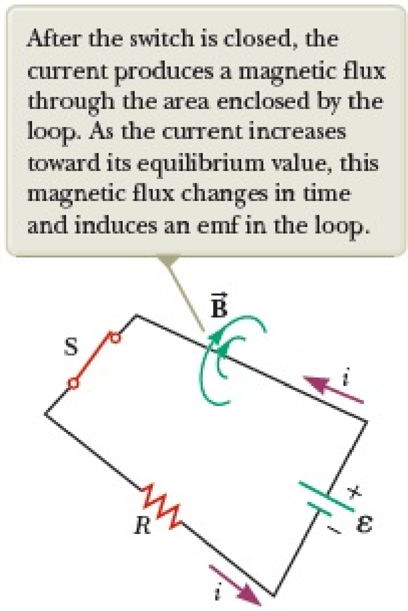


Figure 2.30: *Self-induction in a simple circuit.*

From Equation 2.44, we can also write the inductance as the ratio

$$L = -\frac{\mathcal{E}_L}{di/dt} \tag{2.46}$$

The *SI* unit of inductance is the henry (H), which as we can see from Equation is 1 volt-second per ampere: 1 $= 1\ V \cdot s/A$.

Self-Inductance of a Solenoid

Compute the self-inductance of a solenoid with N turns, length l, and radius R with a current I flowing through each turn. Ignoring edge effects and applying Ampere's law, the magnetic field inside a solenoid is:

$$\vec{B} = \frac{\mu_0 N I}{l}\vec{k} = \mu_0 n I \vec{k}$$

where $n = N/l$ is the number of turns per unit length. The magnetic flux through each turn is

$$\Phi_B = \mu_0 n I(\pi R^2) = \pi \mu_0 n I R^2$$

Thus, the self-inductance is

$$L = \frac{N\Phi_B}{I} = \mu_0 n^2 \pi R^2 l \tag{2.47}$$

We see that L depends only on the geometrical factors (n, R and l) and is independent of the current I.

Self-Inductance of a Toroid

Calculate the self-inductance L of a toroid, which consists of N turns and has a rectangular cross section, with inner radius a, outer radius b, and height h, as shown in Figure 2.31.

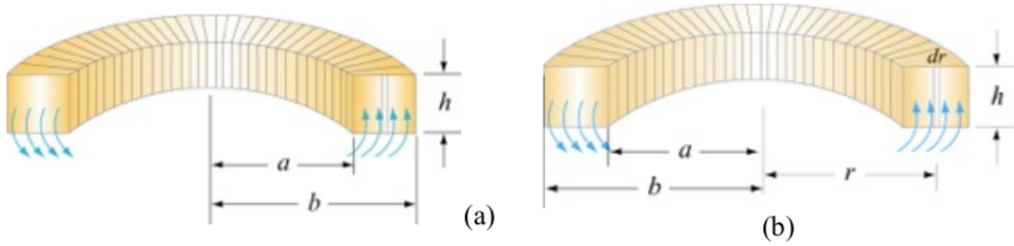


Figure 2.31: *A toroid with N turns*

According to Amperes law, the magnitude of the magnetic field inside the torus is given by:

$$B = \frac{\mu_0 N I}{2\pi r}$$

The magnetic flux through one turn of the toroid may be obtained by integrating over the rectangular cross section, with $dA = h\ dr$ as the differential area element (Figure 2.31),

$$\Phi_B = \int\int \vec{B} \cdot d\vec{A} = \int_a^b (\frac{\mu_0 N I}{2\pi r}) h\ dr = \frac{\mu_0 N I h}{2\pi} ln(\frac{b}{a})$$

The total flux is $N\Phi_B$. Therefore, the self-inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi} ln(\frac{b}{a}) \tag{2.48}$$

Again, the self-inductance L depends only on the geometrical factors. Let's consider the situation where $a >> b - a$. In this limit, the logarithmic term in the equation above may be expanded as

$$ln(\frac{b}{a}) = ln(1 + \frac{b-a}{a}) \approx \frac{b-a}{a}$$

and the self-inductance becomes

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi} \frac{b-a}{a} = \frac{\mu_0 N^2 A}{2\pi a} = \frac{\mu_0 N^2 A}{l} \tag{2.49}$$

where $A = h(b-a)$ is the cross-sectional area, and $l = 2\pi a$. We see that the self-inductance of the toroid in this limit has the same form as that of a solenoid.

2.4.2 Mutual Inductance

Very often, the magnetic flux through the area enclosed by a circuit varies with time because of time-varying currents in nearby circuits. This condition induces an *emf* through a process known as mutual induction, so named because it depends on the interaction of two circuits.

Consider the two closely wound coils of wire shown in cross-sectional view in Figure 2.32.

The current i_1 in coil 1, which has N_1 turns, creates a magnetic field. Some of the magnetic field lines pass through coil 2, which has N_2 turns. The magnetic flux caused by the current in coil 1 and passing through coil 2 is represented by Φ_{12}. In analogy to Equation 2.45, we can identify the mutual inductance M_{12} of coil 2 with respect to coil 1:

$$M_{12} = \frac{N_2 \Phi_{12}}{i_1} \tag{2.50}$$

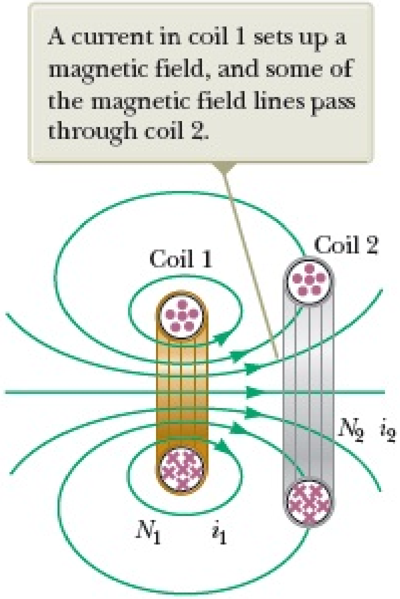


Figure 2.32: *A cross-sectional view of two adjacent coils.*

Mutual inductance depends on the geometry of both circuits and on their orientation with respect to each other. As the circuit separation distance increases, the mutual inductance decreases because the flux linking the circuits decreases.
If the current i_1 varies with time, we see from Faraday's law and Equation 2.50 that the *emf* induced by coil 1 in coil 2 is

$$\mathcal{E}_2 = -N_2 \frac{d\Phi_{12}}{dt} = -N_2 \frac{d}{dt}\left(\frac{M_{12} i_1}{N_2}\right) = -M_{12}\frac{di_1}{dt} \tag{2.51}$$

In the preceding discussion, it was assumed the current is in coil 1. Let's also imagine a current i_2 in coil 2. The preceding discussion can be repeated to show that there is a mutual inductance M_{21}. If the current i_2 varies with time, the emf induced by coil 2 in coil 1 is

$$\mathcal{E}_1 = -M_{21}\frac{di_2}{dt} \tag{2.52}$$

In mutual induction, the *emf* induced in one coil is always proportional to the rate at which the current in the other coil is changing. Although the proportionality constants M_{12} and M_{21} have been treated separately, it can be shown that they are equal. Therefore, with $M_{12} = M_{21}$ equation 2.51 and 2.52 become

$$\mathcal{E}_1 = -M_{21}\frac{di_2}{dt} \qquad \textit{and} \qquad \mathcal{E}_2 = -M_{12}\frac{di_1}{dt} \tag{2.53}$$

These two equations are similar in form of the equation for the self-induced *emf* $\mathcal{E} = -L(di/dt)$.
The unit of mutual inductance is the *henry* (H).

2.4.3 RL Circuits

If a circuit contains a coil such as a solenoid, the inductance of the coil prevents the current in the circuit from increasing or decreasing instantaneously. A circuit element that has a large inductance is called *"an inductor"*. We always assume the inductance

of the remainder of a circuit is negligible compared with that of the inductor. Keep in mind, however, that even a circuit without a coil has some inductance that can affect the circuit's behavior.
Because the inductance of an inductor results in a *back emf*, an inductor in a circuit opposes changes in the current in that circuit. The inductor attempts to keep the current the same as it was before the change occurred. If the battery voltage in the circuit is increased so that the current rises, the inductor opposes this change and the rise is not instantaneous. If the battery voltage is decreased, the inductor causes a slow drop in the current rather than an immediate drop. Therefore, the inductor causes the circuit to be *"sluggish"* as it reacts to changes in the voltage.
In other hand, if the circuit did not contain an inductor, the current would immediately decrease to zero when the battery is removed. When the inductor is present, it opposes the decrease in the current and causes the current to decrease exponentially.

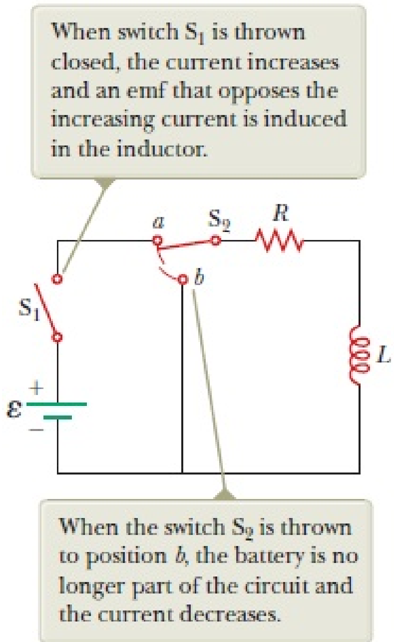


Figure 2.33: *An RL circuit.*

Consider the circuit shown in Figure 2.33, which contains a battery of negligible internal resistance. This circuit is an RL circuit because the elements connected to the battery are a resistor and an inductor. The curved lines on switch S_2 suggest this switch can never be open; it is always set to either a or b. (If the switch is connected to neither a nor b, any current in the circuit suddenly stops.) Suppose S_2 is set to a and switch S_1 is open for $t < 0$ and then thrown closed at $t = 0$. The current in the circuit begins to increase, and a *back emf* (Equation 2.44) that opposes the increasing current is induced in the inductor. With this point in mind, let's apply Kirchhoff's loop rule to this circuit, traversing the circuit in the clockwise direction:

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \tag{2.54}$$

where iR is the voltage drop across the resistor. (Kirchhoff's rules were developed for circuits with steady currents, but they can also be applied to a circuit in which the current is changing if we imagine them to represent the circuit at one instant of time.)

A mathematical solution of Equation 2.54 represents the current in the circuit as a function of time. To find this solution, we change variables for convenience, letting $x = (\mathcal{E}/R) - i$, so $dx = -di$. With these substitutions, Equation 2.54 becomes

$$x + \frac{L}{R}\frac{dx}{dt} = 0$$

Rearranging and integrating this last expression gives

$$\int_{x_0}^{x} \frac{dx}{x} = -\frac{R}{L}\int_0^t dt$$

Which is

$$\ln\frac{x}{x_0} = -\frac{R}{L}t$$

where x_0 is the value of x at time $t = 0$. Taking the antilogarithm of this result gives

$$x = x_0 e^{-Rt/L}$$

Because $i = 0$ at $t = 0$, note from the definition of x that $x_0 = \mathcal{E}/R$. Hence, this last expression is equivalent to

$$\frac{\mathcal{E}}{R} - i = \frac{\mathcal{E}}{R}e^{-Rt/L} \qquad or \qquad i = \frac{\mathcal{E}}{R}(1 - e^{-Rt/L}) \tag{2.55}$$

This expression shows how the inductor affects the current. The current does not increase instantly to its final equilibrium value when the switch is closed, but instead increases according to an exponential function. If the inductance is removed from the circuit, which corresponds to letting L approach zero, the exponential term becomes zero and there is no time dependence of the current in this case; the current increases instantaneously to its final equilibrium value in the absence of the inductance.
We can also write this expression as

$$i = \frac{\mathcal{E}}{R}(1 - e^{-t/\tau}) \tag{2.56}$$

where the constant t is the time constant of the RL circuit

$$\tau = \frac{R}{L}$$

Physically, τ, **the inductive time constant**, is the time interval required for the current in the circuit to reach $(1 - e^{-1}) = 0.632 = 63.2\%$ of its final value $\mathcal{E}/R$. The time constant is a useful parameter for comparing the time responses of various circuits.

2.4.4 Energy in a Magnetic Field

Because the emf induced in an inductor prevents a battery from establishing an instantaneous current, the battery must do work against the inductor to create a current. Part of the energy supplied by the battery appears as internal energy in the resistor, while the remaining energy is stored in the magnetic field of the inductor. If we multiply each term in Equation 2.54 by i and rearrange the expression, we have

$$i\mathcal{E} = i^2R + Li\frac{di}{dt} \tag{2.57}$$

This expression indicates that the rate at which energy is supplied by the battery ($i\mathcal{E}$)equals the sum of the rate at which energy is delivered to the resistor,i^2R , and the rate at which energy is stored in the inductor, $Li(di/dt)$. Thus, Equation 2.57 is simply an expression of energy conservation. If we let U denote the energy stored in the inductor at any time, then we can write the rate dU/dt at which energy is stored as

$$\frac{dU}{dt} = Li\frac{di}{dt}$$

To find the total energy stored in the inductor, we can rewrite this expression as $dU = Lidi$ and integrate:

$$U = \int dU = \int_0^i Lidi$$

or

$$U = \frac{1}{2}Li^2 \tag{2.58}$$

where L is constant and has been removed from the integral. This expression represents the energy stored in the magnetic field of the inductor when the current is i. Note that this equation is similar in form to the equation of energy stored in the electric field of a capacitor, $U = Q^2/2c$ In either case, we see that energy is required to establish a field.

The energy density of a magnetic field is given by

$$u_B = \frac{B^2}{2\mu_0} \tag{2.59}$$

Where μ_0 is the magnetic permeability of the free space (vacuum).

2.4.5 LC Circuits (Electromagnetic oscillation)

Consider a LC circuit in which a capacitor is connected to an inductor, as shown in Figure 2.34.

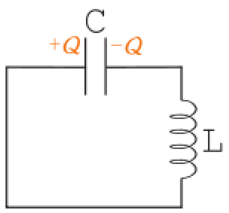


Figure 2.34: *LC Circuit*

An important application is a circuit with an inductor and a capacitor in the same circuit. We will consider here only a simple circuit with a capacitor and inductor in series. Let the capacitor have a capacitance of C, and the inductor a self-inductance of L. Let $\pm Q$ be the charge on the plates of the capacitor. Applying Kirchhoff's modified voltage equation around the circuit gives:

$$V_c = \frac{d\Phi_m}{dt} \tag{2.60}$$

where V_c is the voltage across the capacitor. If $\pm Q$ are the charges on each plate of the capacitor, $V_c = Q/C$. If the self-inductance of the inductor (i.e. the circuit) is L, we have

$$\frac{Q}{C} = L\frac{dI}{dt} \tag{2.61}$$

The current is the rate of change of the charge on the capacitor. If I flows from the positive plate of the capacitor, then $I = -(dQ)/(dt)$. Differentiating both sides with respect to t:

$$\frac{1}{C}\frac{dQ}{dt} = L\frac{d^2I}{dt^2} \tag{2.62}$$

Substituting in for $(dQ)/(dt)$ gives

$$-\frac{I}{C} = L\frac{d^2I}{dt^2} \tag{2.63}$$

Rearranging terms yields the following differential equation:

$$\frac{d^2I}{dt^2} = -\frac{I}{LC} \tag{2.64}$$

The solution $I(t)$ is a function whose second derivative is minus itself. Functions with this property are sinusoidal functions: $I(t) = Asin(\omega t)$. Since $(d^2I)/(dt^2) = -\omega^2 Asin(\omega t)$ we see that

$$\omega = \frac{1}{\sqrt{LC}} \tag{2.65}$$

This is an interesting result. The current in the L-C circuit oscillates with a frequency of $f = \omega/(2\pi) = 1/(2\pi\sqrt{LC})$. This frequency is the resonance frequency of the circuit. If an inductor and capacitor are in series in an antenna, signals at the resonant frequency have a large amplitude. By varying C one can tune into ones favorite radio or TV station.

The general solution to Equation (2.61) is

$$Q(t) = Q_0\ cos(\omega t + \phi)$$

where Q_0 is the amplitude of the charge, $\omega t + \phi$ is the phase, and ϕ is the phase constant. The corresponding current in the inductor is

$$I(t) = -\frac{dQ}{dt} = \omega Q_0\ sin(\omega t + \phi) = I_0 sin(\omega t + \phi)$$

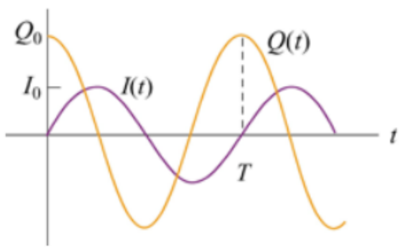


Figure 2.35: *Charge and current in the LC circuit as a function of time*

These are interesting results. The voltage across the capacitor and the current in the circuit oscillate back and forth. The period T of this oscillation is given by

$$\begin{aligned}\frac{T}{\sqrt{LC}} &= 2\pi \\ T &= 2\pi\sqrt{LC}\end{aligned}$$

The frequency of the oscillation is given by $f = 1/T = 1/(2\pi\sqrt{LC})$. This frequency is the **resonance frequency** of the circuit. Note that the quantity $1/\sqrt{LC}$ occurs in the argument of the *sin* (or *cos*) function. It is convenient to define $\omega_0 \equiv 1/\sqrt{LC}$. With this definition, we have for $I(t)$ and $V_c(t)$:

$$\begin{aligned}I(t) &= \frac{Q_0}{\sqrt{LC}} sin(\omega_0 t) \\ V_c(t) &= V_0\ cos(\omega_0 t)\end{aligned}$$

where

$$\omega_0 \equiv \frac{1}{\sqrt{LC}} \tag{2.66}$$

A couple of things to note:

1) The quantity ω_0 has units of $1/time$. It enters in the argument of the sinusoidal functions in the same way as angular frequency would be for an object moving in a circle. Nothing is rotating here, and there are no physical angles. So when we refer to ω as the angular frequency, just think of it as 2π times the frequency.

2) Both the current in the circuit and V_c are sinusoidal functions,but they are out of phase by $90°$. One varies as $sin(\omega t)$ and the other as $cos(\omega t)$.

3) Initially $I = 0$, and all the energy is in the electric field of the capacitor, and the magnetic field in the solenoid is zero. Then at $t = T/2$, there is no charge on the capacitor, the current is maximized, and all the energy is in the magnetic field of the solenoid. The energy is being transfered back and forth between the capacitor (electric field energy) and the inductor (magnetic field energy).

In any real circuit there will be resistance. The energy of the LC circuit will be gradually dissipated by resistive elements. Next we consider what happens if we include resistance in the circuit.

2.4.6 RLC series circuits (Damped oscillator)

Consider now a series circuit that has a capacitor, capacitance C, a resistor, resistance R, and a solenoid, self-inductance L as shown in Figure 2.36. In this case, energy stored in the LC oscillator is NOT conserved. As before, we will assume that the self-inductance of

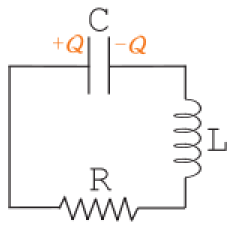


Figure 2.36: *A series RLC circuit*

the whole circuit is approximately that of the solenoid, L. Let's also neglect the resistance of the solenoid. Now, we can apply the physics of circuits to the $R-L-C$-series circuit.

$$\sum (Voltage\ drops) = L\frac{dI}{dt} \tag{2.67}$$

if the path for the voltage changes is in the same direction as the "+" direction of the current. As before, we take the "+" direction of the current I to be away from the positive side of the capacitor. Adding up the changes in voltage in the direction of "+" current we have:

$$V_c - RI = L\frac{dI}{dt} \tag{2.68}$$

V_c is positive, since the path goes from the negative to the positive side of the capacitor (a gain in voltage). The voltage drop across the resistor is $-RI$ because the path is in the direction of the current (i.e. a drop in voltage). As before, substituting $V_c = Q/C$ and $I = -(dQ/dt)$ gives

$$\begin{aligned} \frac{Q}{C} + R\frac{dQ}{dt} &= -L\frac{d^2Q}{dt^2} \\ L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} &= 0 \\ \frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} &= 0 \\ \frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \omega_0^2 Q &= 0 \end{aligned}$$

where $\omega_0 \equiv 1/\sqrt{LC}$. We can guess what the solution to the differential equation above should be. With $R = 0$, the solution is sinusoidal. If $L = 0$, the solution is a decaying exponential. So we can guess that the solution might be a sinusoidal function multiplied by a decaying exponential. There are different ways to solve this differential equation. The solution will be in the form:

$$Q(t) = Q_0\ e^{-\frac{R}{2L}}\ cos(\omega_1\ t + \phi) \tag{2.69}$$

where $e^{-\frac{R}{2L}}$ is an exponential decay term and $cos(\omega_1\ t + \phi)$ is an oscillating term, and

$$\omega_1^2 = \frac{1}{LC} - (\frac{R}{2L})^2 = \omega^2 - (\frac{R}{2L})^2$$

The qualitative behavior of the charge on the capacitor as a function of time is shown in Figure 2.37. Damped oscillator always oscillates at a lower frequency than the natural frequency of the oscillator. Next we will consider an R, L, C circuit that has a sinusoidal voltage source.

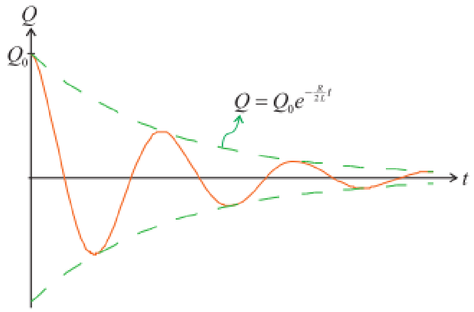


Figure 2.37: *Underdamped oscillations*

2.5 ALTERNATING CURRENT (AC) CIRCUIT

As a final example of LRC series circuits, we will add a sinusoidal voltage source to the circuit.

2.5.1 RL circuit with sinusoidal voltage source

Let's first start with a circuit that has a self-inductance L and a resistance R. The self-inductance is dominated by a solenoid as before. Let the frequency of the sinusoidal voltage source be $f_d = \omega_d/(2\pi)$, and the maximum voltage be V_m.
The same physics is true when there is a source of voltage present:
1) the current is the same through each circuit element since they are connected in series, and
2) the sum of the voltage changes around the loop equals the change of magnetic flux.
Since the voltage source is sinusoidal, after any transient oscillations have damped out, the current in *the circuit will also be sinusoidal with the same frequency as the voltage source*. That is, the current through every element in the circuit can be written as

$$I(t) = I_m sin(\omega_d t) \tag{2.70}$$

Note that I_m is the maximum amplitude that the current will have. We could have also chosen $cos(\omega_d t)$ and the end results would be the same. What we need to determine is the relationship between V_m and I_m, and the relative phase between the sinusoidal current and the sinusoidal voltages. For this, we use the voltage sum law. Equating the voltage changes around the loop to the change in magnetic flux through the loop gives:

$$\begin{aligned} V_d - IR &= L\frac{dI}{dt} \\ V_d &= IR + L\frac{dI}{dt} \end{aligned}$$

where the current $I(t)$ is the same through the resistor and the solenoid. If $I(t) = I_m sin(\omega_d t)$, then the voltage source satisfies

$$\begin{aligned} V_d &= IR + L\frac{dI}{dt} \\ V_d &= I_m R sin(\omega_d t) + L\omega_d I_m cos(\omega_d t) \\ V_d &= I_m(R sin(\omega_d t) + L\omega_d cos(\omega_d t)) \end{aligned}$$

We can simplify this expression by adding the *sin* and *cos* functions. A nice property of sinusoidal functions having the same frequency is that when they are added together the sum is a single sinusoidal function. We demonstrate this property for our circuit by using the trig identity $sin(\omega_d t + \phi) = sin(\omega_d t)cos(\phi) + cos(\omega t)sin(\phi)$.

$$\begin{aligned}
V_d &= I_m(Rsin(\omega_d t) + L\omega_d cos(\omega_d t)) \\
V_d &= I_m\sqrt{R^2 + (L\omega_d)^2}(\frac{R}{\sqrt{R^2 + (L\omega_d)^2}}sin(\omega_d t) + \frac{L\omega_d}{\sqrt{R^2 + (L\omega_d)^2}}cos(\omega_d t)) \\
V_d &= I_m\sqrt{R^2 + (L\omega_d)^2}(cos(\phi)sin(\omega_d t) + sin(\phi)cos(\omega_d t)) \\
V_d &= I_m\sqrt{R^2 + (L\omega_d)^2}\ sin(\omega_d t + \phi)
\end{aligned}$$

The third line follows by noting that R and $L\omega_d$ are two legs of a right triangle. The hypotenuse of this right triangle is $\sqrt{R^2 + (L\omega_d)^2}$, and ϕ is an angle in the triangle. Since the sin function varies between ± 1, the amplitude of the voltage source is $V_m = I_m\sqrt{R^2 + (L\omega_d)^2}$, and the voltage of the source *leads* the current by a phase ϕ, where $tan(\phi) = L\omega_d/R$.

2.5.2 RLC circuit with sinusoidal voltage source

Now let's add a capacitor in series with the resistor and solenoid. We need to determine what the voltage V_c across the capacitor is when a sinusoidal current $I_m sin(\omega_d t$ flows through it. We know $V_c = Q/C$ and $I = +dQ/(dt)$. Here there is a + sign in front of $dQ/(dt)$ since current is flowing into the capacitor. Solving for Q:

$$\begin{aligned}
\frac{dQ}{dt} &= I_m;\ sin(\omega_d t) \\
Q &= \int I_m\ sin(\omega_d t)\ dt \\
Q &= -\frac{I_m}{\omega_d}cos(\omega t)
\end{aligned}$$

Since $V_c = Q/C$, we have

$$V_c = -\frac{I_m}{\omega_d C}cos(\omega_d t) \tag{2.71}$$

Adding the voltage changes around the RLC series circuit loop gives

$$\begin{aligned}
V_d - IR - V_c &= L\frac{dI}{dt} \\
V_d &= IR + L\frac{dI}{dt} + V_c
\end{aligned}$$

Since the current through each element is $I = I_m\ sin(\omega_d t)$ we obtain

$$\begin{aligned}
V_d &= I_m Rsin(\omega_d t) + L\omega_d I_m cos(\omega_d t) - \frac{I_m}{\omega_d C}cos(\omega_d t) \\
V_d &= I_m Rsin(\omega_d t) + I_m(L\omega_d - \frac{1}{\omega_d C})cos(\omega_d t)
\end{aligned}$$

Finally, we can combine the *sin* and *cos* terms into one sinusoidal function as we did before with only the solenoid present.

$$V_d = I_m \sqrt{R^2 + (L\omega_d - \frac{1}{\omega_d C})^2}\, sin(\omega_d t + \phi) \tag{2.72}$$

where now

$$tan(\phi) = \frac{L\omega_d - \frac{1}{\omega_d C}}{R} \tag{2.73}$$

Let's discuss our results.

1. The quantity $\sqrt{R^2 + (L\omega_d - 1/(\omega_d C))^2}$ plays the role of resistance in our series AC sinusoidal circuit. This generalized resistance quantity is called the ***impedance*** of the circuit and usually given the symbol Z.

$$Z \equiv \sqrt{R^2 + (L\omega_d - \frac{1}{\omega_d C})^2} \tag{2.74}$$

   Remember, however, that Z only has meaning for sinusoidal currents and voltage sources. If $L = 0$ and in the absence of a capacitor, $Z = R$. Note that $V_m = I_m Z$.

2. The term $L\omega_d$ is called the **inductive reactance**, and usually labeled as $X_L \equiv L\omega_d$. The inductive reactance has units of resistance (Ohms) and represents the effective inductive resistance of the solenoid for sinusoidal currents.

3. The term $1/(\omega_d C)$ is called the **capacitive reactance**, and usually labeled as $X_C \equiv 1/(\omega_d C)$. The capacitive reactance has units of resistance (Ohms) and represents the effective capacitive resistance of the capacitor for sinusoidal currents.

4. With these definitions, $Z = \sqrt{R^2 + (X_L - X_C)^2}$, and $tan(\phi) = (X_L - X_C)/R$.

5. For high driving frequencies, $X_L = \omega_d L$ is the largest term. The circuit is mainly inductive. For low driving frquencies, $X_C = 1/(\omega_d C)$ is the largest term, and the circuit is mainly capacitative.

6. If $X_L > X_C$ (an inductive "L" situation) the relative phase ϕ between the voltage and the current is positive: voltage leads current. If $X_C > X_L$ (a capacitive "C" situation) the relative phase ϕ between the voltage the current is negative: current leads voltage.

7. Z becomes its smallest when $X_L = X_C$. In this case $Z = R$. The frequency for which this occurs is called the resonant frequency. $X_L = X_C$, i.e. **the resonance condition**, when the driving angular frequency is $\omega_d = 1/\sqrt{LC}$, or $f_d = 1/(2\pi\sqrt{LC})$. At this resonant frequency, the impedance takes on its smallest value and one gets the most current for the least voltage.

There is a geometric way to express the voltage and the impedance using a "*phasor diagram*" as shown in Figure 2.38. R points along the "+x-axis", X_L points along the "+y-axis", and X_C points along the $-y-axis$. Z and ϕ are found by adding the reactances and resistance like vectors.

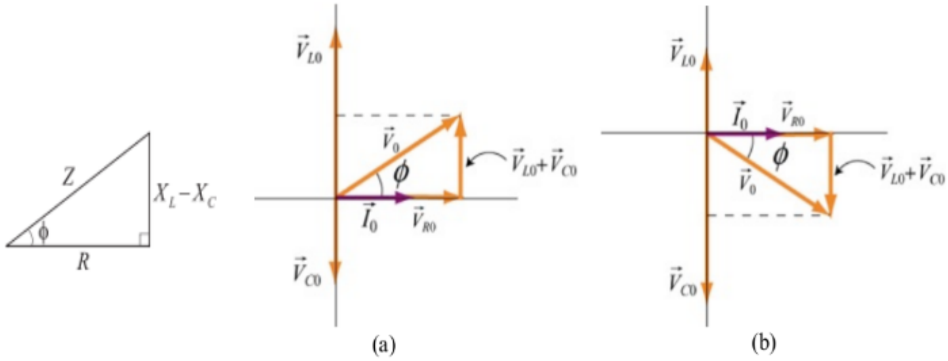


Figure 2.38: *Phasor diagram for the relationships between current and voltage, for the series RLC circuit for (a) $X_L > X_C$ and (b) $X_L < X_C$. For resistive element, I and V_R are in phase. For inductive element, V_L leads I by $\frac{\pi}{2}$, or I lags V_L by $\frac{\pi}{2}$, while for capacitive element, V_C lags I by $\frac{\pi}{2}$, or I leads V_C by $\frac{\pi}{2}$.*

In summary, there are three different types of circuit components. One type are resistive elements, which have $V \propto I$ $(V = RI)$. Another type are inductive elements, which have $V \propto dI/(dt)$ $(V = L(dI/(dt)))$. A third type are capacitive elements, which have $I \propto dV/(dt)$ $(I = (1/C)(dV/(dt)))$. *If the current through the circuit element is* $I = I_m\, sin(\omega_d t)$ then,

$$\begin{aligned} V(t) &= RI_m\, sin(\omega_d t)\ (Resistive) \\ V(t) &= L\omega_d I_m\, cos(\omega_d t)\ (Inductive) \\ V(t) &= -\frac{1}{C\omega_d} I_m\, cos(\omega t)\ (Capacitative) \end{aligned}$$

In series, the voltages (and resistances) add like vectors (phasors) because the sum of two sinusoidals that have the same frequency is itself a sinusoidal with the same frequency.

Note:

(1) Length of a phasor is proportional to the maximum value.

(2) Projection of a phasor onto the vertical axis gives instantaneous value.

(3) Convention: Phasors rotate anti-clockwise in a uniform circular motion with angular velocity.

2.5.3 Power considerations for the series RLC circuit

Energy will only be transfered into the resistive element in the circuit. We derived that the power P transfered into a resistor with resistance R is $P = I^2 R$. A sinusoidal voltage produces a sinusoidal current, so the power varies in time as a sinusoidal squared function. If $I = I_m sin(\omega_m t)$, then the power is:

$$P = I_m^2 R sin^2(\omega_m t) \tag{2.75}$$

and varies in time. We can calculate the average power, P_{ave} by averaging P over one period of oscillation:

$$P_{ave} = I_m^2 R(\frac{1}{T}) \int_0^T sin^2(\frac{2\pi}{T} t) dt \tag{2.76}$$

The average of sin^2 over one cycle equals 1/2. So we have for the average power:

$$P_{ave} = \frac{I_m^2 R}{2} \tag{2.77}$$

It is convenient to express the power in terms of the R.M.S. (Root Mean Square) value of the current (or voltage). The R.M.S. value means the square Root of the average (Mean) value of the Square of the function. For a sinusoidally varying function, the R.M.S. value equals 1/2 the value of the maximum. So,

$$I_{RMS} = \frac{I_m}{\sqrt{2}} \qquad and \qquad V_{RMS} = \frac{V_m}{\sqrt{2}}$$

in terms of the R.M.S. values,

$$\begin{aligned} P_{ave} &= I_{RMS}^2 R \\ &= I_{RMS} \frac{V_{RMS}}{Z} R \\ &= I_{RMS} V_{RMS} cos(\phi) \end{aligned}$$

since $cos(\phi) = R/Z$.

2.5.4 Transformer

A transformer is a device used to increase or decrease the AC voltage in a circuit. A typical device consists of two coils of wire, a **primary** and a **secondary**, wound around an iron core, as illustrated in Figure 2.39. The primary coil, with N_p turns, is connected to alternating voltage source $V_p(t)$. The secondary coil has N_s turns and is connected to a load with resistance R_s. The way transformers operate is based on the principle that an alternating current in the primary coil will induce an alternating emf on the secondary coil due to their mutual inductance.

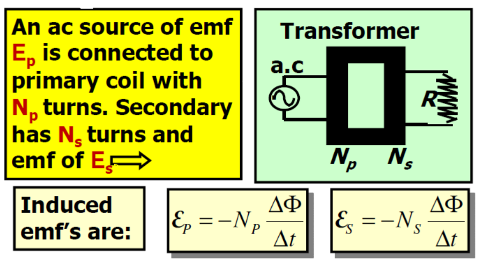


Figure 2.39: *Transformer.*

In the primary circuit, neglecting the small resistance in the coil, Faraday's law of induction implies

$$V_p = -N_p \frac{d\Phi_B}{dt}$$

where Φ_B is the magnetic flux through one turn of the primary coil. The iron core, which extends from the primary to the secondary coils, serves to increase the magnetic field produced by the current in the primary coil and ensures that nearly all the magnetic flux through the primary coil also passes through each turn of the secondary coil. Thus, the voltage (or induced emf) across the secondary coil is

$$V_s = -N_s \frac{d\Phi_B}{dt}$$

In the case of an ideal transformer, power loss due to Joule heating can be ignored, so that the power supplied by the primary coil is completely transferred to the secondary coil,

$$I_p V_p = I_s V_s$$

In addition, no magnetic flux leaks out from the iron core, and the flux $\phi_B B$ through each turn is the same in both the primary and the secondary coils. Combining the two expressions, we are lead to the transformer equation,

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

By combining the two equations above, the transformation of currents in the two coils may be obtained as

$$\frac{V_s}{V_p} = \frac{I_p}{I_s} \quad \Longrightarrow \quad V_p = (\frac{N_p}{N_s})^2 R \ I_p$$

where $(\frac{N_p}{N_s})^2 R$ is an equivalent resistance.
Thus, we see that the ratio of the output voltage to the input voltage is determined by the turn ratio N_s/N_p. If $N_s > N_p$, then $V_s > V_p$, which means that the output voltage in the second coil is greater than the input voltage in the primary coil. A transformer with $N_s > N_p$ is called a **step-up transformer**. On the other hand, if $N_s < N_p$, then $V_s < V_p$, and the output voltage is smaller than the input. A transformer with $N_s < N_p$ is called a **step-down transformer**.

2.6 MAXWELL'S EQUATIONS AND ELECTROMAGNETIC WAVES

2.6.1 Maxwell's Equations

The equations for the electromagnetic fields that we have developed so far, are best expressed in terms of line and surface integrals:

$$\begin{aligned}
\oiint \vec{E} \cdot d\vec{A} &= \frac{Q_{net}}{\epsilon_0} \ (Gauss) \\
\oiint \vec{B} \cdot d\vec{A} &= 0 \ (Gauss) \\
\oint \vec{E} \cdot d\vec{r} &= -\frac{d\Phi_B}{dt} \ (Faraday) \\
\oint \vec{B} \cdot d\vec{r} &= \mu_0 I_{net} \ (Ampere)
\end{aligned}$$

Maxwell realized that the equations above are inconsistent with charge conservation. In particular, there is a problem with the equation referred to as Ampere's Law:

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{net} \tag{2.78}$$

where I_{net} is the net current that passes through the path of the line integral $\oint \vec{B} \cdot d\vec{r}$. We can demonstrate the problem by considering the magnetic field that is produced by a charging parallel plate capacitor. Let the capacitor have circular plates with a radius R, and a plate separation d. Let the wires that connect to the center of the plates extend to $\pm\infty$. Suppose the right plate has charge $+Q(t)$ and the left plate a charge of $-Q(t)$. Suppose also that charge is flowing into the left plate and out of the right plate.
Suppose for a certain time period the current flowing into the plates is a constant, with value I. This current will produce a magnetic field that will circulate the wire. If $d << R$, the magnetic field a distance R from the wire will have a magnitude of $B \approx \mu_0 I/(2\pi R)$. We derived this from the Biot-Savart Law. Since $d << R$, the magnetic field will also exist and have roughly this value at the edge (in the middle) of the capacitor. That is, a distance R from the axis and a distance $d/2$ from one side.
Now, let's apply Ampere's law for a circular path, radius R, in the middle of the capacitor.

$$\begin{aligned} \oint \vec{B} \cdot d\vec{r} &= \mu_0 I_{net} \\ B(2\pi R) &= \mu_0 I_{net} = 0 \\ B &= 0 \end{aligned}$$

The last line equals zero since there is no physical current going through the circular path. The direct surface of the path lies inside the capacitor (i.e. between the plates). There is no physical current flowing between the plates. If we choose a different surface, a curved surface (like a sock) that goes outside the plates and intersects the wire, then a physical current I would go through this "sock-like" surface. The way Ampere's law is stated above, it appears that one gets different results depending on the surface one chooses for "through the path".
Maxwell realized that there is a term missing from Ampere's law. For now, let's call this term M and reconsider Ampere's law with the surface being a flat circular surface through the middle of the capacitor.

$$\begin{aligned} \oint \vec{B} \cdot d\vec{r} &= \mu_0 I_{net} + M \\ B(2\pi R) &= \mu_0 I_{net} + M \\ \frac{\mu_0 I}{2\pi R}(2\pi R) &= 0 + M \\ M &= \mu_0 I \end{aligned}$$

So the missing term must equal $\mu_0 I$, as if the wire went right through the capacitor. But, there is no physical current between the capacitor plates. There is only an Electric field. We can express I in terms of the electric field between the plates. Since $I = dQ/(dt)$, we have

$$M = \mu_0 I = \mu_0 \frac{dQ}{dt} = \mu_0 \frac{d(\epsilon_0 A E)}{dt}$$

since the electric field between the capacitor plates is $E = Q/(A\epsilon_0)$, where A is the area of the plates. With this substitution, we have

$$M = \mu_0 \frac{d(\epsilon_0 AE)}{dt} = \mu_0\epsilon_0 \frac{d(AE)}{dt} = \mu_0\epsilon_0 \frac{d\phi_E}{dt}$$

where ϕ_E is the electric flux through the flat surface in the middle of the plates. Maxwell realized that if we add this term to Ampere's law, then the law is consistent with any surface that is chosen on the right side. The corrected equation is

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{net} + \epsilon_0\mu_0 \frac{d\phi_E}{dt} \tag{2.79}$$

For the charging parallel plate capacitor the problem is resolved. If one chooses the "sock" surface (outside the capacitor), there is a physical current that goes through the surface. The first term on the right is $\mu_0 I$ and the second term is zero since there is no electric field outside the capacitor. If one chooses the flat surface that goes through the capacitor, the first term is zero since there is no physical current through this surface. However, the second term is $\mu_0 I$ since $\epsilon_0(d\phi_E)/(dt)$ through this surface equals I. Maxwell named the quantity $\epsilon_0 d\phi_E/(dt)$ as "**displacement current**".
Some things to note:

1. At the time, there was no experimental evidence for the displacement current term in $\oint \vec{B} \cdot d\vec{r}$. Maxwell used purely theoretical reasoning to justify its existance.

2. It might not be clear from our arguments how charge conservation enters the reasoning. When the field equations are expressed in differential form, it will become clearer. Without the displacement current Ampere's law would be $Curl(\vec{B}) = \mu_0\vec{J}$. Since the Divergence of a Curl is always zero, we would have $Div(\vec{J}) = 0$. Charge conservation requires that $Div(\vec{J}) = \partial\rho/(dt)$. Since $Div(\vec{E}) = \rho/\epsilon_0$, the changing electric flux term restores charge conservation.

3. In our charging capacitor example, only one of the two terms were non-zero at a time. In general, both terms can be non-zero at the same time. That is, a magnetic field can be produced by a "real current" and the second "displacement current" piece at the same time.

4. You might wonder why the displacement current term was not discovered experimentally, as was Faraday's law. The reason it was not initially observed is that it is much smaller than the $\mu_0 I$ piece since ϵ_0 is relatively small.

5. You might be condemned that the choice of surface may be important in Faraday's Law, and that perhaps a term is also missing from the equation $\oint \vec{E} \cdot d\vec{l} = -(d\phi_B)/(dt)$. There is not. Since there are no sources for $\vec{B}$, any valid surface you pick for applying Faraday's law will give the same answer.

We state here the complete set of source equations (in integral form) for the electric and magnetic fields:

$$\begin{aligned}
\oiint \vec{E}\cdot d\vec{A} &= \frac{Q_{net}}{\epsilon_0} \ (Gauss) \\
\oiint \vec{B}\cdot d\vec{A} &= 0 \ (Gauss) \\
\oint \vec{E}\cdot d\vec{r} &= -\frac{d\Phi_B}{dt} \ (Faraday) \\
\oint \vec{B}\cdot d\vec{r} &= \mu_0 I_{net} + \epsilon_0\mu_0\frac{d\Phi_E}{dt} \ (Ampere + Maxwell)
\end{aligned}$$

These equations are referred to as "**Maxwell's Equations**" for the classical electrodynamic fields. The force equation $F = q\vec{E} + q(\vec{v}\times\vec{B})$ relates the force to the fields. Maxwell added the missing piece in 1864. His contribution unified the electric-magnetic interaction with light (or more generally electromagnetic radiation). Radio, television, and wireless communication were made possible from an understanding of these equations once the displacement current piece was added.

2.6.2 Electromagnetic Radiation

Let's consider solutions to Maxwell's equations in empty space, where there is no charge and no real currents. That is, in a region where all closed surface integrals do not contain any charge, and where there is no real current flowing through any closed path. In this case, Maxwell's equations become:

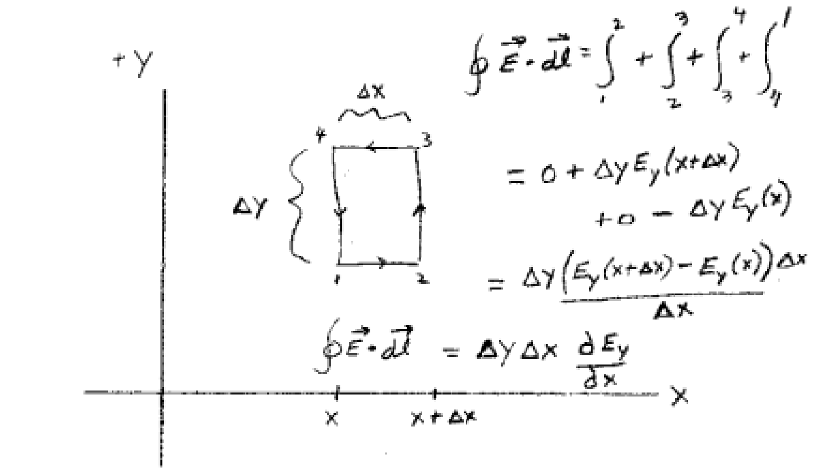
$$\begin{aligned}
\oiint \vec{E}\cdot d\vec{A} &= 0 \\
\oiint \vec{B}\cdot d\vec{A} &= 0 \\
\oint \vec{E}\cdot d\vec{r} &= -\frac{d\Phi_B}{dt} \\
\oint \vec{B}\cdot d\vec{r} &= \epsilon_0\mu_0\frac{d\Phi_E}{dt}
\end{aligned}$$

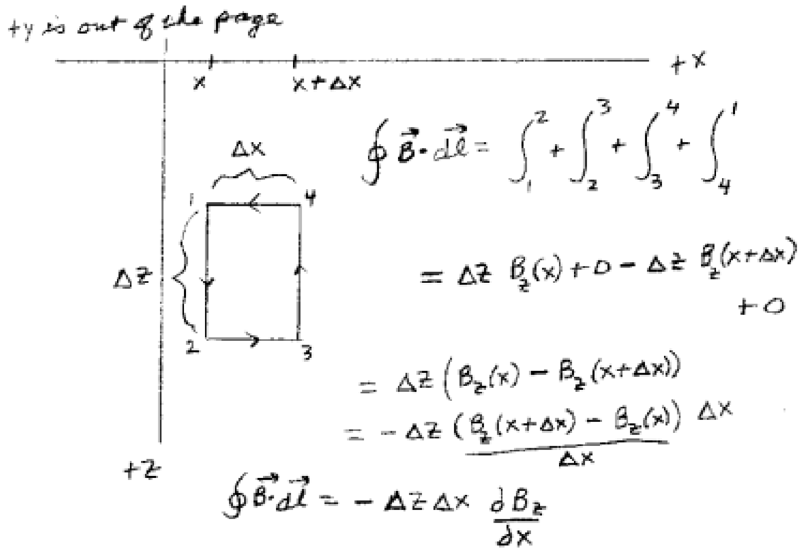
If $\vec{E}$ and $\vec{B}$ do not have any time dependence, then there are zeros on the right side of each equation. This is the "static" case in which all the free charge and currents don't change in time. In this case $\vec{E}$ and $\vec{B}$ can be calculated independent of each other. Let's try and find a simple time-dependent solution for $\vec{E}$ in free space. The simplest case is an electric field that only points in one direction, say the "y" direction $\hat{\jmath}$. Since the surface integral of $\vec{E}$ for a closed surface equals zero, E_y cannot change in the y-direction. So, E_y can only change in a direction perpendicular to the y-axis. So, with little loss of generality, we can choose E_y to vary in the "x" direction. That is:

$$\vec{E} = E_y(x,t)\hat{\jmath} \tag{2.80}$$

We now carry out $\oint \vec{E}\cdot d\vec{r}$ around a small rectangle in the x-y plane. If the sides of the rectangle are Δx and Δy, we have

$$\begin{aligned}
\oint \vec{E} \cdot \vec{dr} &= -\frac{d\Phi_B}{dt} \\
E_y(x+\Delta x)(\Delta y) - E_y(x)(\Delta y) + 0 + 0 &= -\frac{d[(\Delta x)(\Delta y)B_z]}{dt} \\
(\Delta y)[E_y(x+\Delta x) - E_y(x)] &= -(\Delta x)(\Delta y)\frac{dB_z}{dt} \\
(\Delta y)(\Delta x)\frac{dE_y}{dx} &= -(\Delta x)(\Delta y)\frac{dB_z}{dt} \\
\frac{dE_y}{dx} &= -\frac{dB_z}{dt}
\end{aligned}$$





Two of the legs on the left side are zero, since $\vec{dr}$ is in the x-direction, but $\vec{E}$ is in the y-direction. Only the legs in the y-direction contribute to the line integral on the left. On the right, the area for the magnetic flux is in the x-y plane, so only the z-component of $\vec{B}$, B_z contributes. So we see that the z-component of the magnetic field is related to the y-component of the electric field.

The simplest solution will have $\vec{B} = B_z\hat{k}$. Since the surface integral of $\vec{B}$ for a closed surface equals zero, B_z cannot change in the z-direction. Let's consider a solution that has B_z only a function of x and t as with E_y:

$$\vec{B} = B_z(x,t)\hat{k} \tag{2.81}$$

We now carry out $\oint \vec{B}\cdot d\vec{r}$ around a small rectangle in the x-z plane. If the sides of the rectangle are Δx and Δz, we have a similar result as with E_y:

$$\begin{aligned}
\oint \vec{B}\cdot d\vec{r} &= \mu_0\epsilon_0\frac{d\Phi_E}{dt} \\
B_z(x)(\Delta z) - B_z(x+\Delta x)(\Delta z) + 0 + 0 &= \mu_0\epsilon_0\frac{d[(\Delta z)(\Delta x)E_y]}{dt} \\
-(\Delta z)[B_z(x+\Delta x) - B_z(x)] &= \mu_0\epsilon_0(\Delta z)(\Delta x)\frac{dE_y}{dt} \\
-(\Delta z)(\Delta x)\frac{dB_z}{dx} &= \mu_0\epsilon_0(\Delta x)(\Delta z)\frac{dE_y}{dt} \\
\frac{dB_z}{dx} &= -\mu_0\epsilon_0\frac{dE_y}{dt}
\end{aligned}$$

Once again, the z-component of the magnetic field is related to the y-component of the electric field. We can eliminate B_z (or E_y) from these equations by differentiating the first equation with respect to x:

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 B_z}{\partial x\partial t} \tag{2.82}$$

and the second equation with respect to t:

$$\frac{\partial^2 B_z}{\partial x\partial t} = -\mu_0\epsilon_0\frac{\partial^2 E_y}{\partial t^2} \tag{2.83}$$

Combining these two equations gives

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 E_y}{\partial t^2} \tag{2.84}$$

This is a **wave equation**. The solution is wave traveling in the $\pm$ x-direction with a speed of $v = 1/\sqrt{\epsilon_0\mu_0}$, $E_y(x,t) = f(x \pm vt)$ for any function f. A similar "wave equation" results for B_z by differentiating the first equation with respect to t and the second one with respect to x:

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0\epsilon_0\frac{\partial^2 B_z}{\partial t^2} \tag{2.85}$$

These electromagnetic disturbances travel at a speed

$$v = 1/\sqrt{\epsilon_0\mu_0}.$$

When the electromagnetic constants are plugged in, one gets the speed to be

$$v = 1/\sqrt{(4\pi\times 10^{-7})(8.85\times 10^{-12})} \approx 3\times 10^8\ m/s.$$

One has to believe that it is no accident that the two constants ϵ_0 and μ_0 are related to the speed of light as

$$c = 1/\sqrt{\epsilon_0\mu_0}$$

and that light is an electromagnetic disturbance (wave).

Maxwell's discovery that light is part of the electric and magnetic interaction led to a

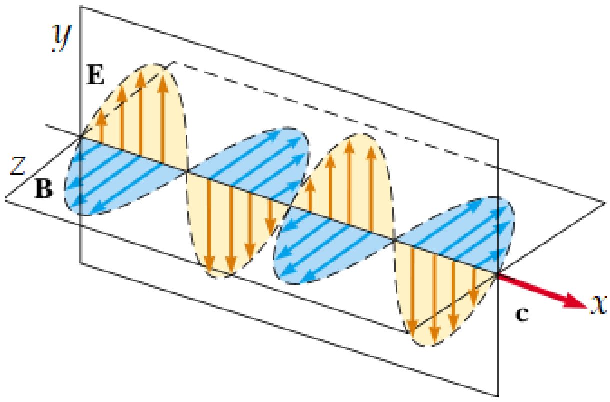


Figure 2.40: *Representation of a sinusoidal, linearly polarized plane electromagnetic wave moving in the positive x direction with velocity c. Note the sinusoidal variations of E and B with x. A time sequence (starting at the upper left) illustrating the electric and magnetic field vectors at a fixed point in the yz plane, as seen by an observer looking in the negative x direction. The variations of E and B with t are sinusoidal.*

dramatic change in human life on earth. With a knowledge of the nature of light, we can understand how electromagnetic radiation can be produced and detected. We can see from the equations above that the traveling wave requires that both $\vec{E}$ and $\vec{B}$ depend on time. That is, radiation will not be produced from stationary charge sources or steady currents. Even a charged object moving at a constant velocity will not radiate, since the object will be at rest in an inertial frame moving (at constant velocity) with the object. So, in order to produce electromagnetic radiation (classically from Maxwell's equations) *charged objects need to be accelerated.* In order to sustain the radiation, charged objects, i.e. electrons, will need to move in a circle or "back and forth".

Thus, most of the radiation produced classically can be characterized by its frequency. Although the radiation might not be exactly sinusoidal in time (AM is modulated in amplitude and FM in frequency), we can refer to the radiation in terms of its frequency. Below we list the names we assign to different parts electromagnetic spectrum.

In the table above, $\lambda = c/f$ and $E = hf$. We can directly measure frequencies if they are below around 10^{10} Hz. We can measure wavelength from around 10 cm to around 1 *nm* using interference effects. Photon energies can be measured if they are greater than a few electron volts.

Table 2.1: Electromagnetic waves. Many of the characteristics of the various types of electromagnetic waves are related to their frequencies and wavelengths.

Name	frequency f (Hz)	wavelength λ	energy E	source
Detection:	electric circuits	interference effects	Photo-absorption	
Radio/TV	$10^3 - 10^6$ Hz			electrons oscillating in wires (AC)
Microwaves	$10^8 - 10^9$ Hz	$10 - 300$ cm		accelerating electrons Molecular Rotations
Infra-Red		3×10^{-5} m		Molecular Vibrations
Visible		$400 - 700\ nm$	$1 - 3$ eV	Atomic Transitions
Ultra-Violet		$10 - 400\ nm$	$3 - 100$ eV	Atomic Transitions
X-rays		$\approx 1\ nm$	$1 - 100$ KeV	large Z inner electron and nuclear transitions
Gamma			> 100 KeV	Nuclear transitions

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